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Risk Sharing Contract**

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Analysis on Airport-Airline Relationship with Risk Sharing Contract

Katsuya Hihara *

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Abstract

As an example of airport-airline vertical contractual relationships, 'Load Factor Guarantee Mechanism' contract, actually agreed at Noto Airport in Japan, stipulates that the airline agrees to serve the airport and the airport agrees to pay to (or receive from) the airline based on the difference between the target load factor set at the start of the period in the contract and a realized one. Airports are often local governments and such payments are controversial since tax money is on the line.

From airport side, such contract ensures the commitment of airline to serve the airport as well as the sharing of up-side profit in high load factor case. From airline side, such contract serves not only as a risk mitigating tool to compensate down-side loss of revenue but as an incentive device to extract airport efforts to avoid large payment hence overcoming its under-efforts seen typically after start of the service at the airport.

To reflect these structures, we revised Hart and Moore hold up model to include more interdependent and joint-venture type relationship between airport and airline. With the revised stochastic model, we show such contract can overcome under-effort problem and restore utilities loss under modified Hart and Moore first best condition. So contractual payment from public airport authority could be justified by such efficiency gains.

Finally we illustrate, by numerical examples, the under-efforts, its recoveries and utilities loss restorations by contract with impacts of expected state of nature improvements, uncertainty increases and "risk aversion" enhancements¹.

key words

Airport-Airline Vertical Relationship, Load Factor Guarantee Mechanism, Hold-Up Problem, Joint-Venture Type Trade, Efficient Pure Risk Sharing

JEL Classification Numbers:

C70,D81,D86,L93

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1 Introduction

Airports and airlines are facing revenue and profit fluctuations under increasingly volatile and unstable business environments. Some of them, especially in local area with low demands or in secondary airports for start-up carriers such as LCC entering into the area, are forming vertical contractual relationship to share their risk and stabilize their financial conditions so that air transport services by those airlines at the airports could be newly introduced or be kept to carry on.

An airport and an airline at one airport are, by nature of the business, jointly making a business project at the airport. Airport are providing airline with airport related service in exchange for landing fee and at the same time airline are providing air transport service to the airport, with or without incentive money from the airport. The two services are not separable in a sense that each service could not exist without the other².

Also they are in a strategic complementary relationship, where one side's effort could improve not only its own but also the other side's contribution to the value of the joint project. For example, airport's effort on improving airport services increase the attractiveness of the airport and help airline's independent effort to bring more passengers, hence both ending up in enjoying more revenues. Therefore further efforts arising from such contractual relationship in addition to such interdependent relationship could have the potential to significantly enhance the values of the project both sides are participating in.

We try to model this type of interdependent vertical contractual relationship between airport and airline from the stand-point of incomplete contract frameworks in contract theory. Specifically the under-effort problem, or hold-up problem, of the participants, entailing utilities loss and its restoration by contract are our main interest. We also try to analyze the risk sharing nature of the contract from the efficient pure risk sharing standpoint.

Our plan of the rest of this paper is as follows. In next section, we briefly discuss the past relevant literature. In section 3, we explain the main contents of the " Load Factor Guarantee Mechanism " contract, which is actually agreed and still binding at Noto airport in Japan. In section 4, we set up a revised Hart and Moore model and explain its characteristics and specific features. We show that under the interdependent, joint-venture type relationship there are under-effort problem as compared with the first best situation.

In section 5, we show that the contract of " Load Factor Guarantee Mechanism " can overcome the under-efforts problem and restore entailing utilities loss under revised first best conditions of Hart and Moore framework.

²Passengers in this case is not directly involved in the trade, since we are focusing on the bilateral relationship between airport and airline from contract theory point of view.

In section 6, we use concrete numerical examples to illustrate the under-effort problems, entailing utilities loss and its restoration of the lost utilities by showing the impact of several factor perturbations, namely improved prospect of state of nature, uncertainty increase, “ risk aversion ” enhancement of parties on such under-effort levels and utilities loss restoration.

2 Past Literature

Because of the fundamental importance of airport-airline relationship, there are a number of literatures in air transport economics that studied the relationship from various standpoints. For example, the recent study from the stand point of consumer welfare analyses is Barbot (2009) and Oum and Fu (2009). Oum and Fu (2009) also studies from the view-points of market competition among many airport-airline vertical relationships and its effects on networking and pricing. Zhang et al. (2010) is the latest and unique research about contractual revenue sharing between airport and airline, and its impacts on pricing and routes.

On the other hand, a lot of literatures are available for hold-up problem, incomplete contact, and other surrounding contract theory frameworks ranging from Hart and Moore (1988), Noldeke and Schmidt (1995), Chung (1991), Aghion et al. (1994), Edlin and Reichelstein (1996), Tirole (1999), Itoh (2003), Bolton and Dewatripont (2005), Itoh and Morita (2009) to Kanemoto (1990), Kanemoto and MacLeod (1992), Gul (2001), Segal (2002), Pitchford and Snyder (2004) and Iyer and Schoar (2008).

But so far it is very difficult for us to find any study on airport-airline vertical relationship from the stand point of hold-up problem under the incomplete contract theory frameworks. We presume one of the reasons for this situation is the airport-airline relationship is more interdependent and complex than simple buyer-seller relationships.

But the basic structure of under-efforts problem and the need to mitigate it with some proper contract could well apply to airport and airline relationship in our view. To reflect these interdependent vertical relationships and capture the basic structure in the context of incomplete contract framework is what we try to pursue in this paper.

3 Noto Airport Load Factor Guarantee Mechanism

Noto airport is a small local airport (only one 2,000m runway) opened in 2003 at Noto peninsula in Ishikawa-Prefecture, central-northern part of Japan. The airport owned by Ishikawa prefecture promises to pay (or receive) contingent payment based on the difference between a realization of load factor θ and the target load factor set at the start of the period by contract, in exchange for airline's commitment of providing air transport service at the airport. The contract is called "Noto Airport Load Factor Guarantee Mechanism."

With this contract, there are 2 scheduled air transport services a day between Noto AP and Tokyo-Haneda AP (530 km away) by ANA group (mostly by A320 with 166 seats). Ticket prices are from about 20,000 yen (160\$) to less than 10,000 yen (80\$).

The basic structure of contractual payments (payment from airport to airline is positive amount) in the first four years from 2003 to 2007 are in Figure 1³. The calculation is done after the end of each one year period and is basically; Payment amount = parameter \times (target load factor - realized load factor).

The parameter is different over several divided ranges of entire load factor possibilities. So the relationship between payment $p(\theta)$ and load factor (θ) in the contract is piece-wise linear functions. Price ceiling is kept at 2×10^8 yen and price floor is set at -2×10^8 yen from the 2nd year on. More detail of the mechanism and its numerical analysis on its payoff structure is in Hihara (2008) and Hihara (2010).

(Figure 1 Noto contract's 4 year contract payoff structure)

From airport perspective, such contract ensures the commitment of airline to serve the airport, while it is also a tool to share up-side profit in high load factor case. From airline side, such contract serves not only as a risk mitigating tool to compensate down-side loss of revenue but also as an incentive device to extract more airport efforts of trying to avoid large payment hence overcoming its under-efforts problem seen typically after start of the service at the airport⁴.

Notice also that the contract has multiple functions. In addition to ensuring commitment and incentive device, it has a mechanism of risk sharing between

³From the fifth year on, the contract is same as that of the fourth year until September 2010.

⁴If the effort is hidden from the other, namely under the asymmetric information framework, this is called moral hazard or hidden action problem in agency frameworks. Here we follow the usual hold up problem framework and assume the efforts are seen from each other, i.e., under symmetric information structure. But the efforts are not verifiable to the third parties.

airport and airline in a mitigating way of the latter's revenue fluctuation. When a realized load factor is low at the route of the airport, the former pays the compensating money specified in the contract depending on the realization of contingent variable. When, on the other hand, a realized load factor is high, the latter pays the "reward" money to the former.

The Noto contract could be seen as one of many cases seen across the world, such as in the case of airport's trying to keep the low demand route at the local airport or in the case of a secondary airport's paying incentive payment for start-up LCCs. With the incentive money from airport, airline then become willing to serve the airport. The payoff structure of Noto case, however, not only covers the downside of low load factor case, where airport pays to airline, but also the upside of high load factor case, where airline pays to airport.

Table 1 is the summary information about the Noto case for the first four years. In the table, we have 1) target load factors that Noto airport and the airline agree on in the contract, 2) model load factors (estimated load factors) that is predicted based on the econometric model about the following year's load factor in Hihara (2008)⁵, 3) actual load factors are those that actually realized, 4) actual number of passengers, and 5) actual contractual payments.

(Table 1 Noto Case Summary Information about here)

In the first three years, airport did a lot of promotion efforts and received money from airline. This betrayed most people's expectations that airport will end up with paying a lot of money to airline. From the fourth year on, there is no money payment because the actual load factors are in the no payment range around target load factors set in the contract⁶.

From Table 1, we can calculate the difference between the estimated load factors and the realized actual load factors and corresponding passenger number difference, for example. This could be one measure of the effect the Noto contract has on the project value of the Noto airport. The corresponding passenger number increases are estimated to bring about as much as 4.5% more revenue. This revenue increase could be attributed at least partly to the contract extracting more efforts from both sides.

In the next section, we try to model these interdependent relationship of the two parties in the context of incomplete contract and hold up problem framework.

⁵In Hihara (2008), the domestic passenger air transport market load factors in Japan from 1950's to 1990's are analyzed as time series data and the structure of (ARIMA(1,1,4)) is estimated. Based on the model, the following years' load factors are predicted by normal distribution. The estimated load factor is the mean of the predicted normal distribution.

⁶This range is the flat line part in the middle of $p(\theta)$ piece-wise linear function of the Figure 1. Such flat area appeared in the third year and widened in the fourth year.

4 The Model

Under the incomplete contract model settings, airport (=AP) and airline (=AL) can be thought to be in a bilateral dual "trading" relationships with relation-specific efforts (or investments). That is AP is buying AL's air transport service at AP for incentive/compensating money, and on the other hand, AL is buying AP's airport service for landing fees. At the same time both AP and AL make relation-specific efforts to increase the number of passengers on air service at the airports, hence to enhance the value of the air service project there.

4.1 Settings

Both airport and airline are risk neutral.

In our analysis, we basically use the canonical settings in Hart and Moore (1988), where airline has only two choices, either to provide service (one) or not to provide it (zero). Service quantity problem in-between zero and one is not relevant here.

