The Effect of Monetary Policy under Price and Wage Stickiness

Momoko Shiraishi
Student ID: 51-178055
Supervisor: Professor Makoto Nirei

Master of Public Policy
Graduate School of Public Policy, The University of Tokyo
2018

Acknowledgements

First of all, I would like to sincerely thank to my supervisor, Professor Makoto Nirei for his constant guidance and encouragement. Without his support, this study would not have been completed. The valuable knowledge and advice I gained from him will continue to help me through my journey in life.

I would also like to express my appreciation to the professors who taught me during my undergraduate years, for providing continuous support and comprehensive advice.

Lastly, I would like to thank the classmates in my study group and all my close friends and family, who supported and encouraged me in all aspects. I am especially grateful to my peer, Kotaro Hamano for his kind help and valuable comments.

Abstract

This paper analyzes pricing mechanisms based on a menu cost model and the effect of monetary policy. There are three main findings of the study: (i) The heterogeneity of the frequency of price change and the size of price change across industrial sectors amplify the monetary non-neutrality in Japan. A particular feature of Japanese statistics is that a wide variance in the frequency of price change contributes to the larger amplification. (ii) Incorporation of intermediate input has also magnified the monetary non-neutrality in Japan. (iii) Lastly, the introduction of real wage rigidity enhances the degree of monetary non-neutrality. The amplification appears because the marginal cost consisting of real wage cannot be adjusted immediately in response to nominal aggregate shock. It is notable here that individuals in a household are incompletely differentiated by ability and have weak wage bargaining power, resulting in a reliance on the aggregate wage level in a firm's price setting, allowing for individual firms' heterogeneous movements. Further, variance of unemployment is positively affected. Ultimately, wage rigidity brings a distortion in the labor market.

Keywords: menu cost, price stickiness, wage stickiness, multi-sector, intermediate inputs

Contents

I. Introduction

From the mid-1990s, Japan had faced a long-lasting deflation period, the so-called "lost two decades." Prices are still staggered and have not achieved the 2%, as targeted, after Bank of Japan implemented a quantitative-qualitative easing as an unconventional monetary policy in 2013. Figure 1 shows the Japanese Consumer Price Index from 1971 to 2018 for all items covered and for four particular items. In a macro scope, the CPI has stayed around 0%, especially after the economic bubble in the mid-1990s. At the same time, on a micro level, prices of individual items do not always move in the same direction. The price change of goods, for example, is relatively greater than that of services. Furthermore, within the group, the index of food products fluctuates more than that of industrial products, for instance. Watanabe and Watanabe (2015) investigate over-the-year percentage changes of prices for 588 individual items composing the CPI. They conclude that prices are sticky, in the sense that the rate changes of about half of the items were around 0%. An analysis of individual prices is necessary to better understand the macro phenomenon.

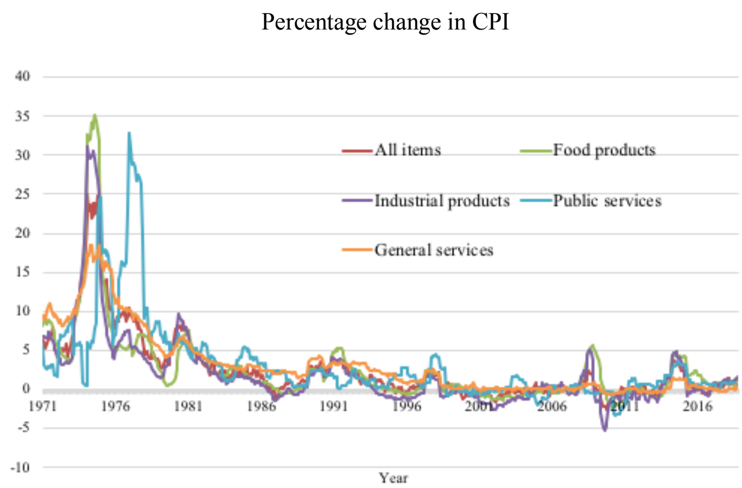

Figure 1: Over-the-year percent change of CPI in Japan from 1971 to 2018. Source: Ministry of Internal Affairs and Communications, Statistics Bureau.

Strategic complementarity is a factor to explain the relation between individual behaviors and aggregate phenomenon. It arises in price decisions as multiple firms set their prices equally, fearful that they may lose customers if they increase their prices independently, resulting in sluggish aggregate inflation. This paper points out the strategic complementarity stemming from mutual dependence on firms' production processes. Figure 2 shows intermediate inputs ratio across sectors in Japan. The ratios in manufacturing industries are higher, and petroleum and coal products are the highest at 78%. Housing and education, on the other hand, have a lower value. The overall average ratio is on an upward trend, from 48% in 2005 to 49.2% in 2011. Morikawa (2017) argues that intermediately input services are growing due to the increase of outsourcing to other firms in production or the fragmentation of production processes along with globalization. Especially in manufacturing industries, external services are provided in production.

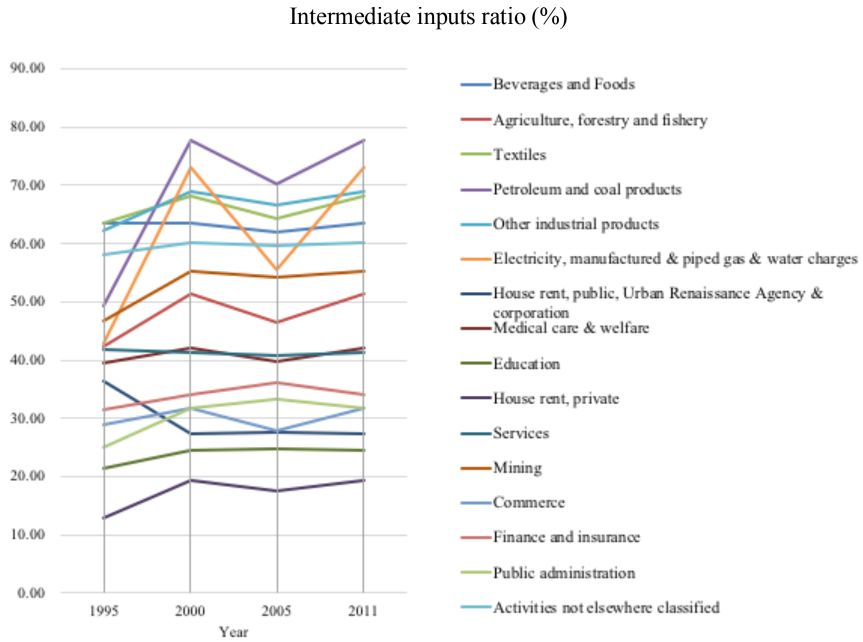

Figure 2: Intermediate inputs ratio across sectors in Japan from 1995 to 2011. Source: Input-out table, Ministry of Internal Affairs and Communications.

In addition to the goods market, this paper examines wage as a price of labor. The average real wage level per an hour for all employees appears in Figure 3.1; it reveals less fluctuation in recent years. On the other hand, the close variation between types of employment appears in Figure 3.2. Wages of general workers are leveling off, while wages for part-time workers has gradually increased since 2000; nevertheless the average wage level is still sluggish.[1] Kawaguchi and Hara (2017) point out a configuration bias

[1] According to type of employment, workers are classified into general workers and part-time workers. Part-time workers are defined as employees working for lower hours or only a couple of days in a week. General workers are the other remaining. Following Kawaguchi and Hara (2017), although in a strict sense, the

as the reason. They point out that the increase of non-regular workers in the mid-2000s (the ratio went from 27.7% in 2001 up to 37.5% in 2015) suppressed the aggregate wage level. They also describe the expansion of non-regular workers as the reason that the rate of returns on human capital scaled down under the diminishing growth economy. Currently, the number of non-regular workers is predicted to grow as the baby boom generation retires and takes up jobs as non-regular workers. The sticky wage in Japan cannot be clarified so easily. Genda (2017) suggests other reasoning, saying that downward wage rigidity induces upward wage rigidity.[2] As mentioned at the beginning, Japan had faced deflation for a long time, which forces firms to hesitate over raising their wages, wary of a future recession. Accordingly, Japanese companies are giving up base-up earning system, replacing it with a performance-based payment system. As a consequence, average contractual earnings have been suppressed. Regulations are also a part of wage rigidity. In medical and nursing services in Japan, for example, laws control the salary levels.

