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The epidemiological macroeconomics with social distancing

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Abstract

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We are inspired by the SIR-macroeconomics model of Eichenbaum, Rebelo, and Trabandt, 2020, and the equilibrium social distancing model of F.M.O Toxvaerd, 2020. In our model, we veer from the analysis in ERT and we take cautiousness into consideration, which is effective for reducing the infection rate. We find that the economic outcome differs significantly. With cautiousness, about fifty percent of the decline of economic activity is mitigated in our benchmark model and the infection "curve" is flattened substantially. We demonstrate that an SIR-macro model embodying cautiousness, which is a contributing factor in preventing infection, implies a different magnitude value of containment. Someone could view our results of voluntary cautiousness as the “Swedish” solution: Sweden avoids government restricting economic activities, and permits people to make their own decisions on consumption, work, and voluntary social distancing. These private incentives and effective voluntary cautiousness may not only end the epidemic on their own but also decelerating the decline in economic activities. Although voluntary cautiousness is more contributing to the reduction of infection than containment policy, in our model. Containment policy is necessary for maximizing social welfare since the social welfare loss caused by death is much larger than the recession.

Keywords: Epidemic, recession, containment policy, SIR macro model, social distancing, cautiousness, vaccine.

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Contents

1 Introduction

The COVID-19 outbreak in 2020, has a huge impact on the whole world. Policymakers are struggling with understanding the mechanism between the rates of infection and economic activities. International Monetary Fund (2020) provides empirical results that the introduction of lockdown is a contributing factor in the recession, but voluntary social distancing in response to rising infections also contributed significantly to the economic contraction. It is necessary to consider the impact of voluntary social distancing on economic activity and infection when design policies. Eichenbaum, Rebelo, and Trabandt (2020), ERT for short from now on, analyze the equilibrium and the response of economic decisions to the epidemic dynamics. ERT assume that the probability of susceptible individuals getting infected depends on people's economic behavior. Since people contact each other when they purchase consumption goods or working.

To extend the model of ERT, inspired by F.M.O Toxvaerd (2020), we share the assumption that individuals can change the infection rate by controlling the level of their exposure when they face the threat of infection. But in contrast, we consider this control as the cautiousness in interaction with others. This cautiousness reduces the infection rate when susceptible people consume, work, and meet with others.

As a result, the epidemic affects aggregate consumption and aggregate labor supply effects, but they are more complicated. As the people expose themselves to the virus when they are working, people react to that risk by curtailing their labor supply. Also, since people expose themselves to the virus when they are consuming, people respond to that risk by reducing their consumption. In addition, the more severe the infection is, higher the intensity of the cautiousness, and it reduces their exposure when they consume and work. People are facing the trade-off among consumption expenditure, hours worked and the cautiousness. The effectiveness of cautiousness helps the economy avoid severe recession since people have an alternative to prevent themselves from infection rather than reducing consumption and labor supply only. Our results show that cautiousness is not only a contributing factor to flatten the infection "curve", which means lower the peak of infection, but also avoid the severe recession.

The impact of cautiousness on infection rate in the SIR-macro model, significantly alters the evolution of the epidemic and reduces its effect on the economy. An alternative to analyze this impact is devoting attention to the most basic SIR-macro model. This model is simplified from the possibility of developing vaccination and treatment, and we compare our SIR-macro model to the one in ERT. In contrast to the basic SIR-macro model in ERT, our basic SIR-macro model leads to a less severe recession and fewer deaths. The first-year decline on the average aggregate consumption since the epidemic outbreak decreases from 4.7 percent in the SIR-macro model in ERT to 2.0 percent which is roughly a half. The reduction of economic activities and the effectiveness of cautiousness decrease the infection peak from 5.3 to 1.4 percent and the proportion of the population that becomes infected from 54 to 35 percent. Also, the total death toll in the U.S. decreases from 880 thousand to 580 thousand.

Nevertheless, the competitive equilibrium in our basic SIR(susceptible-infected-recovered)-macro model is not Pareto efficient. Since infected people do not entirely internalize the effect of the spread of the disease caused by their consumption and work. The optimal conditions of the self-decision of infected people on the consumption expenditure and labor supply by infected people are different from those of maximizing social welfare. People who are infected consume and work more in the competitive equilibrium than the social optimal scenario, which increases the probability of susceptible people getting infected when a susceptible individual is involved in economic activities.

To deal with the externality, as in ERT and Getachew (2020), we consider consumption tax as simple containment policies that reduce consumption and labor supply. Since these policies reducing economic interactions among people, recession becomes more severe but they increase social welfare by reducing the number of death caused by the virus. With cautiousness, our results show that optimal containment policies are relatively modest compared to those in ERT.