The time line along which we assume for both parties to transact with each other is Figure 2 and Figure 3. We also assume time discounting is not relevant as in the usual settings. Efforts are assumed to be specific to the relationship and its cost is to be sunk before the realization of the state of nature. Renegotiations are costless and can attain ex post efficient effort levels⁷.

(Figure 2 and Figure 3 time line around here)

In Figure 2, there is no contract at $t = 0$. When air transport service is provided at AP, AP and AL each make independent efforts from $t = 1$ to $t = 2$ before the realization of θ . After θ is realized at $t = 2$, AP and AL renegotiate on share of project value and the price $p(\theta)$ as long as it is efficient. We assume the renegotiation result is decided according to Nash bargaining solution. The payment is paid at $t = 3$ based on the results. They anticipate these mechanisms after $t = 2$ and make effort based on expectations between $t = 1$ and $t = 2$. If no air transport service is provided, no value is realized. Also, if the realized θ does not justify the air service provision, AP and AL have to tear down the relationship and clean up the transactions up to the point.

⁷We assume the costless renegotiation after the contingency realization and the ex post efficiency so that we focus on the ex ante inefficiencies. This is because we think ex ante inefficiencies are very relevant to our problem and also this type of focus on ex ante inefficiencies is the usual settings of incomplete contract frameworks.

Figure 3 is about the situation with contract. AP and AL agree on a contract at $t = 0$. By contract AL makes commitment to serve at AP. Before the realization of θ , AP and AL make independent efforts. Between $t = 2$ and $t = 3$ they renegotiate on p based on the realization of θ . These are according to Hart and Moore (1988) settings.

However, the AP-AL relationship has its own typical characteristics so we change the Hart-Moore model accordingly. The biggest change is the joint-venture type value structure, which we will describe extensively in the next subsection.

Basic idea of joint-venture type value structure is AP and AL are both buyer and seller at the same time. If AL serves AP, AL "buys" airport service from AP for landing fee but at the same time, AP pays incentive money to AL to "buy" air transport service at the airport.

If AL thinks the value from service at the airport is lucrative enough without any incentive payment, then AL is selling air transport service for zero price. But if the service is not so lucrative, then AP may think the incentive payment to buy AL's service is necessary to secure AL's commitment to service AP and to keep the accompanying business revenues, such as parking fee from passengers and concession revenues of land side operations. In that case, AP is "buying" as a buyer the air transport service for positive price from AL in addition to selling airport service for landing fee as a seller.

With this modification, we can model the situation in a local airport to pay incentive money to LCCs beyond the discount of landing fee, just like the Noto case ⁸. Also with this model, we can set such structures in which the valuation of AL's side to serve at the airport is not so lucrative although serving there meets the minimum participation utility level (as we explain later, this could be negative) and the incentive money from AP to AL plays an important role in AL's decision to actually serve AP.

As significant as joint venture type structure is payment aggregation. AP and AL "trade" with or without such a contingent payment contract in a real world. Here our model covers not only the situations without such contract but also the situations with contract. In a sense, payment in our model is aggregated version of all the payment transactions of the real world AP-AL relationship. So

⁸Noto airport does have landing charge. But the frequency is only two services a day. Also landing fee is not dependent on θ but only on MTOW of aircraft in use, which means the fee is basically constant. In addition, landing fee is discounted to one third and its amount is at most about 10% of maximum contingent payment in the contract. So for the sake of simplicity in this study we assume the zero landing fee to stay focus only on the contingent variable payment $p(\theta)$ without loss of generality. The analysis is unaffected and valid even with the consideration of landing fee.

the payment in our model between AP and AL are only through one payment (aggregate of all payments, from landing fee to incentive payment, if any, in both directions between AP and AL) ⁹.

The direction of net payment from AP to AL is set to be positive and that of the other direction is negative. With or without such a contract, the payment is calculated at the end of one business period based on the valuation including the result from contingent realization of the period.

We further assume that if the valuations of both sides, after the realization of state of nature, result in the situation that no trade is efficient, then AP and AL must dissolve the contractual relationship and have to pay for the cost of tearing down the relationship. For the sake of simplicity, this is assumed to wipe out all the values and final valuations for both AP and AL in this case are assumed to be zero. This assumption is necessary for our model to be consistent with typical incomplete contract settings.

4.2 Joint-Venture Type Contract Model

Here we formally introduce joint-venture type relationship. By joint-venture type, we mean that AP as well as AL is providing (or selling) service (for example, AP = providing runway and passenger terminal, AL = providing air transport service), while AP as well as AL are making relation-specific efforts to increase the value of joint venture and to decrease the service providing cost (for example, AP = renovating terminal for more efficiency, AL = introducing smaller and fuel-efficient new type aircraft). AL as well as AP is independently generating revenues and incurring costs out of the joint-venture.

$$U_{AP}(b, \theta) = v_{AP}(b, \theta) - c_{AP}(b, \theta) - p(\theta) - \phi(b) \quad (1)$$

$$U_{AL}(s, \theta) = p(\theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta) - \psi(s) \quad (2)$$

b : effort of airport (=AP 's effort)

$$b \in [0, \bar{b}] \in R$$

s : effort of airline (=AL 's effort)

$$s \in [0, \bar{s}] \in R$$

θ : random variable corresponding to the state of nature (in Noto case, Load

⁹In the case of LCC and local airport owned by local government, for example, the landing fee is often exempted in the first place and that case it is irrelevant. For bigger hub airport, in our model payment is more like a aggregate payment over all payment between AP and AL, including landing fee, check-in counter lease payment, incentive payment from AP, discount on hanger payment and so on.

Factor of the Noto Haneda route)

$$\theta \in [0, 1] \equiv \Theta \in R$$

(b,s): observable by AP and AL but not verifiable by a third party

(e.g., national government including Ministry of Transport or court)

p : price of the reward for the air transport service from airport to airline (minus value means airport is receiving money from airline)

$$p \in R$$

q : the Noto=Haneda airport passenger air service quantity level

$$q \in \{0, 1\}$$

$v_{AP}(b, \theta)$: AP's valuation function of the air transport service at the airport

$c_{AP}(b, \theta)$: AP's valuation function of the airport service (relationship specific contribution function to project by its effort to decrease the trade cost of the airport service to be provided)

$v_{AL}(s, \theta)$: AL's valuation of the airport service at the airport

$c_{AL}(s, \theta)$: AL's valuation function of the air transport service at the airport (relationship specific contribution function to project by its effort to decrease the trade cost of the air transport service to be provided)

$v_{AP} > 0, v'_{AP} > 0, v''_{AP} < 0$ are assumed for each element of (b, θ) .

$c_{AP} > 0, -c'_{AP} > 0, -c''_{AP} < 0$ are also assumed for each element of (b, θ) .

$v_{AL} > 0, v'_{AL} > 0, v''_{AL} < 0$ are assumed for each element of (s, θ) .

$c_{AL} > 0, -c'_{AL} > 0, -c''_{AL} < 0$ are also assumed for each element of (s, θ) .

$v_{AP} - c_{AL} > 0$ and $v_{AL} - c_{AP} > 0$ are the two conditions for the trades within the joint-venture relationship. This implies

$$v_{AP} - c_{AP} + v_{AL} - c_{AL} > 0.$$

$v_{AP} - c_{AP} + v_{AL} - c_{AL}$ is assumed to be strictly concave function bounded above.

$$v_{AP} - c_{AP} > 0 \text{ or}$$

$$v_{AL} - c_{AL} > 0 \text{ are also assumed.}^{10}$$

$\phi(b)$: AP's direct effort costs

$\phi(s)$: AL's direct effort costs

$$\phi > 0, \phi' > 0, \phi'' > 0 \text{ and}$$

$$\psi > 0, \psi' > 0, \psi'' > 0 \text{ are assumed.}$$

In usual settings, buyer 's utility consists of only the valuation of the project and valuation of cost reduction part is not present. Likewise seller 's utility consists only of valuation of cost reduction part and the valuation of the project

¹⁰Notice that $v_{AP} \geq c_{AL} \geq v_{AL} \geq c_{AP}$, for example, does not satisfy the condition of $v_{AL} - c_{AL} > 0$ but do satisfy the two conditions for the trades in the joint-venture, namely $v_{AP} - c_{AL} > 0$ and $v_{AL} - c_{AP} > 0$.

is not present. In our joint-venture settings, both AP and AL are both buyers and sellers and their utilities have both the valuation of the project part and valuation of cost reduction part¹¹.

In the joint-venture type contract model, the service from AP to AL (e.g., airport service) and the service from AL to AP (e.g., air transport service) are not economically separable. Each service cannot exist without the existence of the other. So even though both AP and AL have both "buyer = service receiver" and "seller = service provider" sides but these services and sides cannot be separated.

With these modifications, we have two more degrees of freedom, namely v_{AL} and c_{AP} in the footnote case, to model the situation realistically. For example, if AL 's opportunity cost, or its valuation, of the air transport service, c_{AL} , is high as compared with the airport 's value to AL, v_{AL} (since AL can use its resources in other very high yield route to other airports), then AL 's total net valuation in the joint project of the AP (Noto airport) could be most likely negative $v_{AL} - c_{AL} < 0$ in low load factor case as depicted in Figure 4. This loss can be covered by the contract payment. This is shown in Figure 4 that $p(\theta) \geq c_{AL} - v_{AL}$ in low θ area.

(Figure 4 about here)

This could be a real situation where the AL, with better potential opportunity to other routes to different APs, is reluctant to serve a local AP but the incentive money from the AP plays decisive role in actually serving there.

On the other hand, if AP 's valuation of the air transport service, v_{AP} (such as those from land side revenues or tourism revenues) is quite high as compared with its airport service valuation (opportunity cost), c_{AP} (since there is no other good candidate for serving the AP), then the trade conditions, $v_{AP} - c_{AL} > 0$ and $v_{AL} - c_{AP} > 0$, would more likely to hold. And the incentive money $p(\theta)$ could be very high in low load factor case because of high v_{AP} .

Joint-venture type modification is not just to tolerate negative $p(\theta)$ in price

¹¹If we literally follow Hart and Moore (1988), the model would be as follows assuming AP is buyer and AL is seller.

$$\begin{aligned} U_{AP}(b, \theta) &= v_{AP}(b, \theta) - p(\theta) - \phi(b) \\ U_{AL}(s, \theta) &= p(\theta) - c_{AL}(s, \theta) - \psi(s) \end{aligned}$$

In other joint venture type contract settings, both buyer and seller could share one combined value and one combined cost function like in Kim and Wang (1998). In our case, it is natural AP or AL each has its own valuation function and cost reduction function, since AP and AL have its own independent revenue sources (AP: landing fee and AL: passenger revenue, for example). They are generated and accounted independently though they usually cannot exist without the existence of the other in the joint venture project of the air transport service at the airport (routes).

agglomeration of both directions between AP and AL ¹². But our model can more realistically and directly describe the reasons and hidden dynamics for such $p(\theta)$ in AP-AL relationships.

