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Katsuya Hihara *

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Abstract

We analyze the double moral hazard problem at the joint venture type airport-airline vertical relationship, where two parties both contribute efforts to the joint venture but neither of them can see the other's efforts. With the continuous-time stochastic dynamic programming model, we show that by the de-centralized utility maximizations of two parties under very strict conditions, i.e., optimal efforts' cost being negligible and their risk averse parameters both asymptotically approaching to zero, the vertical contract, which is linear function of the final state with the slope being the product made by their productivity difference and uncertainty (diffusion rate) level index, could be agreed as the optimal sharing rule.

If both parties 'productivities are same, or the diffusion rate of the underlying process is unity, optimal linear sharing rule do not depend on the final state. If their conditions not dependent on final state are symmetric as well, then risk sharing disappears completely. In numerical examples, we illustrate the complex impact of uncertainty increase and end-of-period load factor improvement on the optimal sharing rule, and the relatively simple impact on total utilities level.

Key words

Airport-Airline Vertical Relationship, Load Factor Guarantee Mechanism, Double Hidden Action/Moral Hazard, Risk Sharing, Stochastic Dynamic Programming

JEL Classification Numbers:

C61,C73,D82,D86,L14,L93

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1 Introduction

Faced with revenue and profit level fluctuations, some airports (local governments) and airlines serving the airports are forming vertical contractual relationship to share their risk and stabilize their financial conditions so that air transport services by those airlines to/from the airports could be created or kept.

Hihara (2008) and Hihara (2010) reports that Noto Airport, one of rural airports in Japan, agreed on contract with one airline group to share the demand fluctuation risk so as to derive the commitment of airline's service to the airport. They used load factor as a key indicator. So the mechanism of the contract is called load factor guarantee mechanism. Also Hihara (2010) analyzed by using incomplete contract framework to show that when such vertical risk sharing contract satisfy proper conditions, such contract can overcome the under-effort problem and improve the utilities levels of both parties.

2 Past Literature

Risk sharing study dates back to Borch (1962). He showed that under pure risk sharing situation without any efforts, the optimal sharing rule stipulates that the ratio of marginal utility of two participating parties in different state are constant and the results hold for any probability distribution.

The risk sharing is also the subject of moral hazard analysis, since the principal is paying to the agent based on the realization of uncertain state. The essence of risk sharing is to share the outcome depend on the realization of contingencies by paying from one party to the other(s) based on some rule. The moral hazard case, the effort of the agent is involved and the principal is optimizing his utilities by controlling payment and also agent's efforts indirectly through the payment structure.

There are two approaches to model the uncertainty in the moral hazard situation. First is that random variable with some probability distribution is assigned to the state itself. This approach dates back to Mirrlees(for example, Mirrlees (1979)) and Rogerson (1985) prove that under this settings, the first order approach is permissible if and only if monotone likelihood ratio condition (MLRC) and the convexity of distribution function condition (CDFC) are satisfied for the probability function with the condition of the agent's action.

But remember the first approach is not a dynamic approach. The random variable is assigned to state contingency and the maximization is about all the possible outcomes at one time only.

The second approach is that the difference of the state is modeled by the stochastic differential equation with the usual Wiener process. This is pioneered by Holmstrom and Milgrom (1987). They showed that if the agent has great action space, the optimal reward contract is simple linear function of the final state (But they did not prove the sufficient condition).

Then Schaettler and Sung (1993) proved the general cases with necessary and sufficient conditions in the second approach case. Also Schaettler and Sung (1997) studied the connection between discrete models and continuous model in the second approach case.

Mueller (1996) and Mueller (1998), based on these studies, showed that if the principal can see the agent action, in the first best situation according to his words, the linear optimal contract is structured by the risk aversion parameters of both principal and agent.

These literatures are about the traditional moral hazard situation where one principal pay to one agent for his efforts that the principal cannot observe. The efforts are performed only by agent.

If both principal and agent are making efforts, which cannot be observed from the other party, the situation is so-called double moral hazard problem. In the first approach case, some authors are analyzing the double moral hazard problems. The recent examples are Bhattacharyya and Lafontaine (1995) and Kim and Wang (1998a). To our knowledge, there are no literatures yet to model the double moral hazard situation between airport and airline using the stochastic differential equation.

In the air transport studies, there are a number of studies on vertical relationship between airport and airline. Recent examples are Oum and Fu (2009), Barbot (2009), Feng et al. (2010),Zhang et al. (2010). The airport-airline vertical relationship could contain some double moral hazard problems, since one cannot directly observe the other 's efforts in the relationship.

To our knowledge, there are no literatures yet to model the double moral hazard situation between airport and airline, in the air transport economics field, using the stochastic differential equation.

In this study, we use the second approach (stochastic differential equation with Weiner process) to analyze the double hidden action/moral hazard problems in airport-airline vertical relationships.

3 Double Hidden Action/ Moral Hazard Situation

As stated in Hihara (2010), airport(=AP) and airline(=AL) are engaging in a joint-venture type project, in which both AP and AL are independently making efforts to provide air transport service to the passengers using the air route to/from the airport. We believe air transport service at one airport cannot be provided by airline or airport alone. Only the combination of airline side and airport side can provide the service to passengers / cargo service users.

In this sense, airport's service and airline's service are closely connected with each other and they are not just a buddle of two different services. For example, if airport make more effort to improve the quality of service, this affects the quality of airline and enhance the contribution of airline's service to the joint-venture type project.

Usually in moral hazard problem, only one party's efforts cannot be seen from the other party. But in the case of airport-airline relationship, neither of the parties' efforts can be seen from the other parties. Hence the double moral hazard situation.

Here we try to use the stochastic differential equation approach to model the double moral hazard situation. Namely neither airport nor airline can see the efforts of the other side. The contingent variable is according to the usual stochastic differential equation as in the single moral hazard studies.

4 The Model

The model is based on the single moral hazard situation of Schaettler and Sung (1993, 1997) with modification to the AP-AL joint-venture type project situation. As a usual situation, two parties (in our case, airport and airline, in simple moral hazard case, principal and agent) agree at time 0 on a certain contract characterized by a salary S among them which is payable at time 1 and satisfy the parties ' reservation utility constraint. The salary or sharing rule $S = S(X)$ is random via dependence on the outcome of a stochastic process X, the common observable among the parties.

Here we use the outcome or situation variable $X(t)$ for load factor during the contracting period. The effort of airport is u_P and that of airline is u_L.

4.1 Settings

For notation and settings we basically follow those of Schaettler and Sung (1993) with modification to airport and airline relationship and important simplification along with Mueller (1996).

As usual, at time 0, airport and airline agree on a sharing rule: $S : C \to R$ which specifies a payment between them at time 1. The sharing rule may depend on a stochastic load factor process X defined on the interval $[0,1]$, which is publicly observable.

4.2 Double Hidden Action/Moral Hazard Model for De-centralized Decision Making Environment

Here we construct a de-centralized utility maximization model. That is airport and airline are performing independent utility maximizations subject to the other's being maximizing its own utilities while sharing the same stochastic control process by joint control production function. In this case, the double moral hazard problem becomes as follows[1];

[1] In other model of past literature, principal maximizes the total of both principal's utility and agent's. Then principal maximizes the total utility by choosing sharing rule, agent's effort and principal's effort. The notion that principal may not observe its own effort might to some extent lack reality. In our view, the assumption that the principal cannot see the agent's effort but can see its own effort might have more reality. See the problem (3.3) of Kim and Wang (1998a), for example.

$$U_P = -\exp\{-R\{PX(1) + F_P - S(X) - \int_0^1 c_P(u_P)dt\}\} \tag{1}$$

$$U_L = -\exp\{-r\{LX(1) + F_L + S(X) - \int_0^1 c_L(u_L)dt\}\} \tag{2}$$

$$dX(t) = f(u_P, u_L)dt + \sigma dB \tag{3}$$

These are assumptions we make in our model. They are consistent with those in Schaettler and Sung (1993) and Mueller (1996), with the revision to adjust double moral hazard situation. We only stipulate the revised aspects to airport-airline vertical relationship contract.

$t \in \mathbf{R}[0,1]$: time during the period the contract between airport and airline covers
Ω: space $C \in \mathbf{R}[0,1]$ of continuous functions on interval $[0,1]$ with value in $\mathbf{R}$
W_t: coordinated process on Ω, i.e., $W_t(\omega) = \omega(t)$ for $\omega \in \Omega$
$\mathscr{F}_t^0$: the filtration generated by W until time t
P:Weiner measure on (Ω, W_t^0)
If we define $\mathscr{F}_t$ to be the augmentation of $\mathscr{F}_t^0$ by all null sets of $\mathscr{F}_1^0$, then the filtration $\mathscr{F}_t$ is continuous and the coordinated process $(W_t, \mathscr{F}_t)$ is a Weiner process on the probability space, $(\Omega, \mathscr{F}_1, P)$.
$X(t)$: the load factor of the air transport routes to/from airport at time t during the contract period
U_P: the utility of airport
U_L: the utility of airline
$R \in \mathbf{R}_+$: risk averse parameter of airport
$r \in \mathbf{R}_+$: risk averse parameter of airline
P: parameter to connect load factor at the year end to the revenues of airport
L: parameter to connect load factor at the year end to the revenues of airline
F_P: the portion of revenues, if any, from the source that does not have any connection with load factor, such as fixed income from leasing contracts of terminal building
F_L: the portion of revenues, if any, from the source that does not have any connection with load factor, such as fixed income from advertisement fee of aircraft body painting
$S(X)$: the sharing rule as a payment, from airport to airline (+) and from airline to airport (-), as a function of load factor, X
u_P: the effort of airport (Efforts are represented by the class $\mathscr{U}$ of all $\mathscr{F}_t$-predictable process u_P in some control set $U \in \mathbf{R}$.)
u_L: the effort of airline (Efforts are represented by the class $\mathscr{U}$ of all $\mathscr{F}_t$-predictable

process u_L in some control set $U \in \mathbf{R}$.)
c_P: the cost of effort of airport. It is assumed to be convex function ($c_P' > 0, c_P'' \geq 0$) and not directly dependent on time t or load factor $X(t)$.
c_L: the cost of effort of airline. It is assumed to be convex function ($c_L' > 0, c_L'' \geq 0$) and not directly dependent on time t or load factor $X(t)$.
$f(u_P, u_L)$: the production function converting efforts of airport and airline to the outcome of load factor. It is assumed to be concave ($f' > 0, f'' \leq 0$) and not directly dependent on time t or load factor $X(t)$.
σ: diffusion rate. Here we assume constant diffusion rate.
B: one-dimensional Brownian motion
$\mathscr{W}^P(t)$: the certainty equivalence of airport at time t (defined in the appendix)
$\mathscr{W}^L(t)$: the certainty equivalence of airline at time t (defined in the appendix)

Problem A

$$\max_{S^P(X), u_P, u_L} E[-\exp\{-R\{PX(1) + F_P - S^P(X) - \int_0^1 c_P(u_P)dt\}\}] \tag{4}$$

$s.t.$

$$dX(t) = f(u_P, u_L)dt + \sigma dB \tag{3}$$

$$u_L \in \arg\max_{\hat{u}_L} E[-\exp\{-r\{LX(1) + F_L + S^P(X) - \int_0^1 c_L(\hat{u}_L)dt\}\}] \tag{5}$$

$$\max_{S^L(X), u_P, u_L} E[-\exp\{-r\{LX(1) + F_L + S^L(X) - \int_0^1 c_L(u_L)dt\}\}] \tag{6}$$

$s.t.$

$$dX(t) = f(u_P, u_L)dt + \sigma dB \tag{3}$$

$$u_P \in \arg\max_{\hat{u}_P} E[-\exp\{-R\{PX(1) + F_P - S^L(X) - \int_0^1 c_P(\hat{u}_P)dt\}\}] \tag{7}$$

Notice that there are two moral hazard problems in Problem A. One is that AP is principal and AL is agent. The other is the opposite. Also note that both AP and AL make efforts and have own revenues independently. This is the main points of our joint-venture type relationship between AP and AL. What makes our double hidden action/moral hazard model unique is that AP and AL make its own maximizations independently, namely de-centralized maximizations, in contrast with the centralized maximization such as in Kim and Wang (1998b)[2].