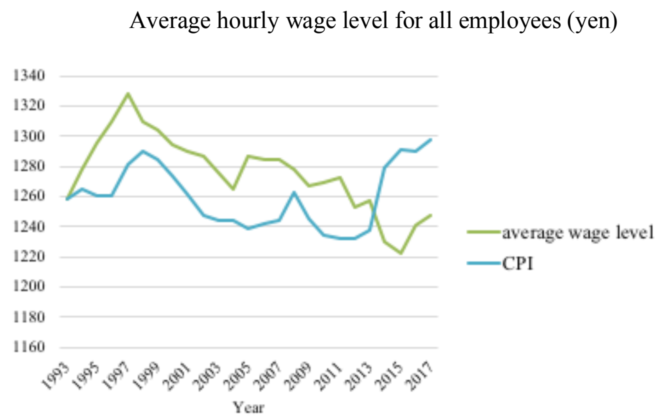

Figure 3.1: Average hourly wage level for all employees in Japan from 1993 to 2017.
Source: monthly labor survey, Ministry of Health, Labor and Welfare.

---

definitions are not exactly the same as (non-) regular workers, in this paper, general workers and regular workers are considered as the same, likewise for part-time workers and non-regular workers.

[2] Usually, upward or downward wage rigidity is mentioned in nominal term, since people tend to behave depending on face value. Also, several arguments in the previous literature introduced in this paper, assume nominal, but this paper does not clarify the difference, since the CPI fluctuates in narrow range.

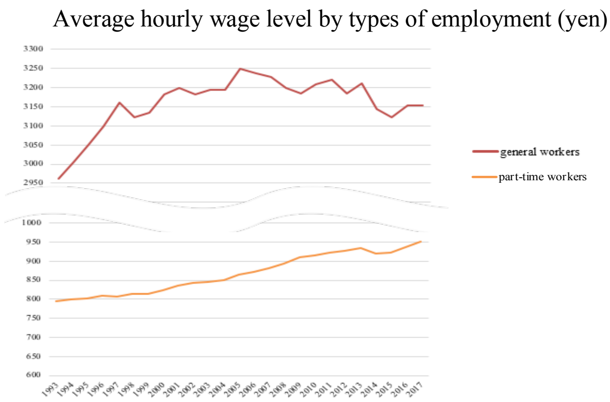

Figure 3.2: Average hourly wage level for general workers and part-time workers in Japan from 1993 to 2017. Source: monthly labor survey, Ministry of Health, Labor and Welfare.

Figure 3.1 also presents the linkage between prices and wages, holding higher correlation.[3] For the cause of such a long deflation in Japan, Yoshikawa (2013) refers to the drop in nominal wage at the end of 1990s as the country switched from a lifetime employment system. On the contrary, Yamamoto and Kuroda (2016) argue that prices and wages might indeed correlate; wage rigidity, however, is not a primary factor to bring out the inflation stagnation, as long as the interaction in individual sectors fades out.

In order to evaluate a monetary policy, nominal rigidity is an important factor as a source of monetary non-neutrality. This paper analyzes the effect of monetary policy on pricing mechanisms, focusing on three features in Japan described above: (i) heterogeneity of individual firms' price setting, (ii) strategic complementarity due to intermediate inputs, and (iii) wage stickiness. Before moving on to the model itself, this paragraph explains basic pricing models, rationalizing nominal rigidity. Pricing mechanisms can be classified into two established theories: time-dependent and state-dependent pricing models. The former, represented by Calvo (1983), assumes that only randomly selected firms can change their prices. In other words, timing to revise a price is determined exogenously. It might be easily imagined from looking at the clothing sector, for example, where prices change periodically over a season. A distinctive feature of the time-dependent model is when firms are able to adjust their prices to their benefit, they care about the future, i.e., are forward-looking. The type of model is called "time-dependent," since firms' re-pricing depends literally on time. On the other hand, in the

[3] The CPI in Figure 3.1 is adjusted to have the same value with the average hourly wage level in the initial year 1993.

state-dependent model, represented by a menu cost model, firms are assumed to bear costs to change their prices. When profits even considering the expense of changing prices, is greater than that without any price change or spending, they revise it. The cost could be literally interpreted as the cost to change prices on menu; yet in a broader sense, it can be the cost to hire consultants or new employees. This type of model is described as "state-dependent," meaning that firms' pricing decision relies on a state, which represents the gap between an actual price and optimal price. The model in this paper is based on menu cost model for price stickiness. Specifically, firms change their price when the gap between the actual price and optimal price gets large enough. Watanabe and Watanabe (2015) analyze the relationship between aggregate CPI inflation and rate of price change of individual items. They find that as the CPI rate change increased, the ratio of individual items changed by around 0% decrease linearly. In higher inflation, the opportunity cost to keeping prices unchanged is thought to be higher, exceeding menu cost, leading firms to change their prices. In this way, a menu cost model can explain the actual phenomena; thus, this paper relies on a menu cost model. Alternately, though it is not nominal, for real wage stickiness, the Calvo-type wage stickiness is applied. In particular, although households offer a wage, whether they get the chance or not, depends on a constant probability. The setting follows a feature in Japan that wage revisions take place relatively at regular time intervals.[4]

II. Literature Review

Caplin and Spulbar (1987) report that firms' price changes are not random; rather, they depend on a stationary distribution of firms' prices relative to desired price. Firms outside of their inaction range[5] at the low end of the distribution, adjust their prices in a manner described as "the selection effect." Based on their model, Golosov and Lucas (2007) introduce not only aggregate shock but also idiosyncratic shock. Consequently, they find that the effects of a monetary shock under a menu cost model is smaller and more transient

[4] According to the survey on the revision in wage conducted by Ministry of Health, Labor and Wealth in 2018, 82.6% of the targeted companies revised or planned to revise wage from January to August.
[5] Literally, an inaction range indicates where firms are not active, i.e., do not change their prices.

than under a Calvo model, since a positive aggregate shock induces the lowest-price firms to increase prices, while the highest-price firms do not adjust prices because the positive aggregate shock offsets the negative idiosyncratic shock. Thus, as a whole, the price level increases more quickly to reflect the aggregate shock. Midrigan (2011) extends the Golosov and Lucas model, inducing multiproduct firms and fat-tailed distribution of prices relative to desired price. He finds that these features lead firms to be less sensitive to monetary shock, meaning that the selection effect caused by the shocks is much weaker and produces real effects as large as those in the Calvo model. Nakamura and Steinsson (2010) develop Golosov and Lucas's model, adding heterogeneity in the frequency and size of price changes across industrial sectors, and intermediate inputs. As a result, they find that these two features amplify the degree of the monetary non-neutrality. For wage stickiness, Erceg, Henderson, and Levin (2000) derive a New Keynesian wage Phillips curve, based on the Calvo-type wage stickiness. Gali (2011) extends the model, using unemployment data since the markup of wages used in Erceg, Henderson, and Levin (2000) is unobservable in reality. Though they do not investigate the degree of the monetary non-neutrality, they do contribute to launching the new framework on wage rigidity. With regard to both price and wage stickiness models, Huang and Liu (2002) suggest that in a time-dependent model, wage stickiness has a greater effect on the degree of the monetary non-neutrality rather than price stickiness. James, Anton and Borja (2018) report sticky wages generate more non-neutrality than sticky prices, in the most model specifications in state-dependent. The model in the present paper is based on Nakamura and Steinsson (2010) and adds two contributions: the use of Japanese data and the introduction of real wage stickiness. The next chapter specifies the model. Chapter IV shows actual data and calibrated parameter values. Chapter V analyzes the result of the effect of monetary policy. Finally, the last chapter provides a conclusion.

III. Model

This chapter specifies the model incorporating three features mentioned above: (i) heterogeneity of individual firms' price setting, (ii) strategic complementarity due to intermediate inputs, and (iii) wage rigidity.

1. Household

In this economy, there exist heterogeneous members differentiated by a pair $(i, j) \in [0,1] \times [0,1]$ in a large household. Specifically, the members' disutility types for labor are a continuum indexed by $i \in [0,1]$, and their ability types are indexed by $j \in [0,1]$. The household as a whole maximizes discounted expected utility, denoted by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} C_t^{1-\gamma} - \omega \int_0^1 \int_0^{L_t(j)} i^{\psi} di dj \right]$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} C_t^{1-\gamma} - \omega \int_0^1 \frac{L_t(j)^{1+\psi}}{1+\psi} dj \right], \tag{1}$$

where C_t denotes the aggregate consumption and $L_t(j)$ is the employment rate among j type workers. β denotes the discount factor and γ is the degree of relative risk aversion. ω is the level parameter of labor disutility and ψ reflects the inverse of Frisch elasticity. In case that a member works, his or her disutility is ωi^{ψ}, where $\psi > 0$. If not working, it takes 0. This economy has a continuum of differentiated goods indexed by $z \in [0,1]$, and let the composite consumption good C_t be an implied Dixit-Stiglitz of these differentiated goods, denoted by $C_t \equiv \left[\int_0^1 c_t(z)^{\frac{\theta_g - 1}{\theta_g}} dz \right]^{\frac{\theta_g}{\theta_g - 1}}$, where $c_t(z)$ is the differentiated good z's consumption, and θ_g reflects the elasticity of substitution between the differentiated goods.