It tolerates, for instance, high positive $p(\theta)$ because AP 's valuation of air transport service (v_{AP}) is quite high because tourism revenue might be quite valuable for AP even in low load factor case. It also tolerates quite negative $p(\theta)$ in high load factor case because in such a case, AL 's valuation of serving the AP (v_{AL}) is quite high and hence more room to share the profit with AP through contract payment. In this case, $v_{AL} - c_{AL} \gg 0 \Leftrightarrow c_{AL} - v_{AL} \ll 0$. And this is shown in Figure 4 that $p(\theta) \geq c_{AL} - v_{AL}$ in high θ area.

Contract of Noto airport load factor guarantee mechanism can be thought as specific performance contract with payment directly contingent on realization of state variable in exchange for the contractual commitment of providing air transport service at the Noto airport.

Noto AP Load Factor Guarantee Mechanism Contract

$$\mathcal{C}\{(p, q) = (p(\theta), 1)\} \tag{3}$$

In Hart and Moore (1988) or Noldeke and Schmidt (1995), they are thinking of a contract, which consists of binary choice of quantity of service (either zero or one) and its corresponding price.

In the case of Noto, by the specific performance contract, AL are committed to provide air transport service ($q = 1$) at Noto airport in exchange for payment $p = p(\theta)$. Notice that AL has no choice of not providing service. Rather they receive $p(\theta)$ from AP or renegotiate on price, depending on the realization of θ and concrete contents of the utility functions of the parties ¹³.

Also we can think $p(\theta) = p_1 - p_0$. In this case, p_0 is set to zero. To make this setting, we adjust, without loss of generality, all the other relevant values in our settings. Notice that either one of the parties utility level can be negative from the assumptions above. So the minimum utility level also could be negative. This can happen when, for example, the whole operation of the AL's all networks are making profit in aggregation, but the local airport routes or start-up routes of the AL (or LCCs) could be in red ink at least for a while. This is one of the main purposes of our model.

¹² p could be negative in usual settings, too. See, for example, Chung (1991) or Noldeke and Schmidt (1995).

¹³With the above assumptions, trade is always efficient. So dissolving the contractual relationship, which occur when the trade is not efficient after the realization of θ , is not relevant here.

In the specific contract above, it specifies that AL provides air transport service at the airport and AP rewards AL by paying $p(\theta)$ in aggregation.

Noto contract in equation (3), however, unlike the usual specific performance contract with constant price level, payment p is a function of θ and changeable amount of payment to be made from AP to stabilize the AL's revenue fluctuations, since low θ makes the p positive, meaning AP is paying to AL with little revenue, and high θ makes the p negative, meaning AL is paying to AP out of larger revenue.

In this sense, the specific performance contract is not just the commitment of AL to provide service at the airport in exchange for fixed amount of reward, but the AL's commitment in exchange for risk sharing in a counterbalancing way by AP. Note that the $p(\theta)$ could be negative. So the AL receives money from AP when the load factor is below the target load factor (beyond the special range of no payment).

4.3 Specific Features of the Model

Note that p could be negative. The reward price in negative means AP is receiving money from AL rather than paying to AL.

In the standard setting of incomplete contract frameworks, not only both parties' efforts but also state contingent variable are unverifiable. Therefore the payment is usually not directly contingent on state variable.

In our model, however, the state variable of load factor is officially announced by the Ministry of Land, Infrastructure, Transport and Tourism. So θ in our model is verifiable. Hence the contract can specify the target load factor and payment is able to be calculated directly from load factor (state variable); $p = p(\theta)$. That was exactly what the airport and airline did in Noto case.

AP and AL still cannot make contract that directly specifies the target of efforts, since they are unverifiable.

By introducing $p = p(\theta)$, AP is assuming part of AL's revenue fluctuation risk and is stabilizing AL's revenue fluctuation by paying money to AL when θ is low and hence AL's revenue is low or by receiving the money when θ is high and hence AL's revenue is high.

Note also that, unlike usual incomplete contract settings, U_{AL} has $v_{AL}(s, \theta)$ as one of its elements in addition to $-c_{AL}(s, \theta)$. This is because in the AL and AP relationship, AL has its own revenue from the air transport service as well as the reward revenue from AP.

In the usual incomplete contract settings, sellers (or employees or service providers) have only one revenue source from the transaction (with or without contract). That is the reward revenue from buyers (or employers or service receivers) represented by p .

In the case of AP and AL relationship, AL always has its own revenue from the air transport service. Off course AP also has its own revenues, such as landing fees, concession revenues from the ground side operations at the airports and parking revenues, out of such air transport services by AL. It could be said that AP and AL "share" the entire revenue from the joint-venture air transport service project at the airport.

We are assuming that the two different revenue sources can be additively separable in the utility function of AP and AL. This means that AL's two elements ($v_{AL}(s, \theta)$ and $-c_{AL}(s, \theta)$) of utility function, for example, are linearly separable. Accordingly, our model in equation (2) captures the unique features of AP-AL' joint-venture type relationship. This type of "double structure" on the both sides of a contract, just like that of AP and AL, is, to our knowledge, never formally set up, although this kind of relationship is, we presume, not so rare in a real business world ¹⁴.

In addition, AP and AL have other types of close relationship. In their value and cost contribution functions, based on the above assumptions, AP and AL are in so called "strategic inter-complementary relationship," where AP's effort level enhance the effect of AL's same level of efforts and vice versa.

We define the project value, $V(b, s)$, as follows (noticing we already assumed $v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta) > 0$),

$$V(b, s, \theta) \equiv v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta) \quad (4)$$

$$V(b, s) \equiv E_{\theta}[V(b, s, \theta) | b, s]. \quad (5)$$

We can show, with the assumptions on differentiability, that,

$$\frac{\partial^2 V(b, s)}{\partial b \partial s} \geq 0. \quad (6)$$

For a reference, if $V(b, s)$ is, contrary to our assumptions here, not differentiable,

¹⁴For example, in franchising contracts of retail convenience store, franchisees (convenience stores) are paying royalty fees to franchisers (brand holding/managing companies) and on the other hand, franchisers are paying incentive or risk mitigating money to franchisees. Another example could be tenants and land owners in commercial shopping mall projects and they could be in the same kind of joint-venture type situation. Land owners are receiving rents from tenants but they could offer some incentive/ risk neutralizing payment mechanism to (potential) tenants to ensure their commitments in the project.

the "increasing difference" condition is as follows ;

$$V(b, s) - V(b', s) \geq V(b, s') - V(b', s') \quad (7)$$

$$\forall \{b, b', s, s'\} \in \{b > b', s > s'\}$$

This property is called "increasing differences," or, in game theory terms, "strategic inter-complementarity." Under the property, if one party increases marginal effort, it does not decrease the marginal contribution by the other party's marginal effort to increase the value of the project, in which both are jointly participating.

Our model of equations, (1) and (2) above, captures, through assumptions, this strategic inter-complementary relationships between AP and AL.

4.4 First Best

In the first best, both parties jointly maximize the total expected utility level.

$$\begin{aligned} & \max_{\{b, s\}} E_{\theta}[U_{AL} + U_{AP}] \\ &= \max_{\{b, s\}} E_{\theta}[v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)] - \phi(b) - \psi(s) \\ &= \max_{\{b, s\}} V(b, s) - \phi(b) - \psi(s) \end{aligned}$$

The first best level of efforts(b^*, s^*) are characterized by FOCs:

$$E_{\theta}[v'_{AP}(b^*, \theta) - c'_{AP}(b^*)] = \phi'(b^*) \quad (8)$$

$$E_{\theta}[v'_{AL}(s^*, \theta) - c'_{AL}(s^*, \theta)] = \psi'(s^*) \quad (9)$$

In the above,

$$v'_{AP}(b^*, \theta) = \frac{\partial v_{AP}(b^*, \theta)}{\partial b}, c'_{AP}(b^*) = \frac{\partial c_{AP}(b^*, \theta)}{\partial b}, v'_{AL}(s^*, \theta) = \frac{\partial v_{AL}(s^*, \theta)}{\partial s}, c'_{AL}(s^*) = \frac{\partial c_{AL}(s^*, \theta)}{\partial s}, \phi'(b^*) = \frac{\partial \phi(b^*)}{\partial b} \text{ and } \psi'(s^*) = \frac{\partial \psi(s^*)}{\partial s}.$$

Note that the first best effort level for AL s^* is increased from the first best situation in usual incomplete contract settings. This is because AL has its own revenue source (i.e., sales from the operation), hence its own valuation function $v_{AL \cdot s}(s^*, \theta) > 0$ in its utility function. This increase the marginal stake to be balanced by the marginal effort costs in equation (9) resulting in the higher s^* .

Here, we further assume that the objective function of the maximization problems is strictly concave and that the problem has interior solution for simplicity.

Therefore the fist best efforts level (b^*, s^*) exists, it is unique and $0 < b^*, 0 < s^*$.

4.5 Hold Up Problem

Assumption 1 - Process

If both parties cannot agree on any ex ante contract, we assume, according to Hart and Moore (1988) and Edlin and Reichelstein (1996), that first AP and AL independently make efforts, wait for the realization of load factor, θ , make renegotiation according to generalized Nash bargaining solution framework about what part of realized value are gained by AP or AL, and at the same time price is also decided according to the bargaining solution¹⁵.

Based on this prospect of the transaction, AP and AL make ex ante utility maximization after taking expectation over the contingency. Also if both parties' renegotiation results in valuation and price that cannot support providing service in the first place, they have to dissolve the relationship, if any, with significant cost as much as wiping out the valuation until then.