Assumption 1

[2] In the usual hidden action/moral hazard problem in our settings, we would have the following problem.

In $f(u_P, u_L)$, u_P and u_L are additively separable and they have no interaction with each other. This implies as follows.

$$\frac{\partial^2 f(u_P, u_L)}{\partial u_P \partial u_L} = 0 \tag{8}$$

Assumption 2
The production function $f(u_P, u_L)$ and cost functions, $c_P(u_P)$ $c_L(u_L)$ are in the

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$$\begin{aligned}
&\max_{S(X),u_L} E[-\exp\{-R\{PX(1)+F_P-S(X)\}\}] \\
&\quad s.t. \\
&\qquad dX(t) = f(u_P,u_L)dt + \sigma dB \\
&\qquad u_L \in \arg\max_{\hat{u}_L} E[-\exp\{-r\{S(X) - \int_0^1 c_L(\hat{u}_L)dt\}\}]
\end{aligned}$$

Notice that only the agent, AL, makes efforts and only the principal has the revenue.

Also if we were in the centralized maximization setting of double moral hazard as in Kim and Wang (1998a) and we moved from non-dynamic to dynamic setting, then we would have the following problem.

$$\begin{aligned}
&\max_{S(X),u_P,u_L} E[PX(1)+F_P-S(X) - \int_0^1 c_P(\hat{u}_P)dt] \\
&\qquad + \lambda E[-\exp\{-r\{S(X) - \int_0^1 c_L(\hat{u}_L)dt\}\}] \\
&\quad s.t. \\
&\qquad dX(t) = f(u_P,u_L)dt + \sigma dB \\
&\qquad u_P \in \arg\max_{\hat{u}_P} E[PX(1)+F_P-S(X) - \int_0^1 c_P(\hat{u}_P)dt] \\
&\qquad u_L \in \arg\max_{\hat{u}_L} E[-\exp\{-r\{S(X) - \int_0^1 c_L(\hat{u}_L)dt\}\}]
\end{aligned}$$

Like our joint-venture type, both parties make independent efforts (u_P, u_L) with cost functions. But only AP has independent revenues$(PX(1)+F_P)$ and AP is maximizing the aggregation of AP's and AL's expected utilities.
The structure that AP is maximizing its effort under the condition of its effort being satisfying IC (Incentive Constrain) in Kim and Wang (1998b) is somewhat unnatural to us. IC means that AP's effort is assumed to be unobservable from AP while AL's effort is assumed to be unobservable from AP, hence the 'Double Moral Hazard' in their model.
In our de-centralized model, AP and AL make independent maximizations for their own expected utility, while what one party cannot observe is only the other party's effort, not its own effort, as can be seen in Problem A.

following relationships. For every $p \in \mathbf{R}$, the function

$$H(p, u_P, u_L) \equiv p f(u_P, u_L) + c_P(u_P) + c_L(u_L)$$

is convex in both u_P and u_L, and has stationary points in both u_P and u_L.

By Assumption 2, we are sure the first order approach (Theorem 4.1 in Schaettler and Sung (1993)) is always valid for the moral hazard problem. By Theorem 4.2 all admissible control u_P and u_L are implemented by the other party using the sharing function S, which is derived by Theorem 4.1 for each control u_P or u_L.

This is because every admissible control is implementable under specific conditions as proven by these theorems in Schaettler and Sung (1993), and we do not have to worry about the situation where we are maximizing over larger sets of control possibilities than actually implementable resulting in one party being unable to implement admissible control, since that control is not optimal within the actual sets of controls.

Proposition 1
Under Assumption 1 and Assumption 2 above, the independent optimal sharing rules in the joint-venture type relationship between AP and AL, both being risk averse, under the de-centralized utility maximizations stated in Problem A, are linear of the final outcome. In this case, part of the drift terms and the diffusion term are weighted by the difference of productivity ratios between the two parities.

Proof (sketch) of Proposition 1

Here we describe only the sketch of the proof. The complete proof is in appendix.

Under Assumption 1, function $f(u_P, u_L)$ can be treated completely separately for u_p and u_L in applying Theorem 3.1, 4.1, and 4.2 for necessary condition and Theorem 5.1 for sufficient conditions in Schaettler and Sung (1993), since there is no interacting relationship between u_P and u_L, implied by the equation (8).

With Assumption 2, all admissible controls are implementable, by Theorem 4.2, through the derived sharing rule from Theorem 4.1. So the first order approach of using the derived sharing rule from Theorem 4.1 is justifiable.

So, for example, in solving equation(5) we can apply these theorems only with respect to u_L, while in solving equation (4), we can apply these theorems only with respect to u_P.

Under this assumption, we can solve these problems by Theorem 3.1 and Theorem 4.1 in Schaettler and Sung (1993). First we use these theorems by solving the maximization problem of equations of (5) and (7) in the conditions part of Problem A. Then with the results, we use again these two theorems to solve the main maximization problem of equations of (4) and (6).

Notice also that all admissible controls, including optimal control u_L^* of the maximization problem of equations of (5) for example, are implementable with Theorem 4.2 under Assumption 2. In this case, with Theorem 5.1, the derived optimal control u_L^* and its entailing sharing rule $S^P(X)$, which are derived by Theorem 3.1 and Theorem 4.1 in maximizing equations of (5), are indeed the optimal control and sharing rule from the other party's maximization (4).

Then we can make optimization of (4) by choosing optimal control of u_P^* and its entailing optimal control $S^{P*}(X)$ by applying Theorem 3.1 nod 4.1 to this maximization again.

Solving the maximization of (6) subject to (3) and (7) have the same processes.

By doing algebra in all the derivations above, we get the following.

$$\begin{aligned} S^{P*}(X) =& PX(1) - LX(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P-L)X(0)\} \\ & - \int_0^1 c_P(u_P^*)dt - \frac{R}{2}\int_0^1 (P-L)^2\sigma^2 dt \\ & - \int_0^1 (P-L)\sigma dB \end{aligned} \tag{9}$$

$$\begin{aligned} S^{L*}(X) =& PX(1) - LX(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P-L)X(0)\} \\ & + \int_0^1 c_L(u_L^*)dt + \frac{r}{2}\int_0^1 (L-P)^2\sigma^2 dt \\ & + \int_0^1 (L-P)\sigma dB \end{aligned} \tag{10}$$

If the integrations in these equations are executed, we have the followings.

$$S^{P*}(X) = (P-L)(1-\sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} - c_P(u_P^*) - \frac{R}{2}(P-L)^2\sigma^2 \tag{11}$$

$$S^{L*}(X) = (P-L)(1-\sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} + c_L(u_L^*) + \frac{r}{2}(L-P)^2\sigma^2 \tag{12}$$

In this way, we can clearly see that the derived optimal sharing rules are linear function of the final state, which is the load factor at the end of the contract period, $X(1)$.

In deriving these results, we have the following relationships.

$$\frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*, u_L)}{\partial u_P}} = P \tag{13}$$

$$\frac{c'_L(u_L^*)}{\frac{\partial f(u_P, u_L^*)}{\partial u_L}} = L \tag{14}$$

Observe that P can be considered the productivity ratios consisting of ratio of marginal cost to marginal production. The same could be considered for L in the above.

By seeing the results in the equations(9), (10), (13) and (14), part of the drift terms and the diffusion term are weighted by the difference of productivity ratios between the two parities.

These prove the Proposition 1[3].

[3]Notice that in our model AP and AL are both risk averse and the optimal sharing rules are linear in the final outcome in dynamic problem setting. This contrasts with the other double hidden action/moral hazard models ’ studies. In Kim and Wang (1998a), for example, only agent is risk averse and principal is risk neutral and their findings are that the optimal contracts do not approach the linear contract as agent ’s risk aversion approach zero in non-dynamic, one time setting. In Romano(1994) and Bhattachryya and Lafontaine(1995), both principal and agent are risk neutral and they show that there always exists a linear contract, among other possible optimal contracts, that implements a second-best outcome in non-dynamic, one time setting.

By putting these results back in the utility functions of (1) and (2) for each party and making arrangements, then we get the following.

$$U_P^* = -\exp\{-R\{LX(1) + \frac{1}{2}\{F_P + F_L + \mathscr{W}^P(0) - \mathscr{W}^L(0) + (P-L)X(0)\} + \frac{R}{2}\int_0^1 (P-L)^2\sigma^2 dt + \int_0^1 (P-L)\sigma dB\}\} \quad (15)$$

$$U_L^* = -\exp\{-r\{PX(1) + \frac{1}{2}\{F_P + F_L + \mathscr{W}^L(0) - \mathscr{W}^P(0) + (L-P)X(0)\} + \frac{r}{2}\int_0^1 (L-P)^2\sigma^2 dt + \int_0^1 (L-P)\sigma dB\}\} \quad (16)$$

In this form, we can see the sharing rule more clearly. The actual effort cost compensation part has already disappeared, because of the nature of actual compensation for the actual expense.

We recall that $PX(1) + F_P$ and $LX(1) + F_L$ are independent revenue terms for airport and airline, the load-factor dependent part and the part independent of load factor respectively. So at the optimal controls, the utility functions consist of the revenue part at the end of the period dependent on the level of the other party, namely $LX(1)$ for U_P for example.

$\frac{1}{2}\{F_P + F_L\}$ is an averaged part of revenue independent of load factor $X(t)$ common to both parties. $\mathscr{W}^P(0) - \mathscr{W}^L(0) + (P-L)X(0)$ could be thought as adjustment parts from the level of the other side's, i.e., for U_P the adjustment parts are the difference between $\mathscr{W}^P$ and $\mathscr{W}^L$ measured from the level from $\mathscr{W}^L$, for example.

