

小テスト2 解答

• 1:

a. $Pr(F) = 7.8\% + 18.2\% = 26\%$.

b. $Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{7.8\%}{7.8\% + 22.2\%} = 26\%$.

c. Yes because $Pr(F|E) = Pr(F)$.

• 2:

a.

$$E[\bar{X}] = \$30,000, \quad SE = \sigma/\sqrt{n} = \$9000/\sqrt{25} = \$1800,$$

The distribution shape is normal.

b. Let Z be a random variable which follows $N(0, 1)$. By the CLT,

$$Pr(\bar{X} - 30000 > 3000) = Pr\left(\frac{\bar{X} - 30000}{1800} > \frac{3000}{1800}\right) \approx Pr\left(Z > \frac{5}{3}\right) = 0.047.$$

• 3: Let X = the number of Republican voters.

a. The probability corresponds to $Pr(X = 1) = 0.6$.

b. The probability corresponds to $Pr(X = 2) + Pr(X = 3) = \binom{3}{2}(0.6)^2(0.4)^1 + \binom{3}{3}(0.6)^3(0.4)^0 = 0.432 + 0.216 = 0.648$.

c. The probability corresponds to $\sum_{x=5}^9 Pr(X = x) = \sum_{x=5}^9 \binom{9}{x}(0.6)^x(0.4)^{9-x} \approx 0.73$.

• 4

a.

$$\mu_X = 0 \times 0.2 + 1 \times 0.6 + 2 \times 0.2 = 1$$

$$\mu_Y = 0 \times 0.6 + 1 \times 0.4 = 0.4$$

$$\sigma_X^2 = 1^2 \times 0.6 + 2^2 \times 0.2 - 1^2 = 0.4$$

$$\sigma_Y^2 = 1^2 \times 0.4 - 0.4^2 = 0.24$$

$$\sigma_{XY} = E(XY) - \mu_X\mu_Y = (1 \times 0.2 + 2 \times 0.2) - 1 \times 0.4 = 0.2$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{0.2}{\sqrt{0.4}\sqrt{0.24}} \approx 0.645.$$

b.

$$\mu_X = 0 \times 0.1 + 1 \times 0.5 + 2 \times 0.4 = 1.3$$

$$\mu_Y = 1 \times 0.6 + 2 \times 0.4 = 1.4$$

$$\sigma_X^2 = 1^2 \times 0.5 + 2^2 \times 0.4 - 1.3^2 = 0.41$$

$$\sigma_Y^2 = 1^2 \times 0.6 + 2^2 \times 0.4 - 1.4^2 = 0.24$$

$$\sigma_{XY} = (1 \times 0.3 + 2 \times 0.2 + 2 \times 0.24 + 4 \times 0.16) - 1.3 \times 1.4 = 0$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = 0.$$

c.

$$\mu_X = 0 \times 1/8 + 1 \times 3/8 + 2 \times 3/8 + 3 \times 1/8 = 3/2$$

$$\mu_Y = 1 \times 2/8 + 2 \times 4/8 + 3 \times 2/8 = 2$$

$$\sigma_X^2 = 1^2 \times 3/8 + 2^2 \times 3/8 + 3^2 \times 1/8 - (3/2)^2 = 3/4$$

$$\sigma_Y^2 = 1^2 \times 3/8 + 2^2 \times 4/8 + 3^2 \times 2/8 - 2^2 = 5/8$$

$$\sigma_{XY} = (2 \times 2/8 + 3 \times 1/8 + 4 \times 2/8 + 6 \times 1/8 + 3 \times 1/8) - 3/2 \times 2 = 0$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = 0.$$

d. For part **b**, $\rho = 0$, and X and Y are independent. For part **c**, $\rho = 0$, yet X and Y are not independent.

e. **1.** True **2.** False