In the generalized Nash bargaining solution, we can assume AP's share is α ($0 < \alpha < 1$) portion of and AL's share is $(1 - \alpha)$ of the total value from the realized contingency, which is as follows¹⁶;

$$U_{AP}^{hu}(b, s) \equiv \alpha \{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}_{\{\theta \in \Theta\}} - \phi(b)$$

$$U_{AL}^{hu}(b, s) \equiv (1 - \alpha) \{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}_{\{\theta \in \Theta\}} - \psi(s)$$

Anticipating these processes ahead, AL and AP are ex ante maximizing their utilities by expectations over their effort level;

$$\begin{aligned} \bar{U}_{AP}^{hu}(b, s) &= E_{\theta}[\alpha \{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}] - \phi(b) \\ &= \alpha V(b, s) - \phi(b) \end{aligned}$$

$$\begin{aligned} \bar{U}_{AL}^{hu}(b, s) &= E_{\theta}[(1 - \alpha) \{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}] - \psi(s) \\ &= (1 - \alpha)V(b, s) - \psi(s) \end{aligned}$$

So the maximization is as follows;

$$\begin{aligned} \max_b \{\bar{U}_{AP}^{hu}(b, s)\} &= \max_b \{E_{\theta}[\alpha \{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}] \\ &\quad - \phi(b)\} \end{aligned}$$

¹⁵This assumption is, as already stated, not because we think this is realistic but because by the renegotiation the ex post efficiency is guaranteed so that we can focus on the ex ante inefficiencies, which are very relevant in our model.

¹⁶The price p in the Nash bargaining case is set to satisfy the relationship; $v_{AP} - c_{AP} - p = \alpha(v_{AP} - c_{AP} + v_{AL} - c_{AL})$

$$\max_s \{\bar{U}_{AL}^{hu}(b, s)\} = \max_s \{E_\theta[(1-\alpha)\{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}] - \psi(s)\}$$

This utility maximizing results in efforts level (b^{**}, s^{**}) , which are obtained by the FOCs;

$$\alpha E_\theta[v'_{AP}(b^{**}, \theta) - c'_{AP}(b^{**}, \theta)] = \phi'(b^{**}) \quad (10)$$

$$(1 - \alpha) E_\theta[\{v'_{AL}(s^{**}, \theta) - c'_{AL}(s^{**}, \theta)\}] = \psi'(s^{**}) \quad (11)$$

On the other hand, we think about the effort making game between AP and AL with their payoff functions being $\bar{U}_{AP}^{hu}(b, s)$ and $\bar{U}_{AL}^{hu}(b, s)$ respectively. Nash equilibrium (b^{hu}, s^{hu}) in this game is defined as follows;

$$\bar{U}_{AP}^{hu}(b^{hu}, s^{hu}) \geq \bar{U}_{AP}^{hu}(b, s^{hu}) \quad \forall b \in [0, \bar{b}] \quad (12)$$

$$\bar{U}_{AL}^{hu}(b^{hu}, s^{hu}) \geq \bar{U}_{AL}^{hu}(b^{hu}, s) \quad \forall s \in [0, \bar{s}] \quad (13)$$

If we further assume $\bar{U}_{AP}^{hu}(b, s)$ and $\bar{U}_{AL}^{hu}(b, s)$ are strictly concave function and its maximization problem has interior solution, there is a unique solution. In this case, Nash equilibrium (b^{hu}, s^{hu}) , if any, satisfies the above first order condition in equations (10) and (11).

Proposition 1 Hold Up Problem

If the process follows Assumption 1, then there exists Nash Equilibrium (b^{hu}, s^{hu}) that is less than the first best levels of effort;

$$(b^{hu}, s^{hu}) < (b^*, s^*). \quad (14)$$

Proof of Proposition 1

The proof is a little complicated so we put it in Appendix A.

So without any contractual agreement between AL and AP in our settings, there is an under-effort problem by both sides. This means their welfare levels are not efficient in comparison with the first best level.

5 First Best Possibility by Price-Contingent-Variable Specific Performance Contract

Hart and Moore are thinking of a contract, which consists of binary choice of quantity of service (either zero or one) and its corresponding price. We describe such a contract as $\{ \mathcal{C}\{(p_0, q_0) = (p_0, 0); (p_1, q_1) = (p_0+k, 1)\}(k : \text{constant}) \}$. In their settings, $p_1 = p_0 + k$. Also they specify the first best conditions for specific performance contract $\mathcal{C}\{(p, q) = (p_1, 1)\}$ in proposition 1 and proposition 3 in Hart and Moore (1988).

5.1 First Best Condition in Hart and Moore (1988)

As specified in the Proposition 1 and Proposition 3 in Hart and Moore (1988), if there exists some constant $k(= p_1 - p_0)$ for which the condition $v(b, \theta) \geq k \geq c(s, \theta)$ holds for all θ, b and s , then the specific performance contract $\{ \mathcal{C}(p, q) = (p_1(= p_0 + k), 1) \}$ achieves the first best effort levels.¹⁷

This means that if the above conditions hold, specific performance contract of doing trade with price $p_1 = p_0 + k$ ($\mathcal{C}(p, q) = (p_1(= p_0 + k), 1)$) always guarantee the first best level of efforts on both sides.

Here we modified the original conditions in Hart and Moore (1988) to suit the context of our settings. First AL is by contract already committed to provide service and no choice of not providing it. In this sense the contract is specific performance contract with no room for choice (But AL can dissolve the contract afterwards).

Second, there is only one contingent variable with common distribution function for both parties. In Hart and Moore (1988) settings, each party has its own independent probability distribution. Therefore, as to the probability distribution, our settings are one special case (both parties share a single common probability distribution) of Hart and Moore (1988).

But the same is that the renegotiation by non-verifiable message sending game to change the price is assumed to be possible. In their case contingency realization results in trade or no trade case with entailing price renegotiation results. In our case, doing trade is always efficient by assumption and parties renegotiate price after the contingency realization.

The price resulting from renegotiation is not always satisfying the first best

¹⁷The message game in this case is by using non-verifiable messages. Also although there are 3 other cases Proposition 3 in Hart and Moore (1988), the three cases are not relevant in our specific contract case. This is because valuation v_{AP} and v_{AL} c_{AP} and c_{AL} are all contingent on b, s, θ .

conditions. Next we cover under what conditions the price after renegotiation always achieves the first best efforts level.

5.2 First Best Condition of AP-AL Specific Performance Contract

In our settings about the specific performance contract of Noto case in equation (3), we can adjust Hart and Moore's first best conditions above so as to reflect the special relationship between AP and AL, the Joint-Venture Type contract.

Proposition 2

If the condition $v_{AP}(b, \theta) - c_{AP}(b, \theta) \geq p(\theta) \geq c_{AL}(s, \theta) - v_{AL}(s, \theta)$ holds for all θ , then the specific performance contract with risk sharing function $\{ \mathcal{C}(p, q) = \mathcal{C}(p(\theta), 1) \}$ achieves the first best effort levels.¹⁸

Proof of Proposition 2

$v_{AP}(b, \theta) - c_{AP}(b, \theta)$ and $c_{AL}(s, \theta) - v_{AL}(s, \theta)$ have the same functional properties as v and c in Hart and Moore (1988) by assumptions. We can think of $v_{AP}(b, \theta) - c_{AP}(b, \theta)$ as $\check{v}_{AP}(b, \theta)$ and $c_{AL}(s, \theta) - v_{AL}(s, \theta)$ as $\check{c}_{AL}(s, \theta)$. So the Proposition 2 with \check{v}_{AP} and \check{c}_{AL} states the same conditions with Proposition 1 and 3 in Hart and Moore (1988). Then the contents of Region (2) in APPENDIX A in Hart and Moore (1988) holds for the \check{v}_{AP} and \check{c}_{AL} , although in our case the contingency is only one common θ .

In this non-verifiable message sending game, the final outcome is to trade at price $p(\theta)$. So specific performance contract $\{ \mathcal{C}(p, q) = \mathcal{C}(p(\theta), 1) \}$ always corresponds to the final outcome of the game under the settings.

AP and AL under the specific performance contract $\{ \mathcal{C}(p, q) = \mathcal{C}(p(\theta), 1) \}$ in this setting are individually maximizing each utility under the condition of always paying $p(\theta)$ for air transport service that is committed to be provided by the contract.

$$\begin{aligned}\bar{U}_{AP}^c &= E_\theta[v_{AP}(b, \theta) - c_{AP}(b, \theta) - p(\theta)] - \phi(b) \\ \bar{U}_{AL}^c &= E_\theta[p(\theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)] - \psi(s)\end{aligned}$$

¹⁸Here again we assume the message game by using non-verifiable messages in the renegotiation stage just as in Hart and Moore (1988).

So the maximization is as follows.

$$\begin{aligned}\max_b \{\bar{U}_{AP}^c\} &= \max_b \{E_\theta[v_{AP}(b, \theta) - c_{AP}(b, \theta) - p(\theta)] - \phi(b)\} \\ \max_s \{\bar{U}_{AL}^c\} &= \max_s \{E_\theta[p(\theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)] - \psi(s)\}\end{aligned}$$

This utility maximization results in the effort levels (b^c, s^c) , which are obtained by the FOCs;

$$E_\theta[v'_{AP}(b^c, \theta) - c'_{AP}(b^c, \theta)] = \phi'(b^c) \quad (15)$$

$$E_\theta[v'_{AL}(s^c, \theta) - c'_{AL}(s^c, \theta)] = \psi'(s^c) \quad (16)$$

These FOCs are exactly the same with the FOCs in the first best case, namely equations (8) and (9).

Hence,

$$(b^c, s^c) = (b^*, s^*) \quad (17)$$

Therefore the specific performance contract $\{ \mathcal{C}(p, q) = \mathcal{C}(p(\theta), 1) \}$ achieves the first best levels of effort.

Q.E.D.

Notice that in our joint-venture type contract, because the \check{v} and \check{c} have two elements, $v_{AP}, -c_{AP}$ and $c_{AL}, -v_{AL}$ respectively. So $c_{AL} - v_{AL}$ can be negative by assumptions. Therefore emerges the possibility that Noto type zero-sum payment structure contract achieves the first best effort level even though the payment $p(\theta)$ could go into the deep negative range.

These two relevant conditions are illustrated in the Figure 4. The actual contractual piece-wise linear price schedule in the fourth year contract of Noto case is indicated by the piece-wise linear line $p(\theta)$.

As you can see in the Figure 4, the reward price function $p(\theta)$ is satisfying the first best condition for the specific performance contract to achieve the first best level in Proposition 2. The line of $p = p(\theta)$ lies within the upper red line $v_{AP} - c_{AP}$, which is the net valuation of AP and lower decreasing blue line $c_{AL} - v_{AL}$, which is the negative of the net valuation of AL, as indicated in the Proposition 2.