The diffusion part (the part related to dB) and the drift part (the part related to dt) are the risk sharing parts, since the former is the compensation error part for this risky reward with zero expectation ($E[dB] = 0$), and the latter is the risk premium for the risky project. The diffusion part is the compensation error in that the reward is based on the observable outcome information, in our model load factor, $X(t)$, rather than efforts, u_P or u_L.

For the last terms of equations of (15) and (16), the term in parentheses, $(P-L)$ or $(L-P)$, are constant. So the last term in equations of (15) and (16) are linear equation of final results, $X(1)$.

The drift term and diffusion term are both weighted by $(P-L)$, which are productivity ratio between the two parties by equations (13) and (14) in addition to volatility(σ) or variance(σ^2) of the uncertainty.

If we do the (stochastic) integrations in the equations (15) and (16), we get the followings.

$$\begin{aligned} U_P^* = -\exp\{-R\{\{L+(P-L)\sigma\}X(1) \\ +\frac{1}{2}\{F_P+F_L+\mathscr{W}^P(0)-\mathscr{W}^L(0)+(P-L)(1-2\sigma)X(0)\} \\ +\frac{R}{2}(P-L)^2\sigma^2\}\} \end{aligned} \tag{17}$$

$$\begin{aligned} U_L^* = -\exp\{-r\{\{P+(L-P)\sigma\}X(1) \\ +\frac{1}{2}\{F_P+F_L+\mathscr{W}^L(0)-\mathscr{W}^P(0)+(L-P)(1-2\sigma)X(0)\} \\ +\frac{r}{2}(L-P)^2\sigma^2\}\} \end{aligned} \tag{18}$$

In this form too, we can confirm that the optimal sharing rules under the decentralized maximizations stated in Problem A are linear function of the final outcome. Also from the equations (15) and (16) with equations (13) and (14), we can confirm that the drift and diffusion terms are weighted by the difference of productivity ratios between the two parities.

4.3 The Agreeable Sharing Rule

The derived S^{P*} and S^{L*} above are not identical. So it is not possible for airport and airline to actually agree on a sharing rule with this difference remained. We need some kind of framework to structure the agreement of two parties. So we introduce the following Assumption 3.

Assumption 3

In so far as for every number $\varepsilon \in \mathbf{R}_+$, there exist numbers, $\delta_1 \in \mathbf{R}_+$, $a_P \in \mathbf{R}$, $\delta_2 \in \mathbf{R}_+$ and $a_L \in \mathbf{R}$ such that

$|S^{L*}(r) - S^{P*}(R)| < \varepsilon$ and $0 < |R - a_P| < \delta_1$ and $0 < |r - a_L| < \delta_2$,

(namely, $|S^{L*}(r) - S^{P*}(R)| \to 0$ as $R \to a_P$ and $r \to a_L$),

airport and airline can agree on a single sharing rule S^*, which is defined as follows.

$S^{P*}(R) \to S^*$ as $R \to a_P$
$S^{L*}(r) \to S^*$ as $r \to a_L$

We call this S^* as an agreeable sharing rule.

Under Assumption 3, we have the framework for airport and airline to reach a single agreement out of the de-centralized utility maximizations in Problem A above. That is, if their risk aversions approach to some fixed numbers asymptotically and the difference of $|S^{P*}(R) - S^{L*}(r)|$ approaches to zero with them, then the two parties can reach a risk sharing rule contract which is optimal to both parties under the de-centralized utility maximizations in Problem A.

Next is the assumption about the cost differences between airport and airline in the equations (11) and (12).

Assumption 4
We further assume that the optimal efforts of both airport and airline are costless or negligibly minuscule. This means as follows.

$$c_P(u_P^*) = c_L(u_L^*) = 0 \ \ (c_P(u_P^*) \approx 0, \ c_L(u_L^*) \approx 0) \tag{19}$$

Assumption 4 is a stringent assumption. By Assumption 2 and assumptions on c_P, c_L and $f(u_P, u_L)$, we have unique optimal effort levels, u_P^* and u_L^*. The costs of both parties' optimal effort levels are negligible by Assumption 4. Here we are not trying to state this assumption is plausible. But rather we try to clarify that in our de-centralized maximization model, the cost terms are very important to satisfy Assumption 3. As can be easily checked from the equations (11) and (12), only with Assumption 4, we have a possibility for the agreeable sharing rule defined in Assumption 3. In other words, unless Assumption 4 is satisfied, there is no chance for the two parties to agree on a single contract in our model even with the agreement framework of Assumption 3. In other words, unless Assumption 4 is satisfied, there is no chance for the two parties to agree on a single contract in our model.

Proposition 2
If both parties' risk aversion parameters (R and r) approach asymptotically to zero under Assumption 3 and Assumption 4, airport and airline can agree on a sharing rule, which is derived as optimal sharing rules out of the de-centralized utility maximizations in Problem A and is the linear function of the final state of load factor. In this case, the agreeable optimal risk sharing rule S^* is as follows.

$$S^* = (P-L)(1-\sigma)X(1) \\ + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} \quad (20)$$

Proof of Proposition 2
When both parties' risk aversion parameters asymptotically approach to zero, this means that $R \to 0$ and $r \to 0$ ($a_P \equiv 0$ and $a_L \equiv 0$ in Assumption 3)[4]. With Assumption 4 and from equations (11) and (12), we can easily derive S^* as in the equation (20).

With the result, $|S^{L*}(r) - S^{P*}(R)| \to 0$. Hence with Assumption 3, airport and airline can agree on S^* as an optimal risk sharing rule since S^{P*} and S^{L*} are optimal for airport and airline respectively.

Q.E.D.

This result is consistent to a certain degree with Romano (1994) and Bhattacharyya and Lafontaine (1995). Although in a different modeling framework for double hidden action/moral hazard problem (i.e., non-dynamic setting), they all show that there always exists a linear contract, which implements the second best (i.e., under double moral hazard) effort levels when both principal and agent are risk neutral.

However, our result contrasts Kim and Wang (1998a). They show that in non-dynamic setting, the optimal non-linear unique sharing rule under double moral

[4] Notice that risk parameters (R and r) being approaching asymptotically zero is not necessarily equivalent to AP and AL becoming risk neutral. Risk aversion parameters being zero ($R = r = 0$) in CARA utility functions ($U = -exp(-iy), i = \{R, r\}$) means utility functions are all constant ($U = -1$), which is a special, more strict case of risk neutrality in general.

hazard with risk neutral principal and risk averse agent does not approach to the linear contract as the agent 's risk aversion approaches to zero.

In our case, Proposition 2 shows that in dynamic setting, two parties can agree on an optimal linear contract under double moral hazard in our decentralized utility maximizations if both parties' risk aversion parameters asymptotically become zero and Assumption 4 is satisfied.[5]

The equation (20) shows that the slope of the linear contract is the product of the two parts. One is $(P-L)$, which is the difference of productivity of both parties, which can be seen from the equations (13) and (14)[6]. The other is the uncertainty level measured as $(1-\sigma)$. The slope is the procuct of the two parts.

This means that if both parties 'productivities are equal, $P-L=0$, or if the diffusion rate is unity, $1-\sigma=0$, then the slope is zero. So both parties are equal-footing in productivity, then the compensation part dependent on the realized final outcome of the optimal sharing rule disappears.

Also if the uncertainty in the project indicated by the diffusion rate of the Weiner process in our model is unity, then the compensation error part related to the realized final outcome, $-(P-L)\sigma X(1)$, of the optimal sharing rule just cancel out the productivity difference compensation part elated to the realized final outcome, $(P-L)X(1)$, resulting in the zero slope of the linear function of realized final outcome.

In both cases, the remaining optimal sharing rule is just the utility level adjustments consisting only of parts not related to the realized final outcome $X(1)$.

In other cases, the sign of the slope of the term related the realized final outcome depends on the signs of the two pars, the productivity differences $(P-L)$ and uncertainty level measured as $(1-\sigma)$. If $(P-L)>0$ and $(1-\sigma)>0$, and, $(P-L)<0$ and $(1-\sigma)<0$, the slope is plus. On the other hand, ff $(P-L)>0$ and $(1-\sigma)<0$, and, $(P-L)<0$ and $(1-\sigma)>0$, the slope is minus.

[5]On the other hand, Kim and Wang (1998a) showed that under double moral hazard situation, on their model, the optimal sharing rule between risk neutral principal and risk averse agent is not linear contract and that linear contract is not a limiting contract as agent's risk aversion approaches to zero. In our case, the optimal contracts are linear contract and as both parties' risk aversions asymptotically approach to zero, they can agree on a limiting optimal linear contract.

[6]In non-dynamic double moral hazard setting for risk neutral principal and agent case, the slope of the linear optimal sharing rule is the agent 's productivity.

As can be seen from these explanations, the slope of the linear function of the agreeable optimal sharing rule is the product made by the productivity difference $(P-L)$ and the uncertainty level index $(1-\sigma)$.

Based on the contents of Proposition 1 and Proposition 2, we can construct Proposition 3.

Proposition 3
If we assume perfect symmetry, i.e., everything being same between the two contracting parties, in addition to Assumption 1 through 4, the risk sharing rule disappears completely in the double moral hazard situation in this setting.

Proof of Proposition 3
Under the perfect symmetry, everything between two parties is same. This means that among other things

$P = L,$
$R = r,$
$F_P = F_L$
$\mathscr{W}^P(0) = \mathscr{W}^L(0).$

With these put in the equation (20), we have the following results

$S^* \equiv 0$

Hence the risk sharing rule disappears completely with the additional perfect symmetry assumption.

Q.E.D.

5 Numerical Illustration

5.1 Concrete Settings

Here we illustrate some of the structures we used in the preceding chapters. First we show the concrete results for modeling $X(t)$ for load factor during the contract period.

We assume the following concrete structures of functions[7];

$$
\begin{aligned}
f(u_P, u_L) &= u_P + u_L \\
c_P(u_P) &= \frac{k_P}{2}{u_P}^2 - \frac{P^2}{2k_P} \\
c_L(u_L) &= \frac{k_L}{2}{u_L}^2 - \frac{L^2}{2k_L} \\
dX(t) &= (u_P + u_L)dt + \sigma dB
\end{aligned}
$$

We further assume

$X(0) = 0.6886$,
$\sigma = 0.056$,
$u_P = u_L = 0.005$ or 0.025.

The 5 sample paths of $X(t)$ for both $u_P = u_L = 0.025$ (left graph) and 0.005 (right graph) cases are shown in the Figure 1.

(Figure 1 is about here.)

Also we simulated the X(1) results with 10,000 iterations. The simulated results are the Figure 2.

(Figure 2 is about here.)

The mean and the standard deviation of the simulated $X(1)$ for $u_P = u_L = 0.005$ is;
mean $\hat{\mu} = 0.6411$
standard deviation $\hat{\sigma} = 0.0266$.