The household determines consumption behavior under the budget constraint

$$P_t C_t + E_t[D_{t+1} B_{t+1}] \leq B_t + \int_0^1 W_t(j) L_t(j) dj + \int_0^1 \Pi_t(z) dz. \tag{2}$$

Here, P_t is the price level, given by $P_t \equiv \left[\int_0^1 p_t(z)^{1-\theta_g} dz \right]^{\frac{1}{1-\theta_g}}$, where $p_t(z)$ denotes the price of good z. B_{t+1} denotes the state-contingent payoffs of the portfolio of financial assets, which the household purchases at time t and sells at time $t+1$. D_{t+1} is the stochastic discount factor for the payoffs B_{t+1}. $W_t(j)$ is the nominal wage for a worker j. Each member j earns money, and totally the earnings is integrated over j. $\Pi_t(z)$ reflects the profit for firm z, hence the last term in the equation indicates the total profits for all individual firm z.

The household chooses the consumption level of each good $c_t(z)$, so that the level of the consumption aggregate is highest, which implies

$$c_t(z) = C_t \left(\frac{p_t(z)}{P_t}\right)^{-\theta_g}. \tag{3}$$

For the problem maximizing (1) subject to (2), obtain Euler Equation, denoted by

$${C_t}^{-\gamma} = \beta E_t \left[{C_{t+1}}^{-\gamma} {D_{t+1}}^{-1} \frac{P_t}{P_{t+1}} \right]. \tag{4}$$

As to labor supply, assume indivisible labor based on Hansen (1985). That is, each member determines not how long he or she works, but the binary choice, whether to work or not. Then the working time is fixed when deciding to work. A member j works if and only if the utility from the payment for working exceeds its disutility. This implies

$$\omega i^{\psi} \leq \frac{W_t(j)}{P_t} \cdot {C_t}^{-\gamma}.$$

Let $i = N_t(j)$ such that $\omega N_t(j)^{\psi} = \frac{W_t(j)}{P_t} \cdot {C_t}^{-\gamma}$. In fact, here $N_t(j)$ reflects the maximum threshold in labor disutility among the all working members, controlling ability type j, as described in Figure 4.

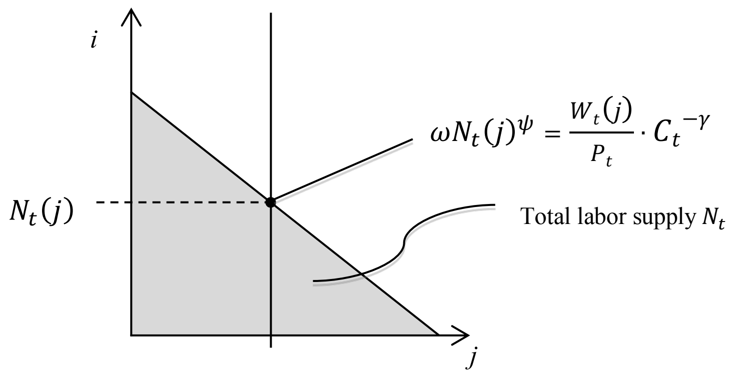

Figure 4: Image of labor supply

After integrating over j, finally we obtain labor supply equation,

$$\omega {N_t}^{\psi} = \frac{W_t}{P_t} \cdot {C_t}^{-\gamma}. \tag{5}$$

Since workers' ability types are differentiated, they have a wage-setting power. At the same time, real wage rigidity is assumed.[6] A fraction of members in the household $1-\alpha$ can reset their real wage, while the other fraction α has to leave their wage unchanged. This implies

$$\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \alpha\left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\theta_w} + (1-\alpha)\left(\frac{W_t^*}{P_t}\right)^{1-\theta_w}, \tag{6}$$

[7]

where $\frac{W_t^*}{P_t}$ denotes an optimal real wage when they luckily obtain the revision chance and θ_w represents the elasticity substitution between differentiated workers.

They set the optimal wage in period t, thinking of the expected utility until they can change their wage next time. Let time $t+s$ be the period, until then the wage continues to be stuck. The labor demand at time $t+s$ for j type members who could reset the wage at time t is

$$L_{t+s|t}(j) = L_{t+s}\left(\frac{W_t(j)}{P_t} / \frac{W_{t+s}}{P_{t+s}}\right)^{-\theta_w}.$$

The labor demand is derived from firms' optimal behavior mentioned below. Under the labor demand and budget constraint, thereby, the members solve the problem, given by

$$\max_{\frac{W_t(j)}{P_t}} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \left[\frac{1}{1-\gamma} C_{t+s|t}^{1-\gamma} - \omega \int_0^1 \frac{L_{t+s|t}(j)^{1+\psi}}{1+\psi} dj\right].$$

And finally, we obtain the optimal real wage level $\frac{W_t^*}{P_t}$, defined by

$$\left(\frac{W_t^*}{P_t}\right)^{1+\theta_w\psi} = \frac{\theta_w}{\theta_w - 1} \cdot \frac{Z_{1,t}}{Z_{2,t}},$$

where

$$Z_{1,t} = \omega\left(\frac{W_t}{P_t}\right)^{\theta_w(1+\psi)} L_t^{1+\psi} + \beta\alpha E_t[Z_{1,t+1}],$$

$$Z_{2,t} = C_t^{-\gamma}\left(\frac{W_t}{P_t}\right)^{\theta_w} L_t + \beta\alpha E_t[Z_{2,t+1}]. \tag{7}$$

[8]

[6] About the assumption of real wage rigidity, Blanchard and Gali (2007) is referred.
[7] The derivation is in Appendix.
[8] The derivation is in Appendix.

In addition, the household optimization implies a transversality condition.

2. Firm

In this economy, each firm produces one good $z \in [0, 1]$. There exist K sectors to which a given firm belongs. Firm z's production function is

$$y_t(z) = A_t(z) L_t(z)^{1-s_m} M_t(z)^{s_m}, \tag{8}$$

where $A_t(z)$ denotes firm z's productivity, $M_t(z)$ reflects the index of intermediate products given by $M_t(z) \equiv \left[\int_0^1 m_t(z, z')^{\frac{\theta_g - 1}{\theta_g}} dz\right]^{\frac{\theta_g}{\theta_g - 1}}$, and s_m represents the intermediate input share. Then, the firm z in sector $k \in K$, maximizes the value of its discounted profits,

$$E_0 \sum_{t=0}^{\infty} D_t \Pi_t(z),$$

where profits in period t are given by

$$\begin{aligned} \Pi_t(z) &= p_t(z) y_t(z) - \int_0^1 W_t(j) L_t(z, j) dj - P_t M_t(z) - \chi_k W_t I_t(z) \\ &= p_t(z) y_t(z) - W_t L_t(z) - P_t M_t(z) - \chi_k W_t I_t(z). \end{aligned} \tag{9}$$

$I_t(z)$ is a binary indicator variable. When a firm changes its price, it equals to one otherwise to zero. Therefore, the forth term in this equation implies menu cost. The rearrange of the equation comes from the labor demand of the firm z, defined by $L_t(z) \equiv \left[\int_0^1 L_t(z, j)^{\frac{\theta_w - 1}{\theta_w}} dj\right]^{\frac{\theta_w}{\theta_w - 1}}$, and the aggregate real wage level, given by $\frac{W_t}{P_t} \equiv \left[\int_0^1 \frac{W_t(j)}{P_t}^{1-\theta_w} dj\right]^{\frac{1}{1-\theta_w}}$. This also implies the labor demand of the firm z for j type workers is

$$L_t(z, j) = L_t(z) \left(\frac{W_t(j)}{P_t} / \frac{W_t}{P_t}\right)^{-\theta_w}. \tag{10}$$

Firm z minimizes its cost, which implies that the firm z's demand for differentiated intermediate good is

$$m_t(z, z') = M_t(z) \left(\frac{p_t(z')}{P_t} \right)^{-\theta_g}. \tag{11}$$

Combining consumer demand equation (3) and input demand equation (12), total demand for good z is

$$y_t(z) = Y_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta_g}, \tag{12}$$

where $Y_t = C_t + M_t$, under $M_t \equiv \int_0^1 M_t(z) dz$. This implies that C_t reflects value-added output, while Y_t reflects gross output. Hence, in analyzing the degree of monetary non-neutrality, C_t is of concerned.