Especially in our model, we can see the situation where, like in the case of AP's

trying to keep the low demand route at the local airport or in the case of a secondary AP's paying incentive payment for start-up LCCs, AL's net valuation $v_{AL} - c_{AL}$ of the project is negative in a bad state of nature (low θ) but by signing a the contract with the incentive payment $p(\theta)$, the adjusted valuation is non-negative ($v_{AL} - c_{AL} + p(\theta) \geq 0$) even in the bad state of nature, while at the same the contract overcomes the under-effort problem with first best conditions holding.

5.3 Utilities Loss Restoration

If the condition for first best holds for the specific performance contract, then the utilities level for both AP and AL are brought back from hold up situation to the fist best situation. Then we can calculate the difference between the two situations and see the utilities loss restoration from hold up (second best) situation to first best situation.

$$\begin{aligned}
ResU &\equiv \bar{U}_{AP}^c(b^*) + \bar{U}_{AL}^c(s^*) - [\bar{U}_{AP}^{hu}(b^{hu}) + \bar{U}_{AL}^{hu}(s^{hu})] \\
&= \bar{v}_{AP}(b^*) - \bar{v}_{AP}(b^{hu}) - (\bar{c}_{AP}(b^*) - \bar{c}_{AP}(b^{hu})) \\
&\quad + \bar{v}_{AL}(s^*) - \bar{v}_{AL}(b^{hu}) - (\bar{c}_{AL}(s^*) - \bar{c}_{AL}(s^{hu})) \\
&\quad - (\phi(b^*) - \phi(b^{hu})) - (\psi(s^*) - \psi(s^{hu}))
\end{aligned}$$

As far as the value of $ResU$ is positive, there is utilities loss restoration. With our assumptions, economically meaningful setting strongly suggests $ResU \geq 0$, although exact results depend on concrete functional forms and concrete parameters. In fact, later we show, with concrete numerical examples that support the first best conditions, $ResU$ is actually positive in the relevant range.

The payment from the public entity to private firm, like government owned local airport to private airline, is often very controversial because tax payers' money is on the line. But the positive value of $ResU$ means that by the contract and its stipulated payment the lost utilities are recovered. So this efficiency gain could be one of the reasons to justify such use of tax money from economic point of views.

6 Numerical Examples

Here we illustrate the theoretical analysis with some concrete models and numbers. We also try to show some relationship between parameter perturbation and the effort levels and utility levels change in our models.

To make this as real as possible in addition to keeping as simple as possible, the price function is using the Noto contract real values, namely, price ceiling 2×10^8 yen and price floor -2×10^8 yen. Henceforth we use simple linear function between the price ceiling and floor (i.e., $p(\theta) \equiv 2 \times 10^8 - 4 \times 10^8 \theta$), which is indicated by dotted line in Figure 4, rather than piece-wise linear function in the actual contracts of Noto case, which is also indicated in line of Figure 4 ¹⁹.

We assume the functional forms and its parameters as follows.

$$\begin{aligned}
v_{AP}(b, \theta) &= B_1 - (B_1 - A_1)e^{\{-\lambda_1(\theta+b)\}} \\
c_{AP}(b, \theta) &= C_1 - (C_1 - D_1)e^{\{-\lambda_2(\theta+b)\}} \\
v_{AL}(s, \theta) &= B_2 - (B_2 - A_2)e^{\{-\lambda_3(\theta+s)\}} \\
c_{AL}(s, \theta) &= C_2 - (C_2 - D_2)e^{\{-\lambda_4(\theta+s)\}} \\
p(\theta) &= E - F\theta, \theta \sim N(\mu, \sigma^2) \\
\phi(b) &= e^b, \psi(s) = e^s \\
A_1 &= 2.4 \times 10^8, B_1 = 4 \times 10^8, C_1 = 0.3 \times 10^8, D_1 = 0.1 \times 10^8, A_2 = 0.7 \times 10^8, \\
B_2 &= 2.5 \times 10^8, C_2 = 2.2 \times 10^8, D_2 = 0.1 \times 10^8, E = 2 \times 10^8, F = 4 \times 10^8
\end{aligned}$$

We further assume that $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda (> 0)$ in all the valuation functions.

Also we use these numbers in the course of our analysis with some variations.
 $\mu = 0.7, \sigma = 2.38, \lambda = 3, \alpha = 0.5001$

If we use these functions and parameters, we get the utilities for both AP and AL²⁰.

¹⁹This is for the sake of maneuverability rather than implying that the efficient contract is linear.

²⁰The expected utilities are calculated here by using the following approximation, since the 0 or 1 are estimated to be at least 6 standard deviation away from the mean value in normal distribution by the study on the Noto contract in Hihara (2008).

$$\int_0^1 U_i(j, \theta) dF(\theta) \approx \int_{-\infty}^{\infty} U_i(j, \theta) dF(\theta)$$

$(i, j) = \{(AP, b) \text{ or } (AL, s)\}$

$$\begin{aligned}
\bar{U}_{AP} &= E_{\theta}[U_{AP}(b, \theta)] = B_1 - D_1 - (B_1 - A_1)e^{\{-\lambda\mu - \lambda b + \frac{\lambda^2\sigma^2}{2}\}} \\
&\quad - (C_1 - D_1)e^{\{-\lambda\mu - \lambda b + \frac{\lambda^2\sigma^2}{2}\}} \\
&\quad - (E - F\mu) - e^b \\
\bar{U}_{AL} &= E_{\theta}[U_{AP}(b, \theta)] = B_2 - D_2 - (B_2 - A_2)e^{\{-\lambda\mu - \lambda s + \frac{\lambda^2\sigma^2}{2}\}} \\
&\quad - (C_1 - D_1)e^{\{-\lambda\mu - \lambda s + \frac{\lambda^2\sigma^2}{2}\}} \\
&\quad + (E - F\mu) - e^s
\end{aligned}$$

From the first order conditions of the first best situation in $\bar{U}_{AP} + \bar{U}_{AL}$ by b and s , we can get the system of equations.

$$(\partial(\bar{U}_{AP}(b) + \bar{U}_{AL}(s)))/\partial b = 0 \text{ and } (\partial(\bar{U}_{AP}(b) + \bar{U}_{AL}(s)))/\partial s = 0$$

Solving the equations, we get the first best effort levels (b^*, s^*) .

$$\begin{aligned}
b^{FB} &= \frac{1}{(\lambda + 1)}(\ln(MN_1)) \\
s^{FB} &= \frac{1}{(\lambda + 1)}(\ln(MN_2)) \\
M &= \lambda e^{\{-\lambda\mu + \frac{\lambda^2\sigma^2}{2}\}} \\
N_1 &= B_1 - A_1 + C_1 - D_1 \\
N_2 &= B_2 - A_2 + C_2 - D_2
\end{aligned}$$

In the no-contract case, we have the following second best (hold up) situations under Assumption 1.

$$\begin{aligned}
\bar{U}_{AP}^{SB} &= \alpha[E_{\theta}[U_{AP}(b, \theta)] + E_{\theta}[U_{AL}(s, \theta)]] - e^b \\
\bar{U}_{AL}^{SB} &= (1 - \alpha)[E_{\theta}[U_{AP}(b, \theta)] + E_{\theta}[U_{AL}(s, \theta)]] - e^s
\end{aligned}$$

From the first order conditions of \bar{U}_{AP} and \bar{U}_{AL} , we have the second best effort levels (b^{SB}, s^{SB}) as follows.

$$\begin{aligned}
b^{SB} &= \frac{1}{(\lambda + 1)}(\ln \alpha + \ln(MN_1)) \\
s^{SB} &= \frac{1}{(\lambda + 1)}(\ln(1 - \alpha) + \ln(MN_2))
\end{aligned}$$

For the Noto contract to mitigate the under-effort problem and to lead the two parties to reach the first best effort levels, the contract must satisfy the condition in Proposition 2. We can check the condition in our settings here by drawing the graphs for $v_{AP}(\theta) - c_{AP}(\theta)$, $v_{AL}(\theta) - c_{AL}(\theta)$, $p(\theta) = E - F\theta$ and $c_{AL}(\theta) - v_{AL}(\theta)$ with θ in our settings.

These graphs are actually in the Figure 4. These graphs are drawn exactly by the functional forms and parameters we set here. The piece-wise linear line in Figure 4 is the real price schedule in Noto contract in the fourth year.

For the piece-wise linear price schedule indicated by line in Figure 4, the condition of Proposition 2 under our settings is equivalent to the following.

$$\begin{aligned} v_{AP}(0) - c_{AP}(0) &= 2.1 \times 10^8 \geq 2 \times 10^8 \\ c_{AL}(0) - v_{AL}(0) &= 1.5 \times 10^8 \leq 2 \times 10^8 \\ c_{AL}(0.9354) - v_{AL}(0.9354) &= -2.16 \times 10^8 \leq -2 \times 10^8 \end{aligned}$$

The point for $(\theta = 0.9354, P = -2 \times 10^8)$ is the beginning point of the lower horizontal line part of the piece-wise linear function of the real Noto contract price schedule. As we can see, all these conditions are met.

Also it is not hard to confirm that the simplified price schedule $p(\theta) = E - F\theta$ indicated by dotted line in Figure 4 we assume here satisfy these conditions, too.

Therefore, by Noto specific performance contract of confirming the commitment of AL's service at the airport in exchange for reward money $p(\theta)$ can lead AP and AL to achieve the first best effort levels according to Proposition 2.

Now that we see the numerical example for the specific performance contract such as Noto contract satisfy the relevant conditions for the contract to lead both AP and AL to reach the first best effort levels. We look into more detail for the structure of our numerical model.

First we like to see the difference between first best effort level and the second best effort level. Notice that b^{FB}, s^{FB} are the function of μ, σ and λ in M above and b^{SB} and s^{SB} are the function of α in addition to the three. So we can draw the graphs for b^{FB} and b^{SB} by μ, σ and λ with appropriate number of α .