The mean and the standard deviation of the simulated $X(1)$ for $u_P = u_L = 0.025$ is;
mean $\hat{\mu} = 0.7013$
standard deviation $\hat{\sigma} = 0.0194$.

[7]The cost function could be negative in our settings like Schaettler and Sung (1993) and Kim and Wang (1998a). This could be explained if both parties are receiving some financial aid from national government, for example. This aid could be lump sum or volume based for the purpose of cost alleviation. In our example, we assume lump some financial aid to reduce the two parties' costs. This could be the case, if local government, which owns and leases land for both airline (office premises) and airport (parking space) surrounding area of the airport, decreases its lease premium by fixed amount, the costs of airline and airport decrease accordingly by the same lump sum amount.

We can see the improvement of final load factor at the end of the period with more efforts of both parties even if the initial load factor is the same.

5.2 Optimal Levels

By the assumptions and these numbers with equations (13) and (14), we can derive the optimal efforts levels and optimal sharing rules.

$$u_P^* = \frac{P}{k_P} \tag{21}$$

$$u_L^* = \frac{L}{k_L} \tag{22}$$

Notice that these solutions satisfy Assumption 4. Namely, we have

$$c_P(u_P^*) = \frac{k_P}{2}\left(\frac{P}{k_P}\right)^2 - \frac{P^2}{2k_P} = 0$$
$$c_L(u_L^*) = \frac{k_L}{2}\left(\frac{L}{k_L}\right)^2 - \frac{L^2}{2k_L} = 0.$$

With the result of the optimal effort levels, we have optimal sharing rules. These are the equations (11) and (12) with concrete parameter values.

$$S^{P*}(X) = (P-L)(1-\sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} - \frac{R}{2}(P-L)^2\sigma^2 \tag{23}$$

$$S^{L*}(X) = (P-L)(1-\sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} + \frac{r}{2}(L-P)^2\sigma^2 \tag{24}$$

If both parties' risk aversions R and r asymptotically approach to 0, both can agree, based on Assumption 3, on the optimal sharing rule of the equation (20) with concrete parameter values as follows.

$$
\begin{aligned}
S^* = {} & (P-L)(1-\sigma)X(1) \\
& + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\}
\end{aligned} \tag{25}
$$

In Figure 3, we show that $|S^{L*} - S^{P*}| \to 0$ as $R \to 0$ and $r \to 0$ under the following numerical settings.

$P = 0.055;\ k_P = 11$
$u_P^* = \frac{P}{k_P} = 0.005$
$L = 0.011;\ k_L = 2.2$
$u_L^* = \frac{L}{k_L} = 0.005$
$X(1) \sim N(0.6411, 0.0266^2)$
$\mathscr{W}^P(0) = 1.1; \mathscr{W}^L(0) = 1$
$F_P = 11; F_L = 10$
$X(0) = 0.668$

(Figure 3 is about here.)

This means airport and airline can agree on an optimal sharing contract S^* with concrete paremater values as their risk aversion parameters asymptotically become zero.

5.3 Diffusion Rate Effects

As the equation (20) and (25) indicate, the optimal agreeable contract , which is linear function of the final state, has the slope of the product of both parties ’ productivity difference $P-L$ and uncertainty (diffusion rate) level index $1-\sigma$.

The productivity difference part of the slope is the part to level the productivity difference dependent on the final state through the sharing rule. By this, the optimal agreeable sharing rule adjusts the revenue differences between two parties generated by the productivity differences even if they share the common final outcome.

The uncertainty (diffusion rate) level index part of the slope is the part to adjust the compensation error after adjusting the productivity differences. Compensation error adjustment is necessary, since the sharing is based on the uncertain outcome not on the efforts actually performed. In our model, the productivity difference adjustment part has the same term $P-L$ in the opposite sign. So the compensation error part in the slope is the form of$1-\sigma$, not just σ.

If the diffusion is zero, then no uncertainty entails. Only the productivity differences matter. So the slope is just the productivity difference adjustment, $P-L$. If the diffusion is unity $\sigma = 1$ in our model, then the compensation error adjustment is just the same as the countering productivity difference. So the slope is zero. In this case, the optimal sharing rule has no part with the final outcome $X(1)$. Only the fixed income part F_P and F_L, and the initial condition parts are involved in the optimal sharing rule. Therefore the optimal agreeable sharing rule has only the attributes of the two parties that are not dependent on the final outcome $X(1)$.

In Figure 4, we illustrate the optimal sharing contract S^* linear in $X(1)$, the load factor at the end of the contract period with the impact of the diffusion rate σ.

(Figure 4 is about here.)

As diffusion rate σ increases, the slope of optimal contract S^* decreases. This mean that as the volatility of Brownian motion increases, the compensation error necessary part in the optimal sharing rule becomes large. If the compensation error overwhelms the positive impact of load factor $X(1)$ impact, then the slope of the linear sharing rule becomes negative.

More specifically, if diffusion rate σ is unity, then the slope is zero. This means that the compensation error part just negate the productivity difference adjustment in the slope, if $\sigma = 1$. In this case, the optimal sharing rule has only the parts that do not depend on the load factor at the end $X(1)$, as we explained above.

If diffusion rate σ is less than 1, the effect of load factor $X(1)$ on the sharing rule is greater than the effect of compensation error, which is the opposite sign of the effect of load factor, resulting from the diffusion rate σ. So the slope of $S^*(X(1))$ is positive. The contradicting effect between that with $X(1)$ and that with σ can be seen from the equation (9) and (10). But on the other hand, if the diffusion rate σ is greater than 1, the negative compensation error effect overwhelms the positive effect of load factor improvement, hence the negative slope of the optimal sharing contract.

Under the agreeable sharing rule, we have the optimal utility functions.

$$
\begin{aligned}
U_P^*(X(1),\sigma) = &-\exp\{-R\{\{L+(P-L)\sigma\}X(1) \\
&+\frac{1}{2}\{F_P+F_L+\mathscr{W}^P(0)-\mathscr{W}^L(0)+(P-L)(1-2\sigma)X(0)\}
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
U_L^*(X(1),\sigma) = &-\exp\{-r\{\{P+(L-P)\sigma\}X(1) \\
&+\frac{1}{2}\{F_P+F_L+\mathscr{W}^L(0)-\mathscr{W}^P(0)+(L-P)(1-2\sigma)X(0)\}
\end{aligned}
\tag{27}
$$

In Figure 5, we show the visualized utility level with the final outcome of load factor $X(1)$ and the diffusion rate σ, setting $R=r=10^{-4}$.

(Figure 5 is about here.)

As we can see from the left graph of Figure 5, the impact of increased diffusion rate on U^{P*} is relatively simple. As diffusion rate increases, so does the impact of $X(1)$, i.e., the slope $\frac{\partial U^{P*}(X(1),\sigma)}{\partial X(1)}$. The slope is always positive. The intercept $U^{P*}(0,\sigma)$, on the other hand, decreases as σ increases.

In the right graph of Figure 5, the impact on U^{L*} is relatively complex in our numerical example. As σ increases, the impact of $X(1)$, i.e., the slope $\frac{\partial U^{L*}(X(1),\sigma)}{\partial X(1)}$, is decreasing. If $\sigma \geq 1.25$, the slope becomes negative and continue to decrease. The intercept $U^{L*}(0,\sigma)$, on the other hand, increases as σ increases.

Now we turn to the total utilities level for both AP and AL. With the above results, we can gauge the impact of increased uncertainty and the end-of-period load factor improvement on the total utilities level. To specify these impacts, we can calculate the derivatives of the total utilities with respect to σ and $X(1)$.

$$U_A^* = U_P^* + U_L^*$$
$$\frac{\partial U_A^*}{\partial X(1)} = U_P^*[-R\{\{L + (P - L)\sigma\}\}] + U_L^*[-r\{\{L + (P - L)\sigma\}\}] > 0$$
$$\frac{\partial U_A^*}{\partial \sigma} = U_P^*(-R)\{(P - L)(X(1) - X(0)) + U_L^*(-r)\{(L - P)(X(1) - X(0))\} \approx 0$$

Notice that since $R \approx 0$ and $r \approx 0$, we have the last results.

Figure 6 is the graph for $U_A^* = U_P^* + U_L^*$ with $X(1)$ and σ. The slope of U_A^* with respect to $X(1)$ is positive and the slope with respect to σ is almost zero.

(Figure 6 is about here.)

The impact of uncertainty increase is almost zero with some perturbations, since we already assuming that the risk averse parameters approach asymptotically to zero. The impact of the final outcome improvement, on the other hand, has a positive effedct on the total utilities level with some perturbations.

Because of risk averse parameters approaching to zero, the conflicting effects caused by diffusion rate changes on each party's utility through the optimal agreeable sharing rule as depicted in Figure 5 are almost smoothed out in the total utilities level as depicted in Figure 6. With some perturbations, σ changes have almost no effect on the total utilities level in Figure 6.

6 Concluding Remarks

We show that, under the double hidden action/moral hazard situation of de-centralized utility maximizations based on the stochastic load factor process under additively separable efforts assumption and convexity assumption about the relationship between production and cost function, the optimal sharing structures are the linear function of the final state for both parties. In this case, the drift and diffusion terms are weighted by the difference of productivity between the two parities.

If we further assume that the costs of optimal effort are negligible and also assume both parties' risk aversion parameters approach asymptotically to zero, then we show that both parties can agree on a single optimal contract, which is also a linear function of final load factor.

Our finding is to some degree consistent with some of double hidden action/ moral hazard situation analysis of preceding literatures. Namely, if both parties are risk neutral, the linear function of the end-state is among the optimal sharing rules.

However, our result contrasts with Kim and Wang (1998a). They show, in non-dynamic setting, the optimal non-linear unique sharing rule under double moral hazard with risk neutral principal and risk averse agent does not approach to the linear contract as the agent ’s risk aversion approaches to zero.

We find that in dynamic setting, two parties can agree on an optimal linear contract under double moral hazard in our decentralized utility maximizations if both parties ’risk aversion parameters asymptotically become zero and optimal effort are negligible, as we alread stated.

The optimal agreeable contract, which is linear function of the final state, has the slope of the product of both parties ’productivity difference and uncertainty (diffusion rate) level index.

If the productivities are same between the two parties in our setting, there is no need to adjust their productivity difference by the optimal sharing rule. So the slope is zero. In this case, the optimal sharing rule has no part with the final outcome.

If the diffusion rate is unity in our model, then the compensation error adjustment is just the same as the countering productivity difference adjustment. So the slope is zero. In this case, the optimal sharing rule has no part with the final outcome.

In addition, if everything is symmetric, then the risk sharing rule disappears completely.

With numerical examples, we show the sample paths for load factor processes generated according to underlying Weiner process. The improving effects of effort increases of both parties on the end-of-period load factors are provided. Also we show the convergence to the agreeable sharing rule as both parties' risk averse parameters get asymptotically to zero.