Assume that firm z's individual productivity follows as

$$log\ A_t(z) = \rho\ log\ A_{t-1}(z) + \epsilon_t(z), \tag{13}$$

where $\epsilon_t(z) \sim N(0, \sigma_{\epsilon,k}^2)$. This equation holds the sector difference in terms of the variance of productivity shock.

The monetary authority focuses on nominal value-added output $S_t = P_t C_t$, which follows as

$$log\ S_t = \mu + log\ S_{t-1} + \eta_t, \tag{14}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$.

This problem is solved by value function iteration method, and thereby the Bellman equation for this problem is

$$V\left(\frac{p(z)}{P}, A(z), \frac{W}{P}, \frac{S}{P}\right) = \max_{p(z)} \left\{ \Pi^R(z) + E\left[D^R V\left(\frac{p(z)'}{P}, A(z)', \frac{W'}{P}, \frac{S'}{P} \right) \right] \right\},$$

where $V(\cdot)$ denotes firm z's value function, $\Pi^R(z)$ is firm z's profits in the real term, D^R is the real stochastic discount factor. The firm z chooses an optimal price $\frac{p(z)'}{P}$ given a state, i.e., we obtain the policy function.

3. Equilibrium

In equilibrium, optimality conditions of household; (3) (4), (5), and a transversality condition, optimality conditions of firms including (10) and (11), market clearing, and the process of $\frac{W_t}{P_t}$ (6) (7), $A_t(z)$ (13) and S_t (14) are satisfied.

4. Solving Method

In this problem, a stationary distribution of optimal price is affected by an aggregate shock. In order to deal with the issue, define the rule of distribution beforehand, given by

$$\frac{P_t}{P_{t-1}} = \Gamma\left(\frac{S_t}{P_{t-1}}\right).$$

This is a finite-dimensional approximation method developed by Krusell and Smith (1998). In detail,

i. Specify a finite grid of points for the state variables, $\frac{p(z)}{P}, A(z), \frac{W}{P}$ and $\frac{S}{P}$.

ii. Propose a function $\Gamma\left(\frac{S_t}{P_{t-1}}\right)$ on the grid.

iii. Given the proposed Γ, solve for the firm's policy function by value function iteration on the grid.

iv. Check whether the policy function and Γ are consistent. If so, stop the iteration and calculate other values of the equilibrium using the policy function and Γ. Otherwise update Γ and go back to step iii.

IV. Calibration

1. Price Data

This subsection provides a statistics analysis on inflation in Japan, using the micro data of the CPI in retail price survey by Ministry of Internal Affairs and Communications. Prices of goods and services, and house rent are taken in 167 municipalities around Japan. Specifically, for goods and services, approximately 27,000 stores are targeted, and for house rent, roughly 28,000 houses. For accommodation fees, prices are taken at 320 hotels in 99 cities. A monthly study is held in an area, selected depending on a feature of an item.[9] In this way, the CPI originally consists of about 250,000 pieces of price data. Due to data accessibility, however, this paper covers not the all of the price data, but instead an average statistic in 61 larger cities;[10] however, it is still sort of micro data in comparison with aggregate CPI. Prices of around 500 items from January 2010 to June 2018 are targeted, and frequency and size of price change are obtained.[11] Frequency of price changes reflects the fraction of cities with changing price. Following Saita and Higo (2007), price change is defined as when a current price differs from a price in the previous month, except for April 2014. At that time, consumption tax increased by 3%; hence, the price change is recognized as a change outside of the $2.86 \pm 0.5\%$ range. The size of price change is also deducted by 2.86% in that case. In estimating aggregate statistics, like at sector-level, frequency and size of price change are weighted using CPI weighting.[12] Table 1 shows the sector weight, frequency and size of price change for the totality (one-sector model), for each of the 21 sectors, and for each of the 9 sectors.[13] For the multi-sector case, the 9 sectors are condensed from the 21 sectors, and a 9-sector model is applied in the solution described below due to computation restrictions.[14]

[9] An item whose price is regionally highly different, for example, is surveyed at more stores.

[10] Cities whose sample size is similar are targeted.

[11] Originally, the CPI consists of 588 or 585 items, but this research excludes a certain measure of items. The criterion in selecting items in detail is in Appendix.

[12] For price data of 2010–2014, CPI weighting of 2010 criteria is introduced, while for that of 2015–2018, CPI weighting based on 2015 criteria is used.

[13] Imputed rent sector, which is originally present, is excluded here because there is no market for imputed rent.

[14] The house rent sectors in both public and private are excluded, and to hold the characteristics of each sector, sectors having similar statistics results are combined.

Table 1: Basic statistics of price change

Model			Sector Weight (%)	Frequency of Price Change (%)	Average Size of Price Change (%)
One Sector Model			-	37.40	6.51
21 Sector Model	Goods	Fresh food, raw meats & cut flowers	8.65	83.25	8.74
		Other agricultural, aquatic & livestock products	1.13	58.48	3.27
		Food products	21.68	43.43	5.29
		Textiles	2.92	26.27	10.44
		Petroleum products	5.39	68.25	2.13
		Other industrial products	17.55	38.35	8.19
		Electricity, manufactured & piped gas & water charges	8.47	45.01	3.11
		Publications	0.60	0.44	7.14
	Public Services	School lunch	0.49	7.60	1.55
		House rent, public, Urban Renaissance Agency & corporation	0.51	51.51	0.49
		Domestic duties	2.09	2.20	11.91
		Medical care & welfare	1.04	2.25	9.37
		Forwarding & communication	1.47	0.76	11.36
		Education	0.31	1.20	5.35
		Culture & recreation	0.09	8.45	19.21
	Private Services	Eating out	7.78	5.98	7.17
		House rent, private	4.20	82.20	0.36
		Domestic duties	6.66	3.24	7.97
		Medical care & welfare	0.55	3.49	6.43
		Education	4.73	5.16	7.50
		Communication, culture & recreation	3.68	9.22	9.61
			100		
9 Sector Model	Goods	Fresh food, raw meats & cut flowers	9.14	83.25	8.74
		Other agricultural, aquatic & livestock products	1.20	58.48	3.27
		Food products	22.90	43.43	5.29
		Textiles	3.09	26.27	10.44
		Petroleum products	5.69	68.25	2.13
		Other industrial products	18.53	38.35	8.19
		Electricity, manufactured & piped gas & water charges	8.94	45.01	3.11
	Public Services		5.81	2.35	10.11
	Private Services		24.71	5.49	7.83
			100		

Figure 5.1, Figure 5.2 and Figure 6 show the frequency and size of price change for each of the 21 sectors, distinguished by price increase or decrease. The frequency of price change is, as a whole, hugely different across sectors. It is relatively for goods than for services. Individually, prices of fresh food, raw meats & cut flowers change very often. In both the public and private house rent sector, it fluctuates greatly, but the size of change is quite small. In comparison with Saita and Higo (2007), who surveyed the frequency and size of price change across the same sectors in 1989 to 2003, in this study, the frequency of price change is higher, whereas the size of price change is analogous to the prior study. Regarding the size of price change, in both studies overall, changes are lower for goods than for services.