Epidemics end if the population achieves "herd immunity", in the SIR and the basic SIR-macro models. The "herd immunity" means that an adequately high fraction of the population gains immunity. Without the vaccine, it is only one way which is becoming infected and recovered to acquire immunity. However, absent effective medical treatments, this method engages in the death of people caused by the virus.

Although it is able to restrain the infection by introducing stringent, permanent containment policies. Three problems occur with this approach. First, the permanent containment policies lead to an enduring economic depression. Second, the population cannot reach such herd immunity. Hence, infections would recur once containment was lifted. Third, lifting containment has only a limited impact on the rebound in economic activities if the epidemic persists. People reduce consumption and labor supply and adopt cautiousness until health risks vanish.

The optimal policy in our model is to establish the fraction of the population that acquires immune, and differ dramatically from the one in ERT. This policy engages gradually raising containment measures as cautiousness wanes and infections decline and deliberately relaxing them as the population approaches the critical immunity level.

Also, our model captures the people's responses to the possibility of effective treatment and vaccine being discovered more precisely. With the possibility of effective treatment, people would like to participate more in economic activities since the expectation of loss caused by being infected becomes smaller. These results imply that the expectation of an effective treatment encourages susceptible people to run the risk of infection, i.e. people become more impatient. Less severe recession but more infection, the death toll, and lower cautiousness. The optimal containment policy in the treatment model is roughly parallel to the one in the basic SIR-macro model.

With vaccines as a possibility, people would like to engage less in economic activities. Since the lifetime utility of remaining susceptible increases which means the expected utility associated with being infected are smaller, i.e. people become more impatient. More severe recession but less infection, the death toll, and higher cautiousness. The difference of the optimal policy between the basic SIR-macro model and the model with vaccines is great. Optimal policy is introducing severe containment measures as soon as possible to minimize the number of deaths. Since the anticipation of a vaccine decreases the necessity of building herd immunity that prevents the recurrence of an epidemic.

The most general version of our model considering the probabilistic development of vaccines and treatments is discussed in Section 4, and also considered as our benchmark SIR-macro model.

Why our optimal containment policy is different from the one in ERT? The reason is susceptible people adopt cautiousness to prevent them from getting infected when they are involved in economic activities. In contrast, susceptible people in the model of ERT can only lower the infection rate by less consumption and hours worked. Hence, containment policy is effective in ERT for the reduction of infection and death toll, accompanied by a more severe recession. The main purpose of containment policy in our model is reducing death toll. In order to prevent a recurrence of the epidemic without vaccines, it needs sufficient of the population to gain immunity by getting infected and recovered. The cautiousness effectively lowers the peak of infection and flatten the curve of infection, which postpones the time of acquiring "herd immunity". These also explain why the optimal containment policy in our model persists longer.

Compared to the basic SIR-macro model with optimal containment policies in ERT, our optimal containment policies in the basic SIR-macro model lead to a less severe recession and fewer death toll. The average aggregate consumption decline in the first year of the epidemic decreases from 17.1 percent in the SIR-macro model of ERT to 3.7 percent which is roughly a quarter. The effectiveness of cautiousness reduces the infection peak from 3.2 percent to 1.3 percent. And the share of the population that becomes infected declines from 43 percent to 33 percent. Also, the total death toll in the U.S. caused by the virus declines from 710 thousand to 544 thousand. These results imply an intuition that is not traditional and reveals the contribution of cautiousness adopted by people. The effectiveness of cautiousness on infection rate changes the pattern and intensity of optimal containment policy.

Compare our benchmark model to the SIR-macro model with treatments and vaccines in ERT, with optimal containment policy. Optimal containment policy in our benchmark model avoids relatively severe recession. The average aggregate consumption expenditure decline in the first year of the epidemic decreases from 16.6 percent in the SIR-macro model of ERT with treatments and vaccines to 6.8 percent which is roughly a half. The effectiveness of cautiousness reduces the infection peak from 3.4 to 1.2 percent. Also, the share of the population that becomes infected declines from 48 to 35 percent. In addition, the death toll in the U.S. caused by the virus declines from 792 thousand to 579 thousand. Optimal strategy for the policymaker is to immediately introduce containment, but containment policies and the recession are relatively less severe.