We show that the complex effect of increased diffusion rate on the linear function of the agreeable optimal sharing rule. The impacts on total utilities level are relatively simple. The impacts of diffusion rate increase on total utilities are almost zero. The increase of final outcome has a positive effect on the total utilities level.

As one example of the next steps, we can look into the other double hidden action / moral hazard model of utility maximizations, which is other than our de-centralized model. This direction is in line with the past literature. However, in the stochastic process model, this would require much more mathematical sophistication.

Also we can try to relax the restrictive assumptions on the functions, including f and c_P and c_L. For example, we assume they are not function of time t. This would also require much more rigorous theoretical treatments. Since even in our simple model with restrictive functions, we can show some results, we should try to balance clarity and reality when pursuing in this direction.

In the context of network industries of routes and airports, another important next step could be to try to link the abstract efforts in our model to concrete tools of the aviation and airport industries, such as pricing, routing and frequencies, so that we can more directly see the interaction within the airport and airline vertical relationship in more realistic settings.

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A Appendix A Proof of Proposition 1

As already stated, under Assumption 1, function $f(u_P, u_L)$ can be treated completely separately for u_p and u_L in applying Theorem 3.1 and 4.1 for necessary condition and Theorem 5.1 for sufficient conditions in Schaettler and Sung (1993), since there is no interacting relationship between u_P and u_L, i.e., equation (8). So for example, in solving equation(5) we can apply these theorems only to u_L, while in solving equation (4) we can apply these theorems only to u_P.

We show the proof of the derivation of $S^P(X)$ only, since that of $S^L(X)$ is basically same. Also to avoid the just duplication, we only describe the main part of the derivation in the following equations.

$$\max_{S^P(X),u_P,u_L} E[-\exp\{-R\{PX(1)+F_P-S^P(X)-\int_0^1 c_P(u_P)dt\}\}] \tag{4}$$

$$s.t.$$

$$dX(t) = f(u_P,u_L)dt+\sigma dB \tag{3}$$

$$u_L \in \arg\max_{\hat{u}_L} E[-\exp\{-r\{LX(1)+F_L+S^P(X)-\int_0^1 c_L(\hat{u}_L)dt\}\}] \tag{5}$$

First, we apply Theorem 3.1 and 4.1 in Schaettler and Sung (1993) to the maximization problem of equation (5) under equation (3) holding u_P constant. Notice that in $dX(t)$ we have also u_L.

As usual, we assume the $S^P(X)$ has the following structure.

$$S^P(X) = S_1^P(X) + \int_0^1 \alpha(s,X)ds + \int_0^1 \beta(s,X)dX_s$$

From equation (5), structure of $S^P(X)$ and $X(1) = X(0)+\int_0^1 f(u_P,u_L)dt+\int_0^1 \sigma dB$ with some arguments suppressed, the utility U_L becomes as follows;

$$\begin{aligned} U_L = &- exp\{-r(LX(0)+F_L+S_1^P(X) \\ &-\int_0^1 r(Lf(u_P,u_L)+\alpha+\beta f(u_P,u_L)-c_L(u_L))dt \\ &-\int_0^1 r(L+\beta)\sigma dB\} \end{aligned}$$

By applying Theorem 3.1 in Schaettler and Sung (1993), there exist $\mathscr{F}_t$ -adapted process $\mathscr{V}_t$ and $\nabla\mathscr{V}_t$, $\int_0^1 \mid \nabla\mathscr{V}_t\sigma \mid^2 dt < \infty$ a.e., such that $u_L^* \in \mathscr{U}$ is optimal control if and only if

$$
\begin{aligned}
&\nabla \mathscr{V}_t\{f(u_P,u_L^*) - r\sigma^2(L+\beta)\} \\
&\quad + \mathscr{V}_t\{-r(L+\beta)f(u_P,u_L^*) - r\alpha + rc_L(u_L^*) + \frac{1}{2}r^2\sigma^2(L+\beta)^2\} \\
&\quad \equiv \max_{u_L in \mathscr{U}}\{\nabla \mathscr{V}_t\{f(u_P,u_L) - r\sigma^2(L+\beta)\} \\
&\qquad\quad + \mathscr{V}_t\{-r(L+\beta)f(u_P,u_L) - r\alpha + rc_L(u_L) + \frac{1}{2}r^2\sigma^2(L+\beta)^2\}
\end{aligned}
$$

for almost all $(t,\omega) \in [01] \times \Omega$. Furthermore, the value process $\mathscr{V}_t$ has an Ito differential of the form

$$
\begin{aligned}
\mathscr{V}_t = \mathscr{V}_0 - \int_0^t &[\nabla \mathscr{V}_s\{f(u_P,u_L^*) - r\sigma^2(L+\beta)\} \\
&+ \mathscr{V}_t\{-r(L+\beta)f(u_P,u_L^*) - r\alpha + rc_L(u_L^*) \\
&+ \frac{1}{2}r^2\sigma^2(L+\beta)^2\}]ds + \int_0^t \nabla \mathscr{V}_s dX_s.
\end{aligned}
$$

$\mathscr{V}_t$, $\nabla \mathscr{V}_t$ and $\hat{\nabla} \mathscr{V}_t$ are value functions of stochastic process as defined in Appendix A in Schaettler and Sung (1993).

Using $\hat{\nabla} \mathscr{V}_t \equiv \nabla \mathscr{V}_t - \mathscr{V}_t r(L+\beta)$, we can get the c_L related part as follows;

$$
\hat{\nabla} \mathscr{V}_t f(u_P,u_L) + \mathscr{V}_t rc_L(u_L)
$$

FOC for maximization for this, we have the following;

$$
\frac{\hat{\nabla} \mathscr{V}_t}{r\mathscr{V}_t} = -\frac{c_L'(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}}
$$

As usual, we define the following certainty equivalent value process $\mathscr{W}_t$ as follows;

$$
\mathscr{W}_t = -\frac{1}{r}\ln(-\mathscr{V}_t)
$$

With some arrangements, we get the following equation.

$$
\begin{aligned}
d\mathscr{W}_t = c_L(u_L^*)dt + \frac{r}{2}\big(\frac{\hat{\nabla} \mathscr{V}_t}{r\mathscr{V}_t}\big)^2\sigma^2 dt + \frac{\hat{\nabla} \mathscr{V}_t}{r\mathscr{V}_t}\sigma dB \\
-(\alpha dt + \beta dX) - LdX
\end{aligned}
$$

So we get $S^P(X)$ as follows.

$$
\begin{aligned}
S^P(X) = &-(\mathscr{W}_1^L - \mathscr{W}_0^L) + S_1^P(X) - L(X(1) - X(0)) \\
&+ \int_0^1 c_L(u_L^*)dt + \frac{r}{2}\int_0^1 \left(\frac{c_L'(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}}\right)^2 \sigma^2 dt \\
&+ \int_0^1 \frac{c_L'(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}} \sigma dB
\end{aligned}
$$

Here we use $\mathscr{W}_t^L$ instead of $\mathscr{W}_t$ for notational clarity in the following further derivation of $S^P(X)$.

By the Theorem 5.1 in Schaettler and Sung (1993), this $S^P(X)$ satisfies the sufficient condition of maximizing the equation (4) with holding u_P as a constant.

Rearranging the terms and defining newly $F^P(X)$, α' and β', we get the following.

$$
\begin{aligned}
S^P(X) = &-(\mathscr{W}_1^L - \mathscr{W}_0^L) + S_1^P(X) - L(X(1) - X(0)) \\
&+ \int_0^1 [c_L(u_L^*) + \frac{r}{2}\int_0^1 \left(\frac{c_L'(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}}\right)^2 \sigma^2 - \left(\frac{c_L'(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}}\right) f(u_P,u_L^*)]dt \\
&+ \int_0^1 \frac{c_L'(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}} dX \qquad \text{(A.1)} \\
\equiv\ & F^P(X) + \int_0^1 \alpha' dt + \int_0^1 \beta' dX \qquad \text{(A.2)} \\
F^P(X) \equiv\ & -(\mathscr{W}_1^L - \mathscr{W}_0^L) + S_1^P(X) - L(X(1) - X(0))
\end{aligned}
$$

Now we turn to the maximization of the equation (4) by choosing u_P, which is now a controlling variable rather than a constant. We put $S^P(X)$ in the equation (4), rearrange terms and have the following equation.

$$
\begin{aligned}
U_P = -\exp\{ & -R(PX(0) + F_P - F^P(X)) \\
& + \int_0^1 R\{(\beta' - P) f(u_P, u_L^*) + c_P(u_P) + \alpha'\} dt \\
& + \int_0^1 R(\beta' - P)\sigma dB\}
\end{aligned}
$$

We apply Theorem 3.1 again here. Then there exist $\mathscr{V}_t^P$ and $\nabla \mathscr{V}_t^P$, $\int_0^1 \mid \nabla \mathscr{V}_t^P \sigma \mid^2 dt < \infty$ a.e., such that $u_P^* \in \mathscr{U}$ is optimal control if and only if

$$
\begin{aligned}
& \nabla \mathscr{V}_t^P \{ f(u_P^*, u_L^*) + R\sigma^2(\beta' - P)\} \\
& \quad + \mathscr{V}_t^P \{ R\{(\beta' - P) f(u_P^*, u_L^*) + \alpha' + c_P(u_P^*)\} + \frac{1}{2} R^2 \sigma^2 (\beta' - P)^2\} \\
& \quad \equiv \max_{u_P in \mathscr{U}} \{ \nabla \mathscr{V}_t^P \{ f(u_P, u_L^*) - R\sigma^2(\beta' - P)\} \\
& \qquad + \mathscr{V}_t^P \{ R\{(\beta' - P) f(u_P, u_L^*) + \alpha' + c_P(u_P)\} + \frac{1}{2} R^2 \sigma^2 (\beta' - P)^2\}
\end{aligned}
$$

for almost all $(t, \omega) \in [0,1] \times \Omega$. Furthermore, the value process $\mathscr{V}_t^P$ has an Ito differential of the form

$$
\begin{aligned}
\mathscr{V}_t^P = \mathscr{V}_0^P - \int_0^t [& \nabla \mathscr{V}_s^P \{ f(u_P^*, u_L^*) + R\sigma^2(\beta' - P)\} \\
& + \mathscr{V}_t^P \{ R\{(\beta' - P) f(u_P^*, u_L^*) + \alpha' + c_P(u_P^*)\} \\
& + \frac{1}{2} R^2 \sigma^2 (\beta' - P)^2\}] ds + \int_0^t \nabla \mathscr{V}_s^P dX_s].
\end{aligned}
$$

We can construct the following.

$$
\hat{\nabla} \mathscr{V}_t^P \equiv \nabla \mathscr{V}_t^P + R \mathscr{V}_t^P (\beta' - P) - R \mathscr{V}_t^P \frac{c'{}_L(u_L^*)}{\frac{\partial f(u_P, u_L^*)}{\partial u_L}}
$$