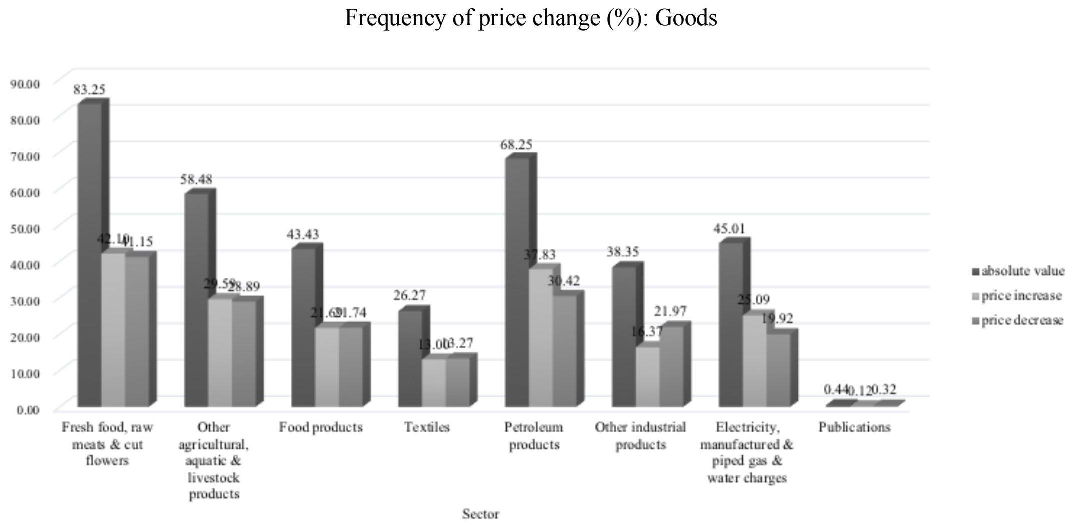

Figure 5.1: Frequency of price change in goods sector

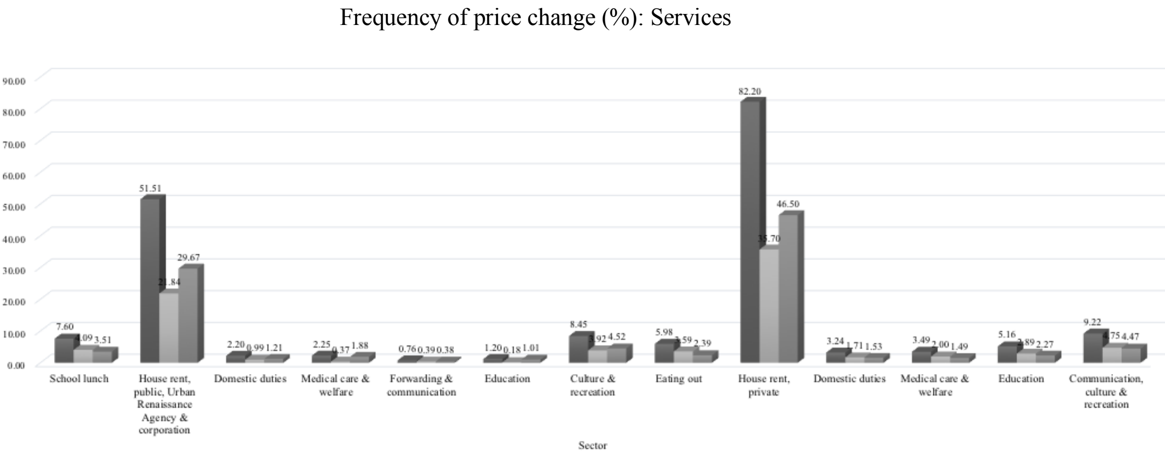

Figure 5.2: Frequency of price change in services sector

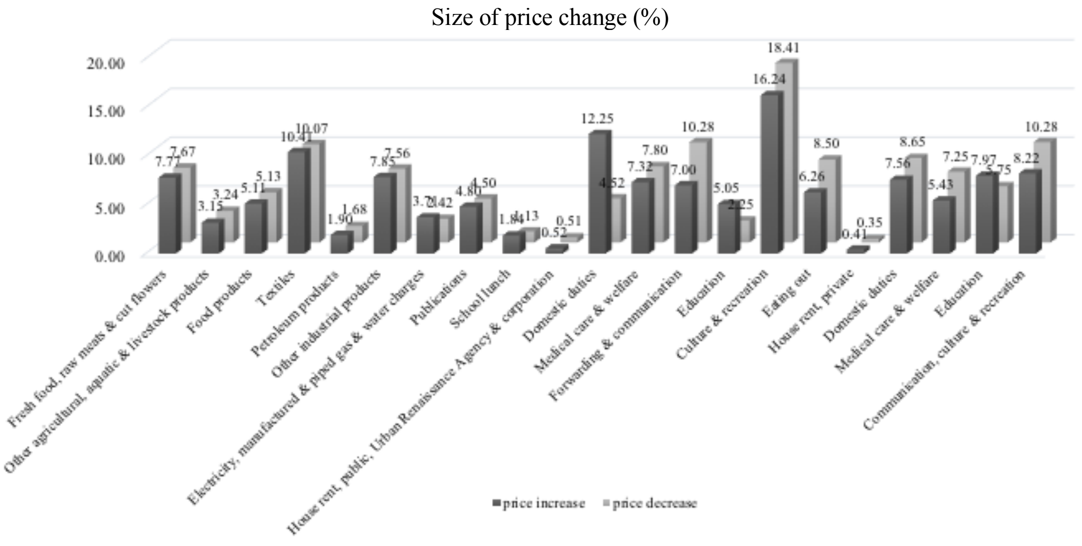

Figure 6: Average size of price change

2. Calibration

This section describes parameters used in the model. First, from the statistics of frequency and size of price change in Japan in the previous section, parameter values, menu cost, and the variance of idiosyncratic productivity shock are obtained. Specifically, these values are determined so that the frequency and the size of price change calibrated in the model match the actual data. Table 2 shows the parameters for a one-sector model, a multi-sector model (a 9-sector model), and a one-sector model with wage rigidity. Following Nakamura and Steinsson (2010), in the table, $\Delta p\ cost$ denotes the average cost of changing prices in a year as a fraction of steady state revenue.[15]

Although the two do not have a perfect one-to-one relationship, menu cost and frequency of price change co-move highly, and variance of productivity shock relates comparatively with the size of price change. Therefore, χ itself is lower in sectors with higher frequency of price change. On the other hand, the variance of a productivity shock is larger in sectors with greater price change.

For the other parameters, the monthly discount factor is $\beta = 0.96^{1/12}$. The degree of relative risk aversion is $\gamma = 1$. The level parameter of labor disutility ω is

[15] $\Delta p\ cost$ is given by $\Delta p\ cost\ = f \cdot \frac{\theta_g - 1}{\theta_g} \cdot \chi_k \cdot \frac{1}{Y_{ss}}$.

determined so that the labor level at a steady state is $1/3$.[16] The parameter implying the inverse of Frisch elasticity is set as $\psi = 1$ based on Kuroda and Yamamoto (2007). They find that since the 1990s the Frisch elasticity in Japan is roughly 0.7 to 1. The elasticity of substitution between differentiated goods is set as $\theta_g = 8$ according to Shirota (2006), who finds that in Japan it takes a value of approximately 8 to 20, depending on the model specifications. Here $\theta_g = 8$ is selected, in order to obtain lesser difference than the U.S. result in Nakamura and Steinsson (2010), which was under $\theta_g = 4$, and so as to compare them easily. The speed of convergence is set as $\rho = 0.7$ for a computation reason. The parameters in the equation of the nominal aggregate process are estimated using Japanese quarterly seasonally-adjusted data of nominal GDP from January 2011 to June 2018, obtained from national accounts of the Japan database by cabinet office, government of Japan. The mean growth rate of the nominal aggregate demand is then set as $\mu = 0.0015$, and the standard deviation of the nominal GDP is $\sigma_\eta = 0.0038$.

[16] In the model with wage stickiness, L denotes the employment rate as described in chapter III. Here the value $1/3$ is considered to be scaled out. In fact, the Japanese average unemployment rate between 2011 and 2018 is 3.3%.

Table 2: Menu cost and variance of productivity shock $(\times 10^{-2})$

Model			$S_m = 0$ $\Delta p\ cost$	$S_m = 0$ σ_ϵ	$S_m = 0.66$ $\Delta p\ cost$	$S_m = 0.66$ σ_ϵ
One Sector Model			0.252	4.21	0.065	4.28
Multi Sector Model	Goods	Fresh food, raw meats & cut flowers	0.093	8.75	0.017	7.34
		Other agricultural, aquatic & livestock products	0.046	2.47	0.012	2.47
		Food products	0.142	3.38	0.014	2.78
		Textiles	0.644	5.89	0.094	4.90
		Petroleum products	0.013	1.64	0.004	1.63
		Other industrial products	0.366	5.12	0.128	4.25
		Electricity, manufactured & piped gas & water charges	0.085	2.50	0.011	2.11
		Public Services	0.364	5.99	0.203	2.25
		Private Services	0.283	4.22	0.316	3.89
With Wage Stickiness			2.063	0.02	31.340	0.32

The parameter of the cost share of intermediate input comes from a Japanese input-output table organized by Ministry of Internal Affairs and Communications. Table 3 shows the share of intermediate inputs across sectors and the CPI weight for each sector. Note that sectors here are not in one-to-one correspondence with the sectors classified in price statistics in Table 1 or in Table 2. The overall intermediate inputs ratio is approximately 58%, after weighted with CPI weighting. Compared to 52% in the U.S., we know Japan has a stronger connection in production. Since the parameter used in the model equals the cost share of the intermediate inputs, not the average intermediate goods ratio, the value is multiplied by the markup. Consequently, the parameter of the cost share of intermediate input in Japan is set as $S_m = 0.66$.

