小テスト 4 解答

- **1**:
  (a) Let

$$\begin{aligned} X_1 &= \text{the strength of the half from old process,} \\ X_2 &= \text{the strength of the half from new process.} \end{aligned}$$

Let $D = X_2 - X_1$. Then, the 95% confidence interval is

$$\begin{aligned} & [\bar{D} - t_{.025}\frac{s_D}{\sqrt{n}},\ \bar{D} - t_{.025}\frac{s_D}{\sqrt{n}}] \\ &= [20 - 2.57 \times \sqrt{\frac{320}{6}},\ 20 + 2.57 \times \sqrt{\frac{320}{6}}] \\ &\approx [20 - 18.8,\ 20 + 18.8], \end{aligned}$$

where there are 5 d.f. in calculating SE. (Here, $s_D = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2}$.)

Since the $H_0 : \Delta \equiv \mu_2 - \mu_1 = 0$ lies outside the 95% confidence interval $[20 - 18.8,\ 20 + 18.8]$, the difference is statistically **discernible** (or **significant**) with 5% level.

(b) Since the null hypothesis is $H_0 : \Delta = 0$, $\bar{D} = 20$ and the estimated SE$= 7.3$, the t-statistic is

$$t = \frac{\text{estimate} - \text{null hypothesis}}{\text{estimated SE}} = \frac{20 - 0}{7.3} = 2.74.$$

Since d.f.$= 5$, we can look up the tail area above 2.74 in the t-table:

$$.010 < \text{(one-sided) p-value} < .025.$$

- **2**:
  (a) The 95% confidence interval for the change is

$$\begin{aligned} & [(P_2 - P_1) - z_{.025}\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}, \\ & \quad (P_2 - P_1) + z_{.025}\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}] \\ &\approx [-0.06 - 1.96 \times 0.018,\ -0.06 + 1.96 \times 0.018] \\ &\approx [-0.06 - 0.036,\ -0.06 + 0.036] \\ &= [-6\% - 3.6\%,\ -6\% + 3.6\%]. \end{aligned}$$

Since the $H_0 : \Delta \equiv \pi_2 - \pi_1 = 0$ lies outside the 95% confidence interval $[-6\% - 3.6\%,\ -6\% + 3.6\%]$, the difference is statistically **discernible** (or **significant**) with 5% level.

(b) Since the null hypothesis is $H_0 : \Delta \equiv \pi_2 - \pi_1 = 0$, $P_2 - P_1 = -0.06$ and the estimated SE$= 0.018$, the t-statistic is

$$t = \frac{\text{estimate} - \text{null hypothesis}}{\text{estimated SE}} \approx \frac{-0.06 - 0}{0.018} = -3.33.$$

Since $n = 200$ is large, we can look up the tail area above 3.33 in the normal table:

$$\text{(one-sided) p-value} \approx 10^{-3}.$$

That is, the sampling data shows little credibility for the null hypothesis that there is no change in the percentage who favor the acceptance of marijuana from 1980 to 1985.

- **3**: Let the fitted line be

$$\hat{Y}_i = b_2 X_i.$$

Then

$$e_i = Y_i - b_2 X_i$$

and

$$RSS = \sum {e_i}^2 = \sum (Y_i - b_2 X_i)^2.$$

The first-order condition for a minimum is

$$\frac{dRSS}{db_2} = -2 \sum X_i (Y_i - b_2 X_i) = 0.$$

and so

$$b_2 = \frac{\sum X_i Y_i}{\sum {X_i}^2}.$$

The second derivative is

$$2 \sum X_i^2.$$

This is positive, confirming that we have found a minimum.

4: $d.f. = n - 2 = 58$. From the t-table, we find that

$$t_{crit,5\%} = 2.00,\ t_{crit,1\%} = 2.66.$$

(a) $t = \frac{b_2}{s.e.(b_2)} = \frac{-0.20}{0.07} = -2.86$. So we reject H_0 at the 1% level. There is no need to mention the 5% test because rejection at the 1% level automatically means rejection at the 5% level.

(b) $t = \frac{b_2}{s.e.(b_2)} = \frac{-0.12}{0.07} = -1.71$. So we do not reject H_0 at the 5% level. There is no need to mention the 1% test because not rejecting at the 5% level automatically means not rejecting at the 1% level.

- **5**: (i)The coefficient of ASVABC implies that S increases by 0.126 years for every one-point increase in ASVABC. By the calculations $t = \frac{0.126}{0.010} = 12.6$, the t-tests indicate that the coefficients of ASVABC is significant at 1% level. ($t_{crit,1\%} = 2.59$ with $d.f. = n - 5 = 535$.)
  (ii) The 95% confidence interval for the coefficient of SM is

$$[0.049 - 1.96 \times 0.039,\ 0.049 + 1.96 \times 0.039] = [-0.027,\ 0.125].$$

  ($t_{crit,5\%} = 1.96$ with $d.f. = n - 5 = 535$.)