With this, we get the u_P related part as follows;

$$
\hat{\nabla} \mathscr{V}_t^P f(u_P, u_L^*) + \mathscr{V}_t^P R c_P(u_P)
$$

FOC for maximization for this, we have the following;

$$
\frac{\hat{\nabla} \mathscr{V}_t^P}{R \mathscr{V}_t} = -\frac{c_P'(u_P^*)}{\frac{\partial f(u_P^*, u_L^*)}{\partial u_P}}
$$

Here, we define the following certainty equivalent value process $\mathscr{W}_t^P$ as follows;

$$\mathscr{W}_t^P = -\frac{1}{R}\ln(-\mathscr{V}_t^P)$$

With some arrangements, we get the following equation.

$$d\mathscr{W}_t^P = c_P(u_P^*)dt + \frac{R}{2}\{\frac{\hat{\nabla}\mathscr{V}_t^P}{R\mathscr{V}_t} + (\beta' - P)\}^2\sigma^2 dt - \{\frac{\hat{\nabla}\mathscr{V}_t^P}{R\mathscr{V}_t^P} + (\beta' - P)\}\sigma dB \\ - (\alpha' dt + \beta' dX) - PdX$$

By integrating and using $S^P(X) = F^P(X) + \int_0^1 \alpha' dt + \int_0^1 \beta' dX$, we have as follows.

$$\begin{aligned} S^P(X) =& \mathscr{W}_1^P - \mathscr{W}_0^P + F^P(X) + P(X(1) - X(0)) \\ & - \int_0^1 c_P(u_P^*)dt - \frac{R}{2}\int_0^1 \left(\frac{c_L'(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} - \frac{c_P'(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} \right)^2 \sigma^2 dt \\ & + \int_0^1 \left(\frac{c_L'(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} - \frac{c_P'(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} \right) \sigma dB \end{aligned}$$

Notice that the terminal conditions are that $LX(1) + F_L + S_1^P(X) = \mathscr{W}^L(1)$ and $PX(1) + F_P - \{\mathscr{W}^P(1) - \mathscr{W}^P(0) + F^P(X) + P(X(1) - X(0))\} = \mathscr{W}^P(1)$. With these conditions and by expanding $F^P(X)$, we get the following.

$$\begin{aligned} S^P(X) =& PX(1) - LX(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P - L)X(0)\} \\ & - \int_0^1 c_P(u_P^*)dt - \frac{R}{2}\int_0^1 \left(\frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} - \frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} \right)^2 \sigma^2 dt \\ & - \int_0^1 \left(\frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} - \frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} \right) \sigma dB \end{aligned} \tag{A.3}$$

By the same process, we get the following.

$$
\begin{aligned}
S^L(X) =& PX(1) - LX(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P-L)X(0)\} \\
&+ \int_0^1 c_L(u_L^*)dt + \frac{r}{2}\int_0^1 \left(\frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} - \frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} \right)^2 \sigma^2 dt \\
&+ \int_0^1 \left(\frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} - \frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} \right) \sigma dB
\end{aligned} \tag{A.4}
$$

Putting these back in the utility functions of U_P and U_L, the resulting utility functions at the optima are the followings.

$$
\begin{aligned}
U_P^* =& -\exp\{-R\{LX(1) + \frac{1}{2}\{F_P + F_L + \mathscr{W}^P(0) - \mathscr{W}^L(0) + (P-L)X(0)\} \\
&+ \frac{R}{2}\int_0^1 \left(\frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} - \frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} \right)^2 \sigma^2 dt \\
&+ \int_0^1 \left(\frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} - \frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} \right) \sigma dB\}\}
\end{aligned} \tag{A.5}
$$

$$
\begin{aligned}
U_L^* =& -\exp\{-r\{PX(1) + \frac{1}{2}\{F_P + F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P-L)X(0)\} \\
&+ \frac{r}{2}\int_0^1 \left(\frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} - \frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} \right)^2 \sigma^2 dt \\
&+ \int_0^1 \left(\frac{c'_L(u_L^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_L}} - \frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L^*)}{\partial u_P}} \right) \sigma dB\}\}
\end{aligned} \tag{A.6}
$$

Notice that, as we already mentioned, by Theorem 5.1 in Schaettler and Sung (1993), the $S^P(X)$ and $S^L(X)$ above satisfy the sufficient condition of the (relaxed) maximization problems of U_P in the equation (4) and U_L in the equation (6).

During the deriving process of u_L^*, for example, the airport's relaxed problem requires u_L to satisfy the following equation of dynamic programming according to

Theorem 5.1 in Schaettler and Sung (1993).

$$\begin{aligned}0 \equiv & \frac{\partial \mathscr{W}^P(t,X)}{\partial t} + \frac{1}{2}\frac{\partial^2 \mathscr{W}^P(t,X)}{\partial X^2} - \frac{R}{2}\Big(\frac{\partial \mathscr{W}^P(t,X)}{\partial X}\Big)^2\sigma^2 \\ & + \max_{u_L}\Big[\frac{\partial \mathscr{W}^P(t,X)}{\partial X}\{f(u_P,u_L) + R\beta'\sigma^2\}dt \\ & \quad - \big(c_p(u_P) + c_L(u_L) + \frac{R+r}{2}\beta'^2\sigma^2\big)dt\Big]\end{aligned} \tag{A.7}$$

From the maximization for u_L^* and noticing $\beta' = \beta'(t,X)$, we get the following.

$$\frac{\partial \mathscr{W}^P(t,X)}{\partial X} = \frac{c'_L(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_P}} \tag{A.8}$$

The dynamic programming equation above is separable and it is well known how to solve it. Noticing the terminal condition of $\mathscr{W}^P(1) = LX(1) + \frac{1}{2}\{F_P + F_L + \mathscr{W}^P(0) - \mathscr{W}^L(0) + (P-L)X(0)\}$, we get the following.

$$\begin{aligned}\mathscr{W}^P(t,X) =& LX \\ & + \{\frac{R}{2}L^2\sigma^2 - \beta' f(u_P,u_L^*) \\ & - \frac{R-r}{2}\beta'^2\sigma^2 + c_L(u_L^*) + c_P(u_P))\}(t-1) \\ & + \frac{1}{2}\{F_P + F_L + \mathscr{W}^P(0) - \mathscr{W}^L(0) + (P-L)X(0)\}\end{aligned} \tag{A.9}$$

From this, we can derive the following.

$$\frac{\partial \mathscr{W}^P(t,X)}{\partial X} = \frac{c'_L(u_L^*)}{\frac{\partial f(u_P,u_L^*)}{\partial u_L}} = L \tag{A.10}$$

From this result along with the assumptions of convexity of c_L and concavity of f and Assumption 2, the optimal control u_L^* is unique.

Also u_L^* is constant across the period, namely $u_L^*(t) \equiv u_L^*$, since neither c_L nor f depends on time t by assumptions.

The same process can be performed for u_P^*. Then we get the following.

$$\frac{\partial \mathscr{W}^L(t,X)}{\partial X} = \frac{c'_P(u_P^*)}{\frac{\partial f(u_P^*,u_L)}{\partial u_P}} = P \tag{A.11}$$

Therefore the optimal controls, u_P^* and u_L^*, are unique and constant during the period.

Putting these results back in (A.3) and (A.4), we have the following.

$$\begin{aligned} S^P(X) =& PX(1) - LX(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P-L)X(0)\} \\ & - \int_0^1 c_P(u_P^*)dt - \frac{R}{2}\int_0^1 (P-L)^2\sigma^2 dt \\ & - \int_0^1 (P-L)\sigma dB \end{aligned} \tag{A.12}$$

$$\begin{aligned} S^L(X) =& PX(1) - LX(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) - (P-L)X(0)\} \\ & + \int_0^1 c_L(u_L^*)dt + \frac{r}{2}\int_0^1 (L-P)^2\sigma^2 dt \\ & + \int_0^1 (L-P)\sigma dB \end{aligned} \tag{A.13}$$

If the integrations in these equations are executed, we have the followings.

$$\begin{aligned} S^{P*}(X) =& (P-L)(1-\sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} \\ & - c_P(u_P^*) - \frac{R}{2}(P-L)^2\sigma^2 \end{aligned} \tag{A.14}$$

$$\begin{aligned} S^{L*}(X) =& (P-L)(1-\sigma)X(1) + \frac{1}{2}\{F_P - F_L - (\mathscr{W}^P(0) - \mathscr{W}^L(0)) + (P-L)X(0)\} \\ & + c_L(u_L^*) + \frac{r}{2}(L-P)^2\sigma^2 \end{aligned} \tag{A.15}$$

These are exactly the equations (11) and (12) as indicated.
With these results, we have the U_P^* and U_L^* as follows.

$$U_P^* = -\exp\{-R\{LX(1)+\frac{1}{2}\{F_P+F_L+\mathscr{W}^P(0)-\mathscr{W}^L(0)+(P-L)X(0)\} \\ +\frac{R}{2}\int_0^1 (P-L)^2\sigma^2 dt+\int_0^1 (P-L)\sigma dB\}\} \tag{15}$$

$$U_L^* = -\exp\{-r\{PX(1)+\frac{1}{2}\{F_P+F_L+\mathscr{W}^L(0)-\mathscr{W}^P(0)+(L-P)X(0)\} \\ +\frac{r}{2}\int_0^1 (L-P)^2\sigma^2 dt+\int_0^1 (L-P)\sigma dB\}\} \tag{16}$$

By doing the integrations of these equations, we have the followings.

$$U_P^* = -\exp\{-R\{\{L+(P-L)\sigma\}X(1) \\ +\frac{1}{2}\{F_P+F_L+\mathscr{W}^P(0)-\mathscr{W}^L(0)+(P-L)(1-2\sigma)X(0)\} \\ +\frac{R}{2}(P-L)^2\sigma^2\}\} \tag{17}$$

$$U_L^* = -\exp\{-r\{\{P+(L-P)\sigma\}X(1) \\ +\frac{1}{2}\{F_P+F_L+\mathscr{W}^L(0)-\mathscr{W}^P(0)+(L-P)(1-2\sigma)X(0)\} \\ +\frac{r}{2}(L-P)^2\sigma^2\}\} \tag{18}$$

These are exactly the equations (17) and (18) as indicated.

Q.E.D.