(Figure 5 about here)

The Figure 5 left is the graph for the followings. The upper red graph is for first best case, and the lower black graph is for the second best case.

$$\begin{aligned}
b^{FB}(\mu, \sigma) &= \frac{1}{(\lambda + 1)} (\ln(MN_1)) \\
b^{SB}(\mu, \sigma) &= \frac{1}{(\lambda + 1)} (\ln \alpha + \ln(MN_1)) \\
\alpha &= 0.1; \lambda = 3
\end{aligned}$$

The Figure 5 right is the graph for the followings. The upper blue graph is for first best case, and the lower black graph is for the second best case.

$$\begin{aligned}
s^{FB}(\mu, \sigma) &= \frac{1}{(\lambda + 1)} (\ln(MN_1)) \\
s^{SB}(\mu, \sigma) &= \frac{1}{(\lambda + 1)} (\ln(1 - \alpha) + \ln(MN_1)) \\
\alpha &= 0.9; \lambda = 3
\end{aligned}$$

(Figure 6 about here)

The Figure 6 left is the graph for the followings. The upper red graph is for first best case, and the lower black graph is for the second best case.

$$\begin{aligned}
b^{FB}(\mu, \lambda) &= \frac{1}{(\lambda + 1)} (\ln(MN_1)) \\
b^{SB}(\mu, \lambda) &= \frac{1}{(\lambda + 1)} (\ln \alpha + \ln(MN_1)) \\
\alpha &= 0.5001; \sigma = 2.38
\end{aligned}$$

The Figure 6 right is the graph for the followings. The upper blue graph is for first best case, and the lower black graph is for the second best case.

$$\begin{aligned}
s^{FB}(\mu, \lambda) &= \frac{1}{(\lambda + 1)} (\ln(MN_1)) \\
s^{SB}(\mu, \lambda) &= \frac{1}{(\lambda + 1)} (\ln(1 - \alpha) + \ln(MN_1)) \\
\alpha &= 0.5001; \sigma = 2.38
\end{aligned}$$

The vertical differences in these graphs indicate the under-effort level. We can say that the under-effort level situation indicated in lower graphs is overcome by the contract and AP and AL reach the first best effort levels indicated in

upper graphs in these Figures.

Also we can see the effects of perturbation of μ , σ and λ on the effort level by seeing the relevant slopes of these graphs. The exact effects of these can be indicated by the derivatives of b^{FB}, s^{FB} and b^{SB}, s^{SB} with respect to each of μ , σ , λ and α . For the sake of space, we write only some of them here.

$$\begin{aligned}\frac{\partial b^{SB}}{\partial \mu} &= \frac{-\lambda}{\lambda + 1} < 0 \\ \frac{\partial b^{SB}}{\partial \sigma} &= \frac{\lambda^2}{\lambda + 1} > 0 \\ \frac{\partial b^{SB}}{\partial \lambda} &= \frac{1}{(\lambda + 1)^2} [(\lambda + 1) \left\{ \frac{1}{\lambda} - \mu + \sigma^2 \lambda \right\} \\ &\quad - \left\{ \ln \alpha + \ln \lambda - \lambda \mu + \frac{\lambda^2 \sigma^2}{2} + \ln N_1 \right\}] \\ \frac{\partial b^{SB}}{\partial \alpha} &= \frac{1}{\alpha(\lambda + 1)} > 0\end{aligned}$$

μ increasing means that expected level of state of nature improves. Then to reach the same level of utility, you need less effort. Hence the derivative is negative.

σ increasing means that the state of nature is more volatile and more uncertain. The expected level of utility decreases, which means also world for you shrinks under more uncertain environment. Then to reach the same level of utility, you need more efforts. Hence the derivative is positive.

α increasing means that you will share more realized value of the project. Since the stake for the efforts of AP is higher, the effort for the higher stakes is also higher at the optimal. Hence the positive value of the derivative. α has nothing to do with the state of nature.

The effect of λ change on b is a little complicated. λ is just like the risk averse parameter in the CARA utility function in the valuation and cost reduction functions in our settings. The perturbation of it means the objective valuation function is more against risk (i.e., low certainty equivalent), although we assume risk neutral for AP and AL. Its derivative is so complex that by only seeing the derivative we cannot readily tell the sign. From the graphs in Figure 6, the slope of derivative is first positive starting from $\lambda = 0$, then turns to be negative and again turn to be positive between $\lambda = 1.5$ and $\lambda = 2$. We have to be careful to pick the number for λ , since the its effect on the level of efforts is rather big and marginal effect of changing λ is complicated in order to avoid wrong implications from model analysis.

With the derivation of (b^{FB}, s^{FB}) and (b^{SB}, s^{SB}) above, we can put these back

into $\bar{U}_{AP} + \bar{U}_{AL}$ in both first best situation with contract and second best situation without contract. Then by taking the difference of the two situations utility levels $ResU$, we can capture the utility loss of not having proper contract and of not reaching the first best effort levels on both sides but of being restored by making a proper contract satisfying the condition of Proposition 2 in our settings.

$$ResU \equiv \bar{U}_{AP}(b^{FB}) + \bar{U}_{AL}(s^{FB}) - [\bar{U}_{AP}^{SB}(b^{SB}) + \bar{U}_{AL}^{SB}(s^{SB})]$$

The Figure 7 is the graph for the utilities loss from under-efforts by not having proper specific performance contract in our settings that are restored by the contract to going back to the first best effort levels. As you might see from the Figure 7, $ResU$ is in fact positive over the entire range for all the four choice parameters, α , μ , σ and λ in our settings. Figure 7 left is the loss explained by μ and σ . Figure 7 right is the loss explained by μ and λ . The five different sheets of graphs are for 5 different levels of α . α indicates the share balance between the two in the hold up situation. $\alpha = 0.5$ is perfectly balanced between AP and AL. The value near to 0 or 1 indicates less balanced share among AP and AL.

(Figure 7 about here)

The perturbation effects on the utility loss are more complicated than that on effort levels, since functions and their derivatives are more complicated. The effect of μ and σ , however, in our settings could be said, though the exact effects are dependent on other parameters, that their effects on utility loss are same as in the effort levels. The effect of perturbation of λ is also complicated just like in the effect of λ change on effort levels. We can see the effects in Figure 7 right, although the exact effects are dependent on other parameters just like in the case of μ and σ .

The effect of α change is the most complicated one. We can see from the two graphs in Figure 7 that, roughly speaking, the more balanced the share of value between AP and AL is, the less utility loss results from under-efforts without contract. Although the exact effects are dependent on the other parameters, especially λ , the graph reading means the loss is greater for $\alpha = 0.3$ or 0.7 than α around 0.5 . The same caution for λ for effort level is valid here for both α as well as λ .

By seeing the two graphs in Figure 7, we could say that if the factors indicate good prospect (high μ), low uncertainty (low σ), modest "risk averse" (middle λ value) and more balanced share in hold up situation (α more toward 0.5), then the utility loss is not so severe. On the other hand, if the factors indicate poor prospect (low μ), high uncertainty (high σ), high "risk averse" (extreme

λ value) and less balanced share in hold up situation (α more away from 0.5), then the utility loss could be severe and the restoration by contract of the loss is more imminently needed.

7 Concluding Remarks

As an example of airport-airline vertical contractual relationships, 'Load Factor Guarantee Mechanism' contract, actually agreed at Noto Airport in Japan, stipulates that the airline agrees to serve the airport and the airport agrees to pay to (or receive from) the airline based on the difference between the target load factor set at the start of the period in the contract and a realized one. Airports are often local governments and such payments are controversial since tax money is on the line.

From AP perspective, such contract ensures the commitment of AL to serve AP as well as the sharing of up-side profit in high load factor case. From AL perspective, such contract serves not only as a risk mitigating tool to compensate down-side loss of revenue but as an incentive device to extract AP efforts to avoid large payment hence overcoming its under-efforts seen typically after start of the service at the airport.

In this study, we try to model such AP-AL vertical contractual relationship, namely joint-venture type relationship, in hold up problem settings under the incomplete contract theory framework. In such relationship, both AP and AL are buyer and seller at the same time and the relationships are not economically separable. For this, we modified the model of Hart and Moore (1988) so as to follow such joint-venture type contract.

With this modification, we can model the situation in a local airport to pay incentive money to LCCs beyond the discount of landing fee. Also we can construct the structures in which the valuation of AL's side to service at AP is not so lucrative, although serving there meets the minimum participation utility level, and the incentive money from AP to AL plays an important role in AL's decision to actually serve AP.

We show that, without the contractual commitment of providing air transport service to the airport, there is an under-effort problem under the usual incomplete contract settings. The specific performance contracts such as Noto case, however, can achieve the first best levels of efforts and restore utilities losses under the modified first best conditions of Hart and Moore (1988).

We also show, by numerical examples, that if we have good prospect of project, low uncertainty, modest "risk averse" and more balanced share in hold up situation, then the utilities loss is not so severe. On the other hand, if we have poor

project prospect, high uncertainty, high "risk averse" and less balanced share in second best situation, then the utilities loss could be severe and the utilities loss restoration by such contracts is more imminently needed.

With these results, we could justify at least partly from theoretical point of view such incentive/risk mitigating payments by vertical contract from local or secondary airport to LCCs, for example, even when the airport is local government. This is because such contracts can achieve the first best efficient effort levels and restore the first best utilities levels. Also the risk mitigating aspects of zero-sum payment by such contracts could lead to the efficient pure risk sharing.

The next steps would be, among other points, to model the multi-year contract. Since our model is one year setting, multi-year dynamic model would be more realistic. Another possibility is to include the effort externalities. In our model each efforts does not affect the other 's valuations. With effort externalities, AP's efforts can directly enhance the valuation of AL and vice versa. Also we could look into more specific efforts, such as price discount or frequency increase, in the context of network industries. These modifications could enable us to model much more interdependent and plausible relationships between AP and AL.

With such refinements, we have the better possibility to model more closely the complex relationship and dynamic negotiations between APs and ALs in the real world. This could contribute to better public policy analysis on the matter.

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A Appendix A Proof of Proposition 1

First we present the proof of the existence of Nash equilibrium. Then, we show the inequality (14) holds.

Here, to show the existence of Nash equilibrium, we follow the explanation in Fundenberg and Tirole (1991) and Itoh (2003) about the existence of pure-strategy Nash equilibrium in super-modular game.

$V(b, s)$ has increasing difference property by assumption. Therefore, it is relatively easy to see that $\bar{U}_{AP}^{hu}(b, s)$ and $\bar{U}_{AL}^{hu}(b, s)$ also satisfy increasing difference property. We derive this as follows;

$$\begin{aligned}\bar{U}_{AP}^{hu}(b, s) &= E_\theta[\alpha\{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AP}(s, \theta) - c_{AL}(s, \theta)\}] - \phi(b) \\ &= \alpha V(b, s) - \phi(b) \\ \bar{U}_{AL}^{hu}(b, s) &= E_\theta[(1 - \alpha)\{v_{AP}(b, \theta) - c_{AP}(b, \theta) + v_{AL}(s, \theta) - c_{AL}(s, \theta)\}] - \psi(s) \\ &= (1 - \alpha)V(b, s) - \psi(s)\end{aligned}$$

So for example,

$$\begin{aligned}\frac{\partial \bar{U}_{AP}^{hu}(b, s)}{\partial b} &= E_\theta[\alpha\{v'_{AP}(b, \theta) - c'_{AP}(b, \theta)\}] - \phi'(b) \\ \frac{\partial^2 \bar{U}_{AP}^{hu}(b, s)}{\partial b \partial s} &= 0\end{aligned}$$

Hence,

$$\frac{\partial^2 \bar{U}_{AP}^{hu}(b, s)}{\partial b \partial s} \geq 0. \quad (\text{A.1})$$

Therefore $\bar{U}_{AP}^{hu}(b, s)$ and $\bar{U}_{AL}^{hu}(b, s)$ also satisfy increasing difference property.

