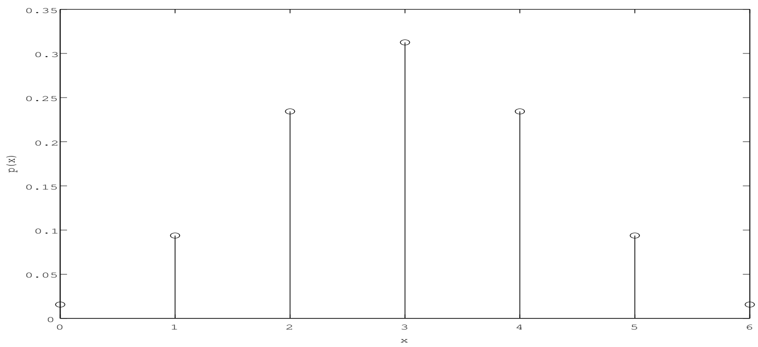
小テスト 1 解答

- **1**:
  a.

$$
\begin{aligned}
p(0) &= \binom{6}{0}(0.5)^0(1-0.5)^6 = \frac{1}{64} \\
p(1) &= \binom{6}{1}(0.5)^1(1-0.5)^5 = \frac{6}{64} \\
p(2) &= \binom{6}{2}(0.5)^2(1-0.5)^4 = \frac{15}{64} \\
p(3) &= \binom{6}{3}(0.5)^3(1-0.5)^3 = \frac{20}{64} \\
p(4) &= \binom{6}{4}(0.5)^4(1-0.5)^2 = \frac{15}{64} \\
p(5) &= \binom{6}{5}(0.5)^5(1-0.5)^1 = \frac{6}{64} \\
p(6) &= \binom{6}{6}(0.5)^6(1-0.5)^0 = \frac{1}{64}
\end{aligned}
$$



b.

$$\mu = \sum_{x=0}^{6} xp(x) = 0\times\frac{1}{64}+1\times\frac{6}{64}+2\times\frac{15}{64}+3\times\frac{20}{64}+4\times\frac{15}{64}+5\times\frac{6}{64}+6\times\frac{1}{64} = 3.$$

$$\begin{aligned}
\sigma &= \sqrt{\sum_{x=0}^{6} x^2p(x) - \mu^2} \\
&= \sqrt{0^2 \times \frac{1}{64} + 1^2 \times \frac{6}{64} + 2^2 \times \frac{15}{64} + 3^2 \times \frac{20}{64} + 4^2 \times \frac{15}{64} + 5^2 \times \frac{6}{64} + 6^2 \times \frac{1}{64} - 3^2} \\
&= \sqrt{\frac{96}{64}} \approx 1.22.
\end{aligned}$$

(We could use the formulas. $\mu = 6\times0.5 = 3$ and $\sigma = \sqrt{6 \times 0.5 \times (1 - 0.5)} \approx 1.22$.)

c.
i. $p(3) = \frac{20}{64} = 0.3125$.
ii. $p(2) + p(3) + p(4) = \frac{15}{64} + \frac{20}{64} + \frac{15}{64} = \frac{50}{64} = 0.7813$.
iii. $p(3) + p(4) + p(5) + p(6) = \frac{20}{64} + \frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{42}{64} = 0.6563$.

- **2**: Let Z be a random variable which follows the standard normal distribution ($N(0, 1)$).

  a: $Pr(X > 13) = Pr(\frac{X-9}{2} > \frac{13-9}{2}) = Pr(Z > 2) = 0.023$.

  b: $Pr(X < 8) = Pr(\frac{X-9}{2} < \frac{8-9}{2}) = Pr(Z < -0.5) = 0.309$.

  c: $Pr(10 < X < 13) = Pr(0.5 < Z < 2) = 0.286$.

  d: $Pr(5.08 < X < 12.92) = Pr(-1.96 < Z < 1.96) = 0.95$.

- **3**:

$$\begin{aligned}
\mu_X &= 0 \times 0.2 + 1 \times 0.6 + 2 \times 0.2 = 1 \\
\mu_Y &= 0 \times 0.6 + 1 \times 0.4 = 0.4 \\
\sigma_X^2 &= 1^2 \times 0.6 + 2^2 \times 0.2 - 1^2 = 0.4 \\
\sigma_Y^2 &= 1^2 \times 0.4 - 0.4^2 = 0.24 \\
\sigma_{XY} &= E(XY) - \mu_X\mu_Y = (1 \times 0.2 + 2 \times 0.2) - 1 \times 0.4 = 0.2 \\
\rho_{XY} &= \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{0.2}{\sqrt{0.4}\sqrt{0.24}} \approx 0.645.
\end{aligned}$$