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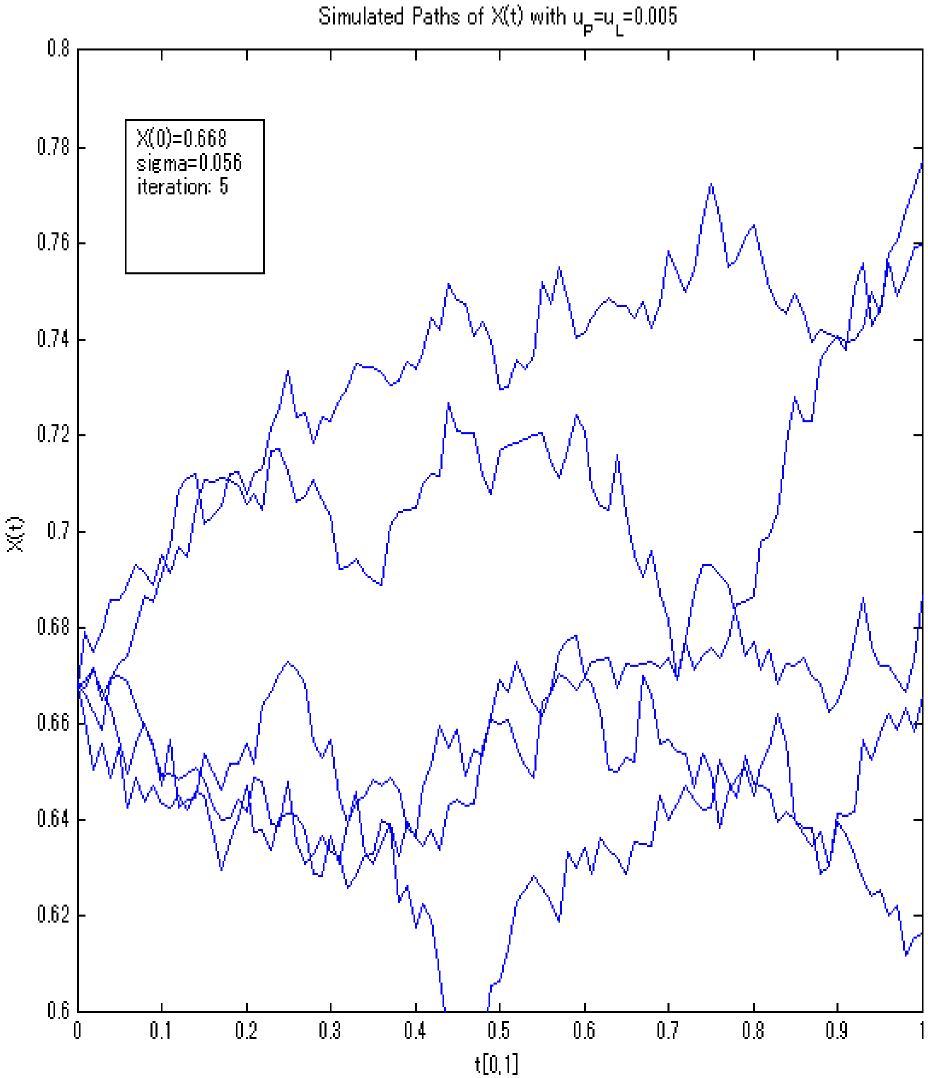
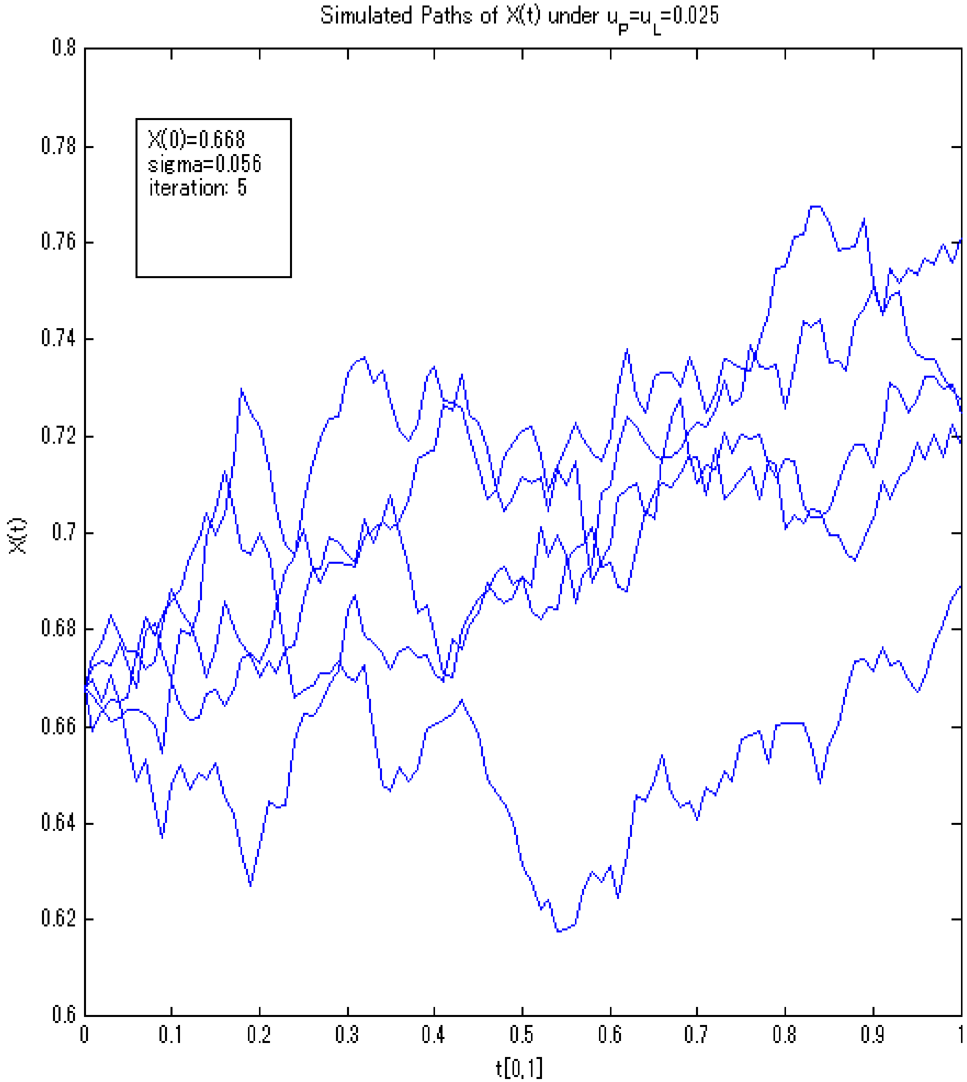
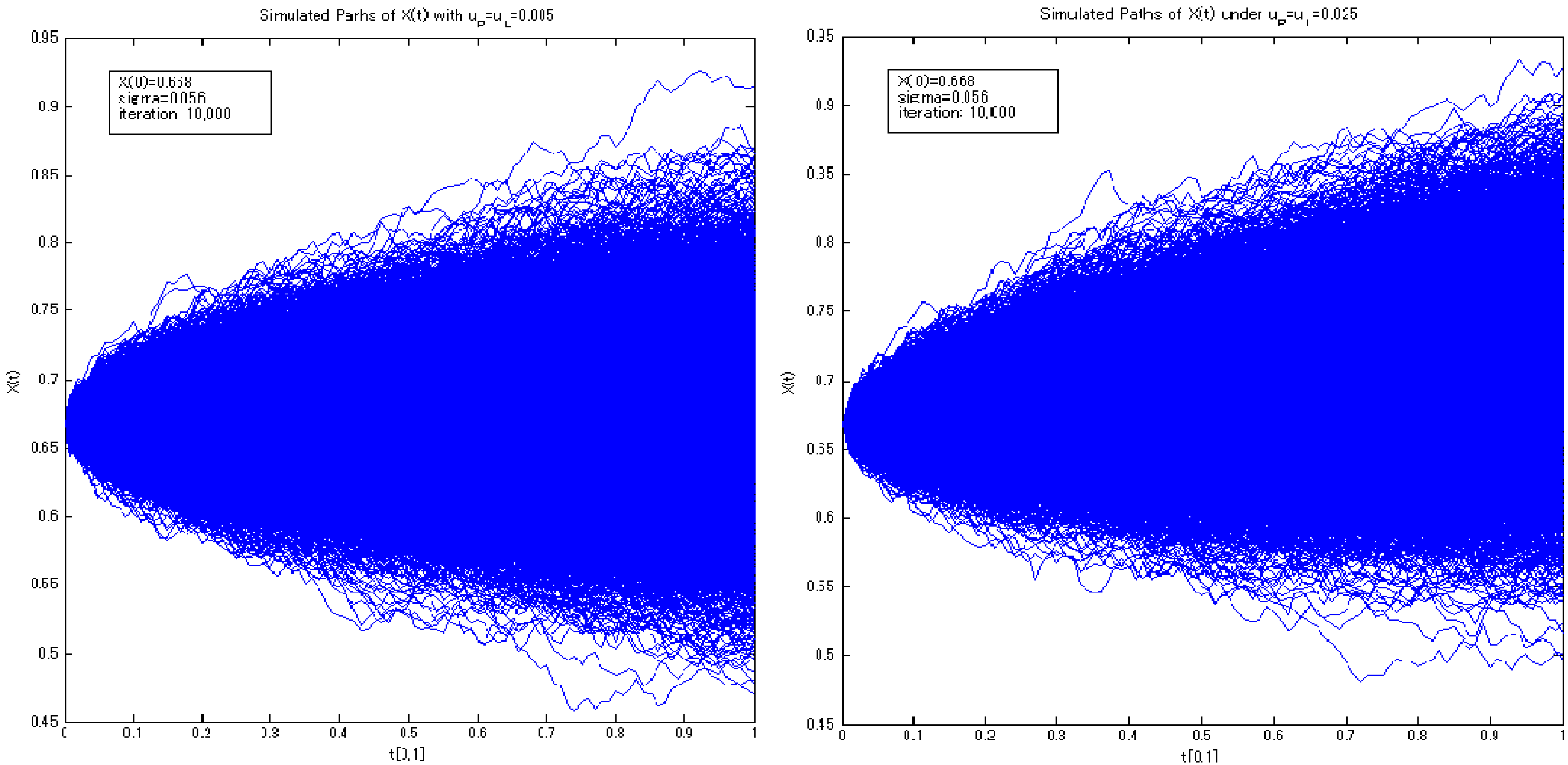
Figure 1 5 Sample Paths for Load Factor State Variable, X(t)





Left graph for u_{P}=u_{L}=0.005, Right graph for u_{P}=u_{L}=0.025

Figure 2 Simulated Paths for Load Factor State Variable, X(t)



Left graph for u_{P}=u_{L}=0.005, Right graph for u_{P}=u_{L}=0.025;
each 10,000 iterations