In addition, b and s , which are elements of pure strategies profile $S\{b \in S, s \in S\}$ for AP and AL, are one dimension. S is easily shown, by assumptions, that it is non-empty and compact. So the game by AP and AL with payoff function $\bar{U}_{AP}^{hu}(b, s)$ and $\bar{U}_{AL}^{hu}(b, s)$ respectively, are super-modular game. Therefore the set of pure strategy Nash equilibria is non-empty and has greatest and least points.

Now we show the inequality (14) holds. Here we follow the simple explanation in Itoh (2003) about under-effort level Nash equilibrium.

Suppose the condition $(b^{hu}, s^{hu}) < (b^*, s^*)$ in equation (14) does not hold.

First we assume $b^* \leq b^{hu}$ in conflict with the condition above. If $s^* \leq s^{hu}$, then

$$\begin{aligned}
0 &> [V(b^{hu}, s^{hu}) - \phi(b^{hu}) - \psi(s^{hu})] - [V(b^*, s^*) - \phi(b^*) - \psi(s^*)] \\
&= \alpha V(b^{hu}, s^{hu}) - \phi(b^{hu}) + (1 - \alpha)V(b^{hu}, s^{hu}) - \psi(s^{hu}) \\
&\quad - [\alpha V(b^*, s^*) - \phi(b^*)] - [(1 - \alpha)V(b^*, s^*) - \psi(s^*)] \\
&\geq \alpha V(b^*, s^{hu}) - \phi(b^*) + (1 - \alpha)V(b^{hu}, s^*) - \psi(s^*) \\
&\quad - [\alpha V(b^*, s^*) - \phi(b^*)] - [(1 - \alpha)V(b^*, s^*) - \psi(s^*)] \\
&= \alpha[V(b^*, s^{hu}) - V(b^*, s^*)] + (1 - \alpha)[V(b^{hu}, s^*) - V(b^*, s^*)] \\
&\geq 0.
\end{aligned}$$

The first strict inequality is because, by assumption, (b^*, s^*) is the unique efficient effort level. The second inequality comes from (b^{hu}, s^{hu}) is Nash equilibrium. The third inequality is since $V(b, s)$ is, by assumption, monotonically increasing function. Hence contradiction.

If $s^* > s^{hu}$, then,

$$\begin{aligned}
0 &> [V(b^{hu}, s^*) - \phi(b^{hu}) - \psi(s^*)] - [V(b^*, s^*) - \phi(b^*) - \psi(s^*)] \\
&> [\alpha V(b^{hu}, s^*) - \phi(b^{hu})] - [\alpha V(b^*, s^*) - \phi(b^*)] \\
&\geq [\alpha V(b^{hu}, s^{hu}) - \phi(b^{hu})] - [\alpha V(b^*, s^{hu}) - \phi(b^*)] \\
&\geq 0.
\end{aligned}$$

The first strict inequality is because, by assumption, (b^*, s^*) is the unique efficient effort level. The second strict inequality comes from $\alpha \in (0, 1)$ and $V(b, s)$ being monotonically increasing function. The third inequality is because $V(b, s)$ satisfies "increasing difference" condition. The last inequality comes from (b^{hu}, s^{hu}) is Nash equilibrium. Hence contradiction.

By symmetry, the contradiction case for $s^* \leq s^{hu}$ goes the same way.

Q.E.D.

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Figure 1

$P(\theta)$ schedule by 'Noto Contract' (+: AP→AL)

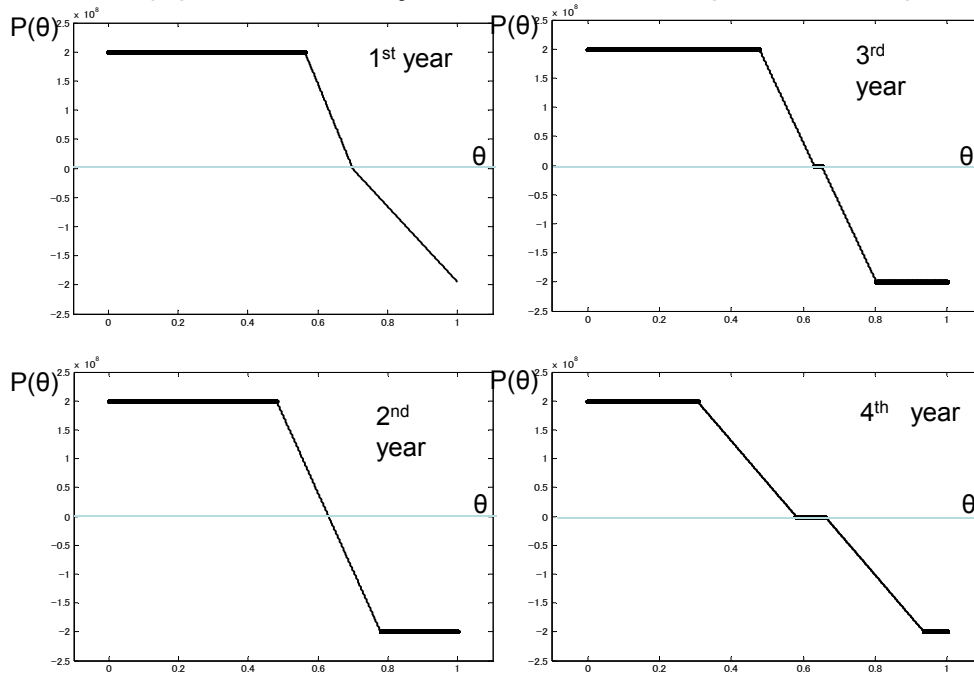


Table 1

Noto Case Summary Information

Yr	Target $\theta(LF)$	Model $\theta(LF)$	Actual $\theta(LF)$	# of Actual Passenger	- $P(\theta)$ AP←AL
1	70%	65%	79.5%	151,015	973.29
2	63%	63.7%	64.6%	155,623	159.80
3	64%	63.5%	66.5%	160,052	200.00
4	62%	65.0%	65.1%	156,654	0

Target $\theta(LF)$ is a load factor AP and AL agreed on in the contract.

Model $\theta(LF)$ is an estimated load factor based on econometric model in Hihara(2008).

Actual $\theta(LF)$ is the load factor that actually realized.

(100,000 ¥)