Table 3: Intermediate inputs ratio across sectors

Sector	Intermediate Inputs Ratio (%)	CPI Weight (%)
Beverages and Foods	63.5	30.8
Agriculture, forestry and fishery	51.5	1.1
Textiles	68.1	2.9
Petroleum and coal products	77.7	5.4
Other industrial products	68.9	18.2
Electricity, manufactured & piped gas & water charges	73.0	8.5
House rent, public, Urban Renaissance Agency & corporation	27.3	0.5
Medical care & welfare	45.6	1.0
Education	24.4	0.3
House rent, private	19.4	4.2
Services	41.4	27.1
Mining	55.2	0.0
Commerce	31.5	0.0
Finance and insurance	34.2	0.0
Public administration	31.7	0.0
Office supplies	100.0	0.0
Activities not elsewhere classified	60.1	0.0
		100.0

The Calvo probability that members in a household keep their wage unchanged and the elasticity of substitution between workers are based on Shintani and Muto (2014). Even though they calibrated in a model under nominal wage rigidity, the same values are used, taking account of Japanese stagnated inflation mentioned in chapter I. They estimate the probability between 0.32 and 0.6. As for the mean, $\alpha = 0.5$ is set as benchmark. The elasticity of substitution between differentiated workers is derived as $\theta_w = 30$ from the model under $\psi = 1$. Table 4 summarizes all of the benchmark parameters used in the model with their data sources.

Table 4: Benchmark parameters

Parameter	Description	Source
$\beta = 0.96^{1/12}$	Monthly discount factor	Nakamura and Steinsson (2010)
$\gamma = 1$	Degree of relative risk aversion	Nakamura and Steinsson (2010)
ω	Level parameter of labor disutility	Nakamura and Steinsson (2010)
$L_{SS} = 1/3$	Labor supply level at the steady state	Nakamura and Steinsson (2010)
$\psi = 1$	Inverse of Frisch elasticity	Kuroda and Yamamoto (2007)
$\theta_g = 8$	Elasticity of substitution between differentiated goods	Shirota (2006)
$\rho = 0.7$	Speed of convergence	Nakamura and Steinsson (2010)
$\mu = 0.0015$	Mean of nominal aggregate demand process	Calibration
$\sigma_\eta = 0.0038$	Standard deviation of nominal aggregate demand	Calibration
$S_m = 0.66$	Cost share of intermediate inputs	Calibration
$\alpha = 0.5$	Probability to leave wage unadjusted	Shintani and Muto (2014)
$\theta_w = 30$	Elasticity of substitution between workers	Shintani and Muto (2014)

V. Result

Figure 7 presents the gamma function, describing a relation of inflation and real aggregate demand given by $\frac{P_t}{P_{t-1}} = \Gamma\left(\frac{S_t}{P_{t-1}}\right)$, obtained by solving the problem. The function is almost linearly upward sloping. Figure 8 and Figure 9 show the profit function and the policy function respectively. Table 5 provides the result of the degree of monetary non-neutrality represented by the variance of output for each model setting. This chapter interprets the results of the three aspects: (i) heterogeneity of frequency and size of price change, (ii) intermediate inputs and (iii) wage stickiness.

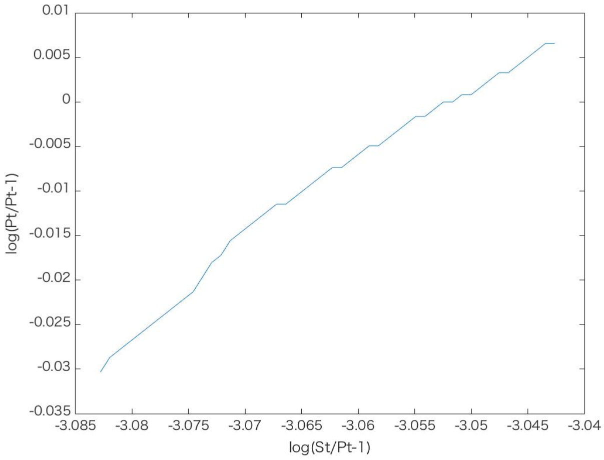

Figure 7: Gamma function[17]

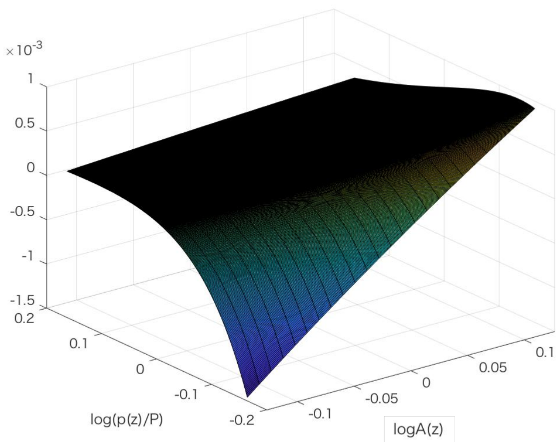

Figure 8: Profit function

[17] All of the functions in Figure 7, 8 and 9 are in one sector model and under $S_m = 0.66$. In every model function forms are similar.

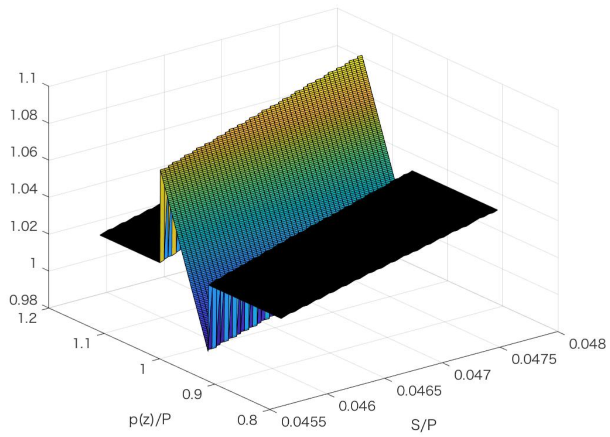

Figure 9: Policy function

Table 5: Monetary non-neutrality, $Var(C_t)$ ($\times 10^{-5}$)

Model	$S_m = 0$	$S_m = 0.66$
One Sector Model	0.0266	0.1486
Multi Sector Model	0.2549	0.5953
With Wage Stickiness	0.3079	0.5736

(i) Heterogeneity

Table 5 shows that the degree of monetary non-neutrality in the multi-sector model is larger than in in the one-sector model, regardless of the presence of intermediate inputs. It qualifies that the heterogeneity of frequency and size of price change amplifies the degree of monetary non-neutrality. The result, in fact, originates in a convex function of variance of output and frequency of price change at any level of variance of productivity shock, which Nakamura and Steinsson (2010) find. When there is heterogeneity, the overall degree of monetary non-neutrality is nearly each sector's weighted average. Thus, based on Jensen's inequality, the variance of output in the multi-sector model becomes higher than when there is only one sector. A characteristic of the convex functions is that given frequency of price change, in lower variance of productivity shock, the convexity is diminished because of a stronger selection effect. In other words, in lower productivity shock, an aggregate shock becomes more influential and many more firms adjust their prices. Consequently, the variance of output takes a lower value.

Comparing the result in Japan with the U.S. case in Nakamura and Steinsson (2010), firstly, the degree of monetary non-neutrality is relatively lower in Japan.[18] Again, controlling for the other elements, the monetary non-neutrality is more amplified under a lower frequency of price change and a larger-sized price change, respectively. In the one-sector model, the frequency of price change is 37% in Japan, while only 21% in the U.S., and the size of price change is 6.5% in Japan, but 9.8% in the U.S. These price statistics account for the relatively lower degree of non-neutrality in Japan.

On the other hand, an increase in the rate of change by incorporating heterogeneity is much higher in Japan. In the U.S., the rate is about three times in the one-sector model. In this result, particularly under $S_m = 0$, the degree of monetary non-neutrality increases by almost ten times. It comes from the characteristic of Japanese price data, especially a wide variety of frequencies of price change. A simple comparison of the standard deviation of frequency of price shows that, in Japan it is 27.0 (%), while in U.S. it is 24.9 (%). Additionally, the CPI weight for items with higher frequency of price change is larger. Besides, the exclusion in this analysis of the house rent sector, which has a higher frequency of price change, the amplification due to heterogeneity would be much larger in the real economy. In a short, the result implies that it is important to consider the heterogeneity of pricing behaviors across sectors in Japan.