Figure 3 Convergence into a Risk Sharing Contract

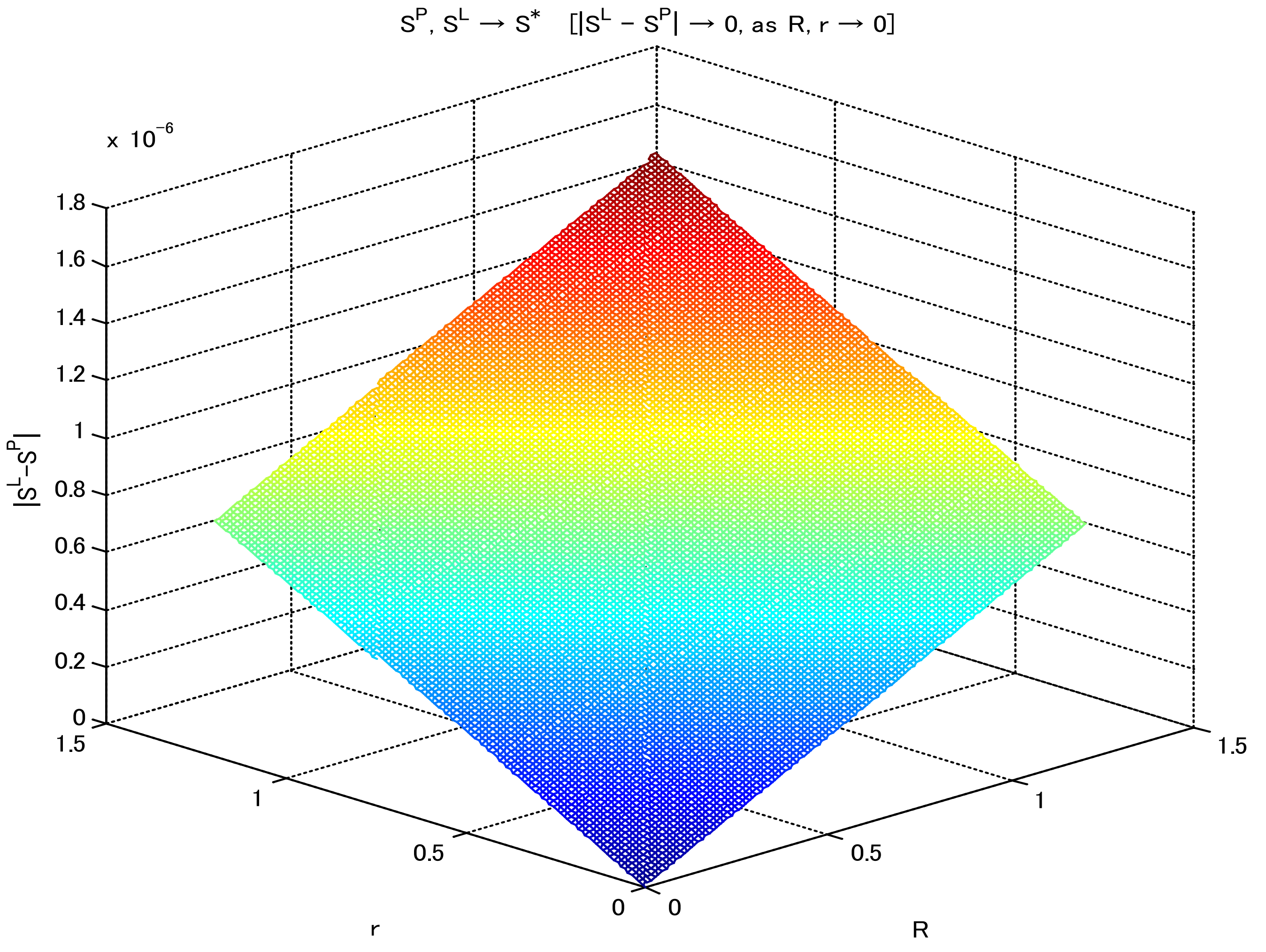


Figure 4 Optimal Contract S* Linear in X(1) with Diffusion Rate (σ)

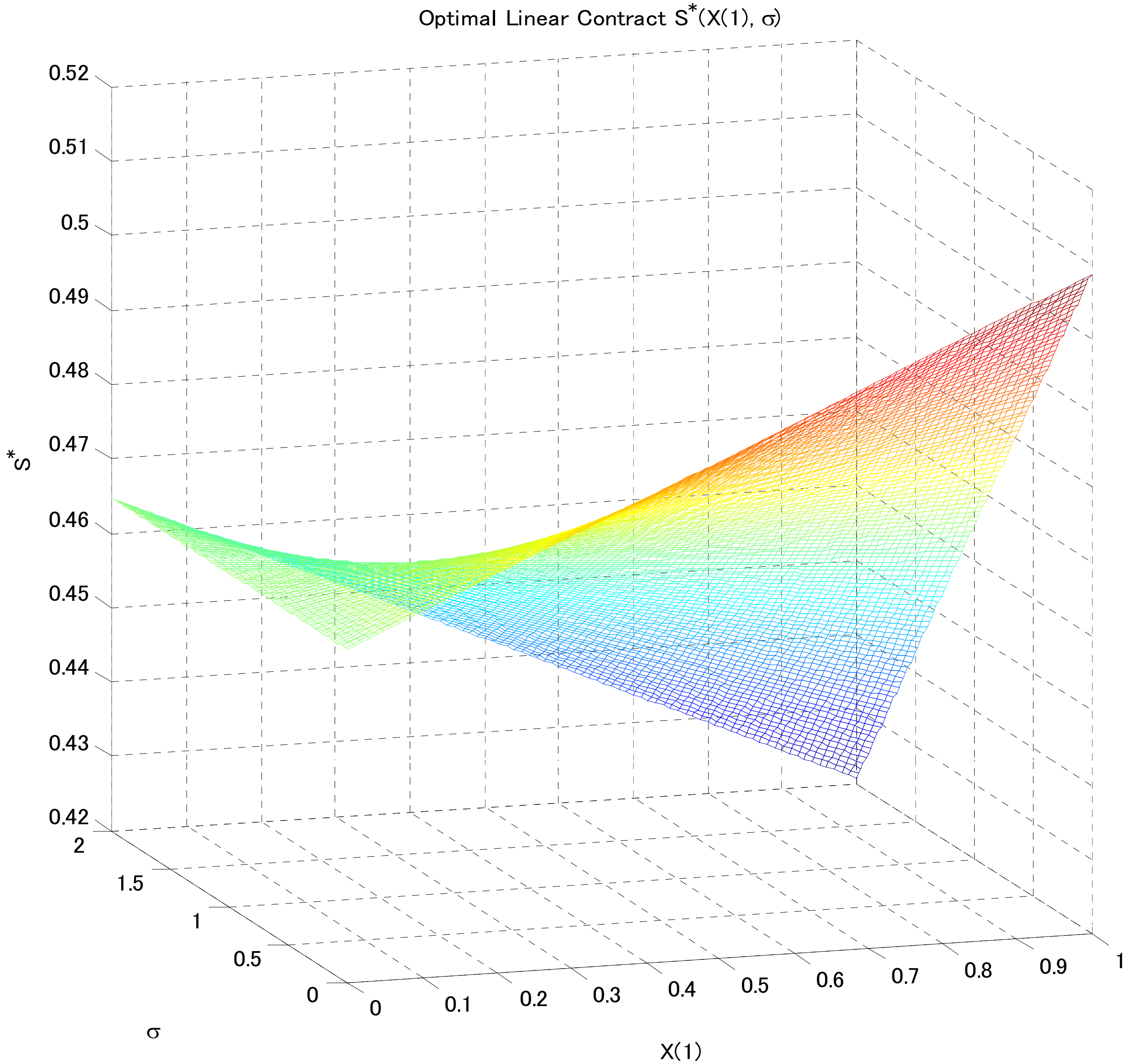


Figure 5 U_P* and U_L* with X(1) and σ

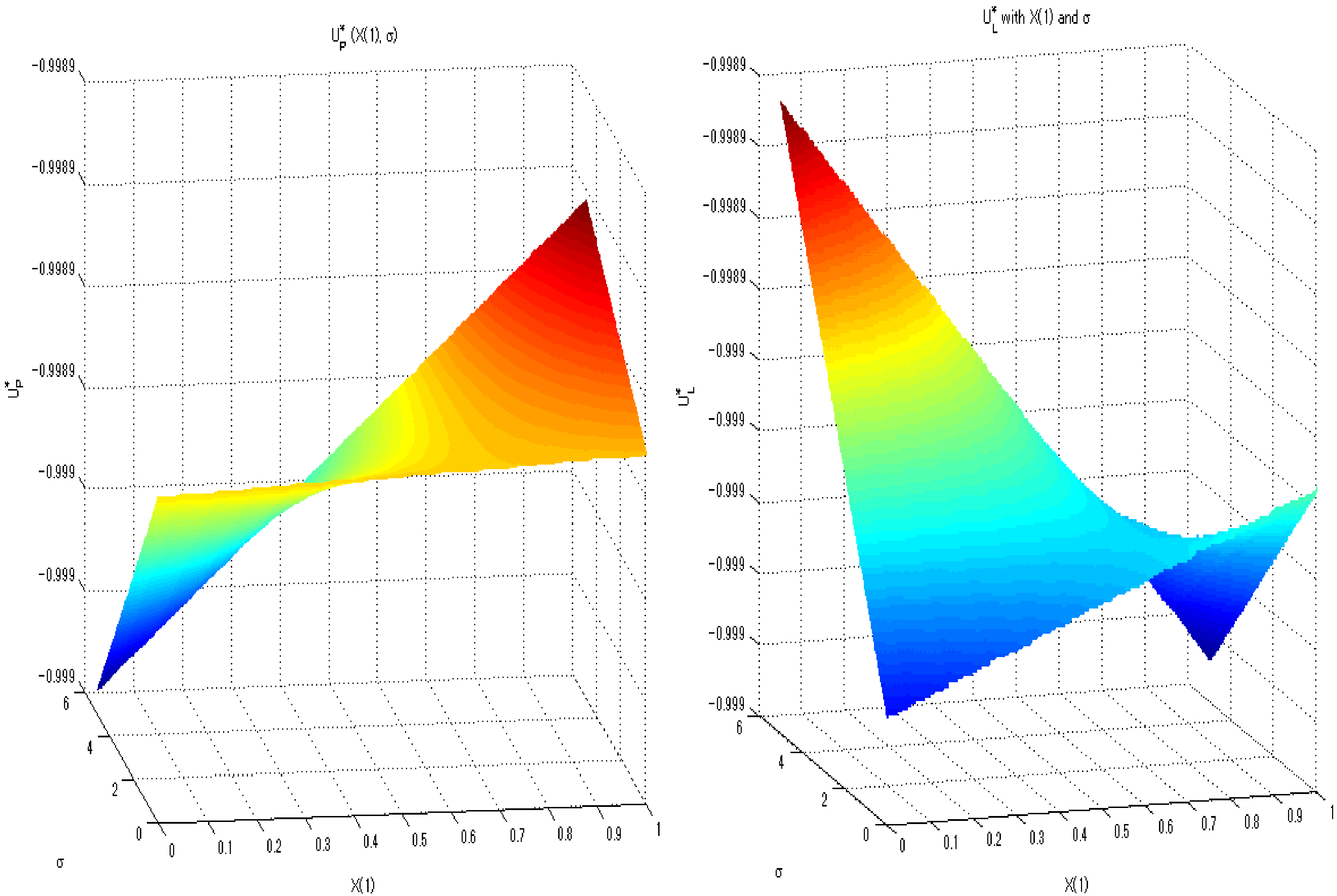


Figure 6. Total Utilities with X(1) & σ