Figure 2

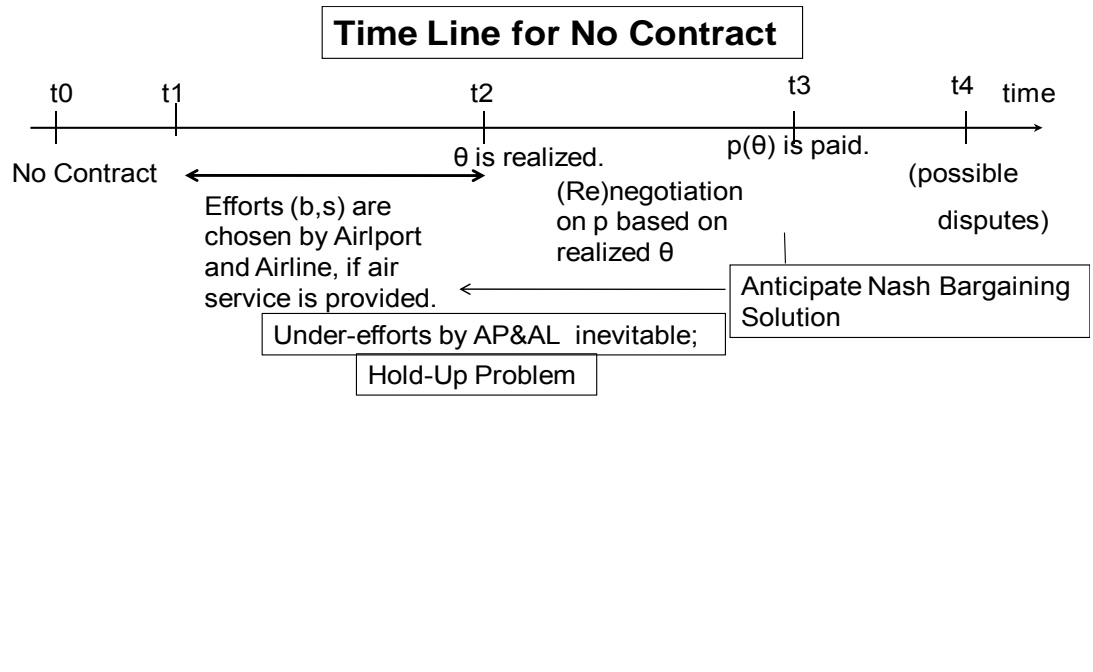


Figure 3

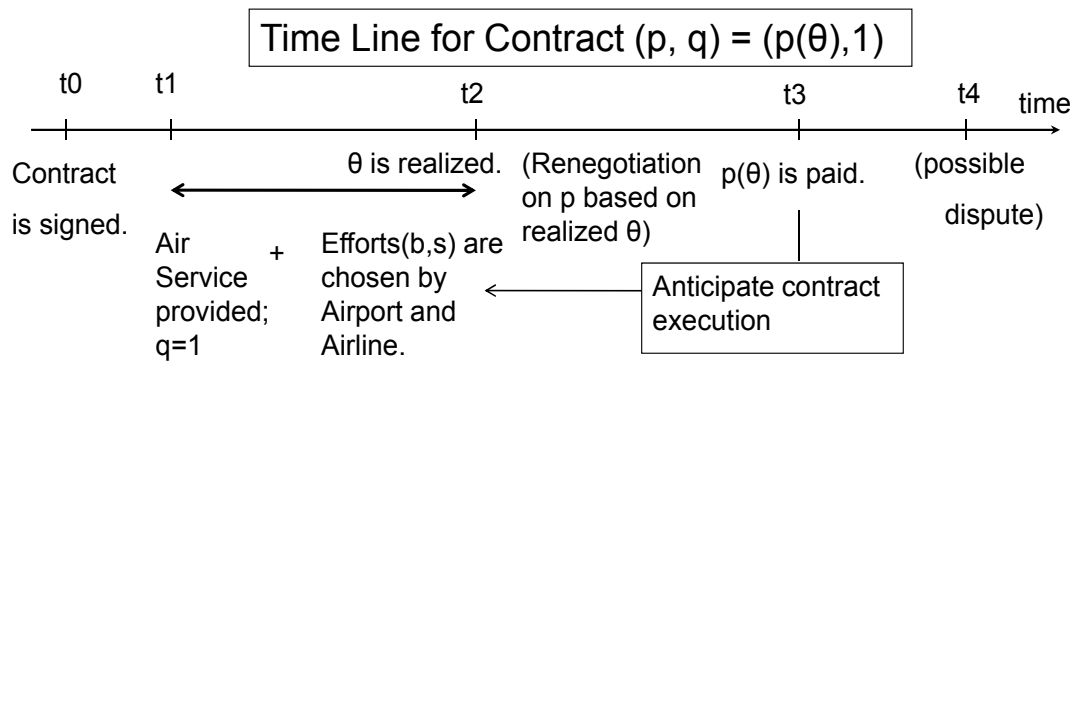


Figure 4

Condition for Overcoming Under-effort Problem by Contract

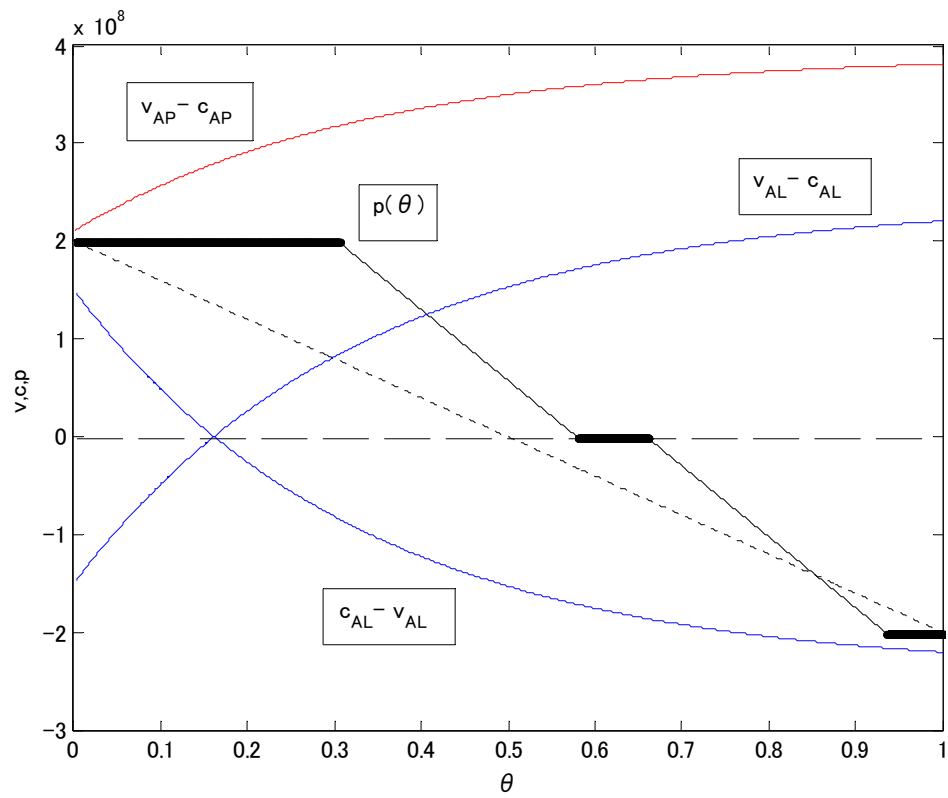
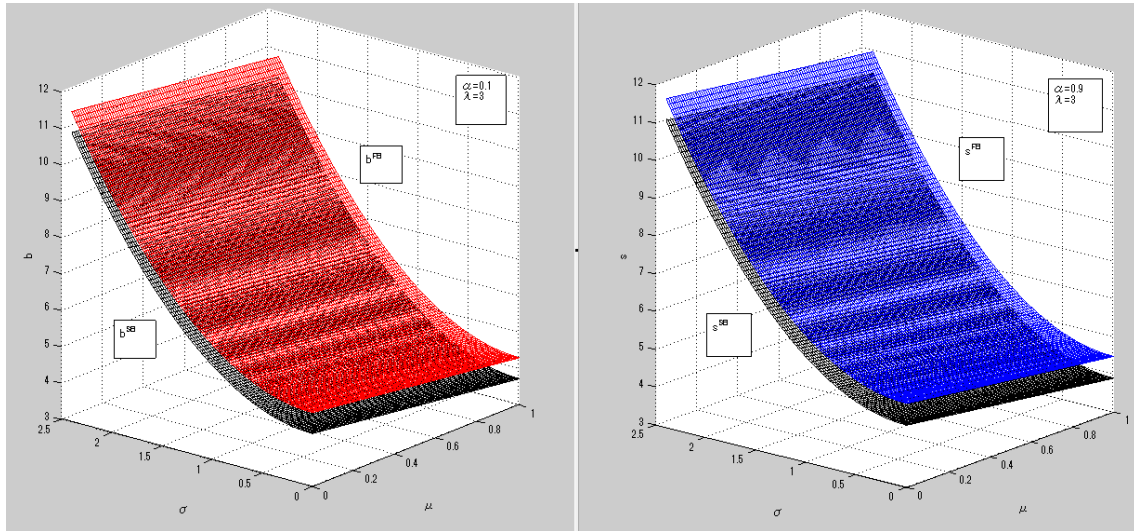
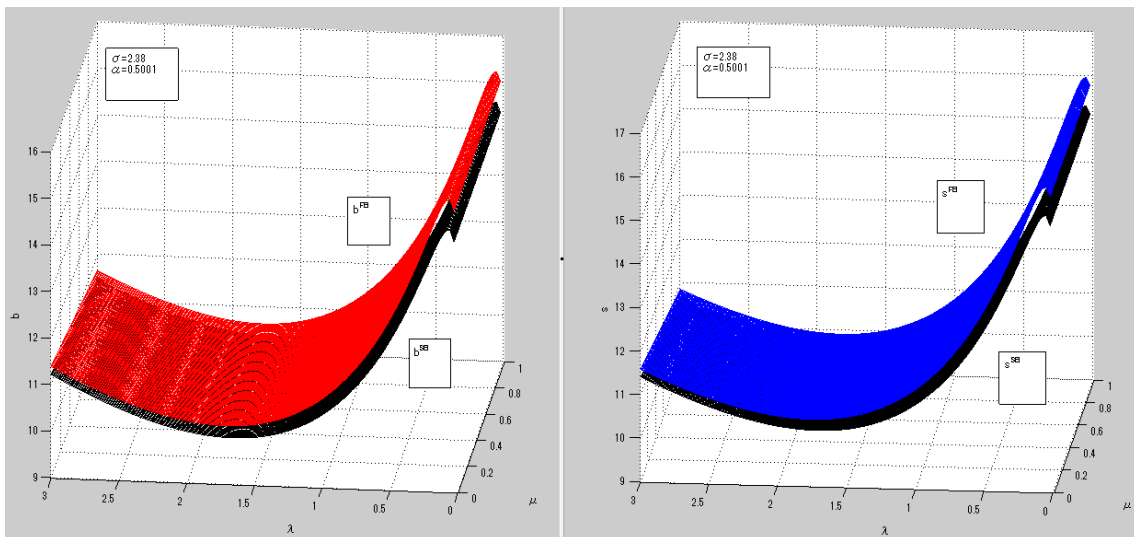


Figure 5 Under-efforts and Recovery by Contract - 1 (b/s by μ & σ)



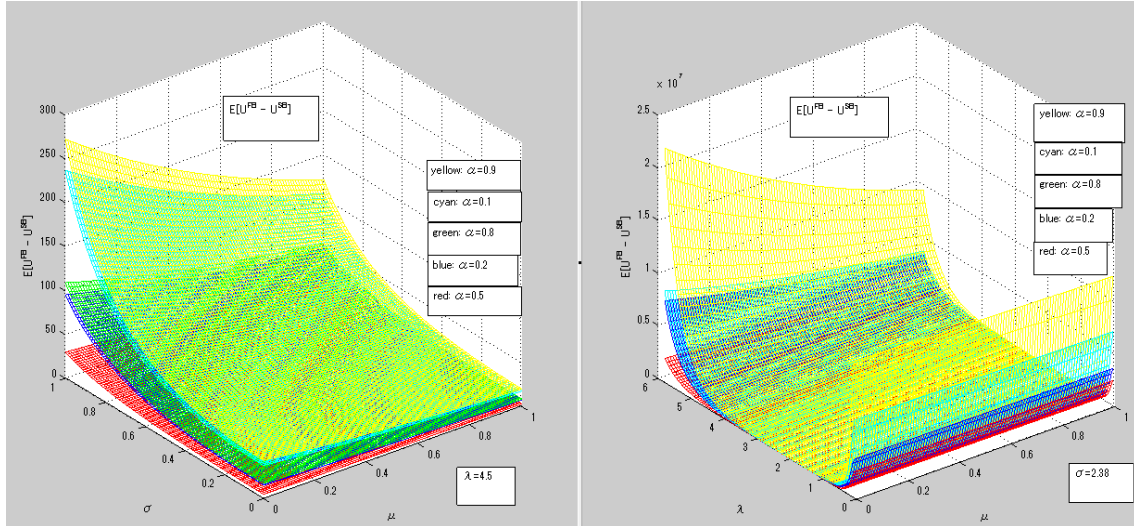
1. The red graph is the first best effort level of AP by contract.
2. The blue graph is the first best effort level of AL by contract
3. The black graph is the second best effort level of AP and AL without contract.

Figure 6 Under-efforts and Recovery by Contract - 2 (b/s by μ & λ)



1. The red graph is the first best effort level of AP by contract.
2. The blue graph is the first best effort level of AL by contract
3. The black graph is the second best effort level of AP and AL without contract.

Figure 7 Utilities Losses from Under-Efforts that are Recovered by Contract
 (left graph: Loss by μ & σ ; right graph: Loss by μ & λ)



The graphs are the difference between first best expected utility level of both AP and AL restored by contract minus second best expected utility level of AP and AL.

Graphs in colors are;

red for $\alpha=0.5$; blue for $\alpha=0.2$; green for $\alpha=0.8$; cyan for $\alpha=0.1$; yellow for $\alpha=0.9$.