(ii) Intermediate Inputs

The amplification of monetary non-neutrality due to the intermediate inputs in production is confirmed for all model settings in Table 5. With the augmentation, the marginal cost structure is strongly associated. In this model, the marginal cost consists of wage. In the model without wage rigidity, the wage term W_t is rewritten with nominal aggregate demand S_t, coming from the labor supply equation. Here under $S_m > 0$, in response to an aggregate shock, the marginal cost is lower than in a case without intermediate inputs, since the term P_t exists and it is not adjusted to the shock immediately, due to the presence

[18] In U.S., in one sector model (mean), under $S_m = 0$, it takes 0.055, and under $S_m = 0.7$, it is 0.182. In 9 sector model, for example, under $S_m = 0$, it is 0.143, and under $S_m = 0.7$, it has 0.576 ($\times 10^{-4}$).

of menu costs. The pricing decision in an economy as a whole determines individual firms' price setting. This is the mechanism of strategic complementarity given by intermediate inputs.[19]

Table 6 shows a more fine-tuned examination. An increase in reliance on other firms clearly amplifies the monetary non-neutrality. As described in chapter I, at present, the intermediate inputs ratio in Japan is on an upward trend, especially in industrial products. The result implies that intermediate inputs have a significant role in the effect of monetary policy.

Table 6: Cost share of intermediate inputs and monetary non-neutrality[20]

Cost Share of Intermediate Inputs	$Var(C_t)$ ($\times 10^{-5}$)
$S_m = 0.0$	0.0266
$S_m = 0.2$	0.0292
$S_m = 0.5$	0.0728
$S_m = 0.66$	0.1486

(iii) Sticky Wages

Table 5 shows that the introduction of real wage rigidity increases the monetary non-neutrality, regardless of whether intermediate goods are included. The rate change is increased at the most, by twelve times in a one-sector model without intermediate goods. Table 7 shows the relation of the degree of monetary non-neutrality for three different parameter values in detail. The greater the value of α, meaning that the real wage is becoming more sticky, the greater the variance of the output. The mechanism behind it is the following. A firms' marginal cost in this model is, again, composed of real wage. Since the real wage is staggered in response to aggregate shocks, firms adjust their price setting, but less so than in a case without real wage rigidity. Nakamura and Steinsson (2010) mention that the strategic complementarity not driven by a firm's own price setting has a larger effect. Real wage rigidity as an aggregate determinant holds the feature, and it has a measureable effect on the degree of the monetary non-neutrality.

[19] Nakamura and Steinsson (2010) report the mechanism in detail.
[20] All of the values are obtained in one sector model.

Table 7 also shows the variance of unemployment, it clearly increases according to the augmenting of wage stickiness. As it turns out, arising wage rigidity distorts the labor market; consequently, unemployment occurs. Unemployment in this model represents quitting a job voluntarily to enjoy leisure time, because the condition that all of the members in a household willingly decide whether or not to work under the condition of labor supply.

Table 7: Wage rigidity, monetary non-neutrality, and variance of unemployment[21]

Probability with Unchanging Wage	$Var(C_t)$ ($\times 10^{-5}$)	$Var(u_t)$[22] ($\times 10^{-2}$)
$\alpha = 0.2$	0.2284	0.3725
$\alpha = 0.5$	0.2866	0.3760
$\alpha = 0.85$	0.3079	0.4028

Further, it is found from the result that, in monetary phenomena, the aggregate wage level has an important role. Kuroda and Yamamoto (2016) argue that wage stickiness is not a primary factor in explaining staggered price in Japan, since at the individual industry levels, wages and prices do not move in the same way, even though, at aggregate level, they strongly correlate. Considering idiosyncratic shock, enabling individual firms or sectors to behave variedly, nominal price rigidity is influenced by aggregate wage rigidity in this model. Members in a household are differentiated by i and j but are still not completely different. Therefore, each person does not have strong wage bargaining power; everyone must rely on the aggregate wage level. Firm z accordingly sets its price, taking into account the aggregate wage. It also should be emphasized that a wage is determined by the ability and disutility of members in the household, in a sense, their communication skills or diligence, regardless of firm or sector. The clarification on heterogeneity across economic actors is an important aspect to better understand aggregate phenomena. To summarize, in short, aggregate wage rigidity can be a factor to explain staggered pricing, allowing for the individual sectors' heterogeneous movements.

[21] All of the values are taken in one sector setting and under $S_m = 0.0$.
[22] Unemployment rate u_t indicates straightforwardly the difference between labor supply N_t and labor demand L_t.

VI. Conclusion

This paper analyzes a pricing mechanism based on a menu cost model and the effect of monetary policy, considering the situation in recent years in Japan. There are three main findings. (i) The heterogeneity of frequency of price change and size of price change across industrial sectors amplifies the monetary non-neutrality in Japan. A particular feature of Japanese statistics is that a wide variance in the frequency of price changes contributes to the larger amplification, as the increase in rate change is ten times as much as a one-sector model, while it is approximately three times in the U.S. (ii) Incorporating intermediate inputs also magnifies the monetary non-neutrality. Considering the intermediate inputs ratio on an upward trend in Japan, the result implies that it will continue to be a significant factor when discussing the effect of monetary policy. (iii) Lastly, the introduction of real wage rigidity enhances the degree of monetary non-neutrality. Specifically, the variance of output is increased at the most, by twelve times. The amplification appears, since in response to nominal aggregate shock, the marginal cost consisting of real wage could not be adjusted immediately. As a result, firms' price becomes more staggered, and it provokes a higher variance of output. It is notable here that individual members in a household do not have a strong wage bargaining power and a firm relies on the aggregate wage level in setting price, allowing for the individual firms' heterogeneous behaviors. In addition to the changing variance in output, the variance of unemployment is positively affected as well. Ultimately, wage rigidity creates a distortion in the labor market.

This research provides certain implications about the effect of monetary policy especially in recent Japan, but it still has several limitations. First, the data used in this paper is not strict micro data. Along with developing the data science, I would expect analysis closer to individual items to be conducted in the future. Further, this research does not cover the complexity in actual society, such as variations in the elasticity of differentiated workers and in the intermediate inputs ratio. An analysis of the connections, both individual-specific and at the aggregate level, is necessary to better understand the real economy. Making everything heterogeneous simplifies the situation; however, I hope that by extracting an essence, refinement will yield efficient implications on real life.

Appendix

A) Price Data Description

<table>
<tr><td>Source</td><td colspan="2">Retail price survey (Ministry of Internal Affairs and Communications)</td></tr>
<tr><td>Target year</td><td colspan="2">From January 2011 to June 2018</td></tr>
<tr><td>Number of items in the CPI basket</td><td>Period 2011-2014
588</td><td>Period 2015-2018
585</td></tr>
<tr><td rowspan="2">Number of target items in this study (coverage (%))</td><td>472
(80.27%)</td><td>461
(78.8%)</td></tr>
<tr><td colspan="2">Items corresponding to the following criteria are excluded.
(i) Seasonal items, specifically, that are surveyed not every month (58/55 (2010/2015) items are excluded)
(ii) Items whose price is the same across cities (Selected based on tables of nationwide uniform prices of charges) (52/64 (2010/2015) items are excluded)
(iii) Items not to have enough data in terms of time span and places (6/5 (2010/2015) items are excluded)</td></tr>
<tr><td>Target cities</td><td colspan="2">The following 61 cities, which are prefectural capitals or whose population is more that 15 thousand; Hakodate, Asahikawa, Aomori, Morioka, Ishinomaki, Akita, Yamagata, Fukushima, Koriyama, Mito, Hitachi, Utsunomiya, Ashikaga, Maebashi, Kumagaya, Kawaguchi, Tokorozawa, Sakura, Urayasu, Hachioji, Tachikawa, Fuchu, Yokosuka, Atsugi, Nagaoka, Toyama, Kanazawa, Fukui, Kofu, Nagano, Matsumoto, Gifu, Fuji, Toyohashi, Tsu, Matsuzaka, Otsu, Hirakata, Higashiosaka, Himeji, Nishinomiya, Itami, Nara, Wakayama, Tottori, Matsue, Fukuyama, Ube, Yamaguchi, Tokushima, Takamatsu, Matsuyama, Imabari, Kochi, Saga, Nagasaki, Sasebo, Oita, Miyazaki, Kagoshima, and Naha.
Note that the following 19 government ordinary cities are excluded, so that the number of collected price data is equalized; Sapporo, Sendai, Saitama, Chiba, Tokyo, Yokohama, Kawasaki, Niigata, Shizuoka, Hamamatsu, Nagoya, Kyoto, Osaka, Sakai, Kobe, Okayama, Hiroshima, Kitakyushu, Fukuoka, and Kumamoto.</td></tr>
<tr><td>Remarks</td><td colspan="2">● For “model items”, that consist of several items, only one represented price data is picked up.
● For public house rent and Urban Renaissance Agency & corporation, the index is calculated with several price data based on a CPI guideline.</td></tr>
</table>

B) Derivation of Real Wage Process

From the aggregate wage level, we know

$$\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \int_0^1 \left(\frac{W_t(j)}{P_t}\right)^{1-\theta_w} dj.$$

Under wage stickiness,

$$\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \int_{1-\alpha}^{1} \left(\frac{W_{t-1}(j)}{P_{t-1}}\right)^{1-\theta_w} dj + \int_{0}^{1-\alpha} \left(\frac{W_t^*}{P_t}\right)^{1-\theta_w} dj.$$

Rewrite as,

$$\begin{aligned}\left(\frac{W_t}{P_t}\right)^{1-\theta_w} &= \int_{1-\alpha}^{1} \left(\frac{W_{t-1}(j)}{P_{t-1}}\right)^{1-\theta_w} dj + (1-\alpha)\left(\frac{W_t^*}{P_t}\right)^{1-\theta_w} \\ &= \alpha \int_{0}^{1} \left(\frac{W_{t-1}(j)}{P_{t-1}}\right)^{1-\theta_w} dj + (1-\alpha)\left(\frac{W_t^*}{P_t}\right)^{1-\theta_w}.\end{aligned}$$

As a result,

$$\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \alpha \left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\theta_w} + (1-\alpha)\left(\frac{W_t^*}{P_t}\right)^{1-\theta_w}.$$

C) Derivation of Optimal Real Wage

Again, members in a household solve this problem,

$$\max_{\frac{W_t(j)}{P_t}} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \left[\frac{1}{1-\gamma} C_{t+s|t}^{1-\gamma} - \omega \int_0^1 \frac{L_{t+s|t}(j)^{1+\psi}}{1+\psi} dj\right]$$

$$\begin{aligned} s.t\ P_{t+s|t}C_{t+s|t} &+ E_{t+s|t}\left[D_{t+s+1|t}B_{t+s+1|t}\right] \\ &\le B_{t+s|t} + \int_0^1 W_{t+s|t}(j)L_{t+s|t}(j)dj + \int_0^1 \Pi_{t+s|t}(z)dz, \\ L_{t+s|t}(j) &= L_{t+s}\left(\frac{W_t(j)}{P_t} / \frac{W_{t+s}}{P_{t+s}}\right)^{-\theta_w}. \end{aligned}$$

Set Lagrangian just associated with $\frac{W_t(j)}{P_t}$,

$$\begin{aligned}\mathcal{L} = E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \Bigg[&-\frac{\omega}{1+\psi}\left(\frac{W_t(j)}{P_t} / \frac{W_{t+s}}{P_{t+s}}\right)^{-\theta_w(1+\psi)} L_{t+s}^{1+\psi} \\ &+ \lambda_{t+s}P_{t+s}\left(\frac{W_t(j)}{P_t}\left(\frac{W_t(j)}{P_t} / \frac{W_{t+s}}{P_{t+s}}\right)^{-\theta_w} L_{t+s}\right)\Bigg].\end{aligned}$$

Obtain FOC,

$$\frac{\partial \mathcal{L}}{\partial \frac{W_t(j)}{P_t}} = \theta_w \left(\frac{W_t(j)}{P_t}\right)^{-\theta_w(1+\psi)-1} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \omega \left(\frac{W_{t+s}}{P_{t+s}}\right)^{\theta_w(1+\psi)} L_{t+s}^{1+\psi}$$

$$+ (1-\theta_w)\left(\frac{W_t(j)}{P_t}\right)^{-\theta_w} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \lambda_{t+s} P_{t+s} \left(\frac{W_{t+s}}{P_{t+s}}\right)^{\theta_w} L_{t+s} = 0.$$

Therefore,

$$\begin{aligned}
\left(\frac{W_t^*}{P_t}\right)^{1+\theta_w\psi} &= \frac{\theta_w}{\theta_w - 1} \cdot \frac{E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \omega \left(\frac{W_{t+s}}{P_{t+s}}\right)^{\theta_w(1+\psi)} L_{t+s}^{1+\psi}}{E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \lambda_{t+s} P_{t+s} \left(\frac{W_{t+s}}{P_{t+s}}\right)^{\theta_w} L_{t+s}} \\
&= \frac{\theta_w}{\theta_w - 1} E_t \sum_{s=0}^{\infty} (\beta\alpha)^s \left(\omega \left(\frac{W_{t+s}}{P_{t+s}}\right)^{\theta_w\psi} L_{t+s}^{\psi} C_{t+s}^{\gamma}\right).
\end{aligned}$$

References

Blanchard, O., and J. Gali. (2007). Real Wage Rigidities and the New Keynesian Model, *Journal of Money, Credit and Banking*, 39(s1), 35-65.

Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics*, 12, 383-398.

Caplin, A., and D. Spulber (1987). Menu Costs and the Neutrality of Money, *Quarterly Journal of Economics*, 102(4), 703-725.

Erceg. C., W. Henderson, and A. Levin. (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts, *Journal of Monetary Economics*, 46(2), 281-313.

Gali, J. (2011). The Return of the Wage Phillips Curve, *Journal of the European Economic Association*, 9(3), 436-461.

Genda, Y. (2017). *Why Wage Is Stagnated, Regardless of Shortage of Workers*. Tokyo: Keio University Press Inc.

Golosov, M., and R. E. Lucas. (2007). Menu Costs and Phillips Curves, *Journal of Political Economy*, 115, 171-199.

Hansen, G. D. (1985). Indivisible Labor and the Business Cycle, *Journal of Monetary Economics*, 16, 309-327.

Huang, K., and Z. Liu. (2002). Staggered Price-Setting, Staggered Wage-Setting, and Business Cycle Persistence, *Journal of Monetary Economics*, 49, 405-433.

James, C., N. Anton, and P. Borja. (2018). Monetary Policy Implications of State-Dependent Prices and Wages, CEPR Discussion Paper 13398.

Kawaguchi, D., and H. Hara. (2017). A Shortage of Workers and Wage Stagnation can be Explained in an Economic Theory. In Y. Genda, *Why Wage Is Stagnated, Regardless of Shortage of Workers* (p101-119). Tokyo: Keio University Press Inc.

Krusell, P., and A. Smith. (1998). Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, 106(5), 867-896.

Kuroda, S., and I. Yamamoto. (2008). Estimating Frisch Labor Supply Elasticity in Japan, *Journal of Japanese and International Economics*, 22(4), 566-585.

Morikawa, M. (2017). Service Sectors and Policy Transition: Overview, RIETI Policy Discussion Paper Serie*s*, 17-P-003.

Nakamura, E., and J. Steinsson. (2010). Monetary Non-Neutrality in a Multi-Sector Menu Cost Model, *Quarterly Journal of Economics*, 125(3), 961–1013.

Saita, Y., and M. Higo. (2007). Price-Setting in Japan: Evidence from CPI Micro Data, Working Paper, Bank of Japan, 07-J-11.
Shintani, K., and I. Muto. (2014). Wage New Keynesian Phillips Curve: Japan to U.S. Comparison, Working Paper, Bank of Japan, 14-J-2.
Shirota, T. (2006). Estimation of Phillips curve Considering Heterogeneity of Elasticity in Price, Working Paper, Bank of Japan, 06-J-16.
Midrigan, V. (2011). Menu Costs, Multiproduct Firms, and Aggregate Fluctuations, *Econometrica*, 79, 1139-1180.
Watanabe, T., and K. Watanabe. (2016). Price Rigidity in Deflation: Reason and Implication, Working Paper, Bank of Japan, 16-J-2.
Yamamoto, I., and S. Kuroda. (2016). Is Wage Deflation a Main Factor of Mild Long-lasting Deflation? In T. Watanabe, *Investigation of Chronic Deflation* (p176-204). Tokyo: Nikkei Inc.
Yoshikawa, H. (2013). *Deflation*. Tokyo: Nikkei Inc.