To obtain transparent intuition for our results, we construct a relatively simple model. This simplicity causes that we are not able to analyze many critical policy issues. For example, our model is simplified from financial markets. And we do not consider the heterogeneity of cautiousness for consumption and work. People should be able to choose different levels of cautiousness on different events. Also, we do not consider the scenario that treatments and vaccines really come. We propose to embody these important issues in future study.

The papers most closely related to ours are Eichenbaum, Rebelo, and Trabandt (2020) and F.M.O Toxvaerd (2020). F.M.O Toxvaerd (2020) describes social distancing as a method that can lower the infection rates on interaction of people and studies equilibrium social distancing.

Our analysis also relates to other recent work such as Farboodi, Jarosch, and Shimer (2020). Consistent with our main mechanism, they analyze US micro-data and argue that a large reduction in social activity by private household occurs even prior to the introduction of public stay-at-home-orders and lockdown policies of economic activity. F. Toxvaerd and Rowthorn (2020) consider the equilibrium and socially optimal vaccination and treatment for the inducement of herd immunity Getachew (2020), studies the optimal social distancing in an SIR based macroeconomics models. Fenichel (2013) compare the incentives and outcomes of social distancing under decentralized, complete control social planner, and restrained social planner which has no control on health class specification, decision-making scenarios. Alvarez, Argente, and Lippi (2020) study the optimal lockdown policy for a policymaker who wants to restrain the death toll in a pandemic while minimizing the economic loss of the lockdown. Bayraktar, Cohen, and Nellis (2020) construct an SIR model considering herd immunity, behavior-related transmission rates, working from home, and indirect externalities of lockdown. Our paper relates to Jones, Philippon, and Venkateswaran (2020) on the cautiousness. Their study analyzes the optimal containment policy in a model where economic activities interact with epidemic evolution. We can view working from home as cautiousness at work. Krueger, Uhlig, and Xie (2020) draw a conclusion that permitting substitution of consumption across sectors with diverse degrees of infection probabilities, the “Swedish solution” of terminating the epidemic without government containment and permitting agents to shift their sectoral behavior on their own can cause a substantial mitigation of the economic and human loss of the COVID-19 crisis.

In our model, we veer from the analysis in ERT and we take cautiousness into consideration, which is effective for reducing the infection rate. We find that the economic outcome differs significantly. With cautiousness, about fifty percent of the decline of economic activity is restrained in our benchmark model and the infection "curve" is flattened substantially. We demonstrate that an SIR-macro model embodying cautiousness, which is a contributing factor in preventing infection, implies a different magnitude value of containment. Someone could view our results of voluntary cautiousness as the “Swedish” solution: Sweden avoids government restricting economic activities, and permits people to make their own decisions on consumption, work, and voluntary social distancing. These private incentives and effective voluntary cautiousness may not only end the epidemic on their own, but also decelerating the decline in economic activities. Although voluntary cautiousness is more contributing to the reduction of infection than containment policy, in our model. Containment policy is necessary for maximizing social welfare since the social welfare loss caused by death is much larger than the recession.

In the next section, we describe a macroeconomic framework. And we develop the SIR and the basic SIR-macro model. In addition, We extend our basic SIR-macro model with the possibility of effective treatment and vaccines is available in section 3. We choose parameters and analyze competitive equilibria in various versions of our model in section 4. Section 5 contains the results of the optimal policy in our model. In section 6, we analyze the benchmark model and the optimal policy and discuss the impact of cautiousness. Section 7 is the conclusion.

2 Model

2.1 The pre-epidemics economy

Our macroeconomic framework based on Eichenbaum et al. (2020) and shares some crucial model ingredients. The population of this economy is normalized to 1 and individuals are identical. Every household maximizes the objective function

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

where $\beta \in (0,1)$ denotes the discount factor, c_t denotes consumption of agents and n_t denotes hours worked. Like ERT, we assume the momentary utility with the form as

$$u(c_t, n_t) = \ln c_t - \frac{\theta}{2} n_t^2.$$

Budget constraint of the representative individual is[1]:

$$(1 + \mu_t) \, c_t = w_t n_t + \Gamma_t.$$

Here w_t denotes the real wage rate, μ_t denotes consumption tax rate, and γ_t is the lump-sum transfers from the government. Following ERT, we regard μ_t as containment policy and consider μ_t as the containment rate.

The first-order condition for the representative household's problem is:

$$(1 + \mu_t) \theta n_t = c_t^{-1} w_t.$$

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[1] We abstract from capital and thus from intertemporal savings decisions, as ERT do.

We think of the production technology as that produce consumption goods using hours Production function:

$$C_t = AN_t.$$

The firm maximize its profits Π_t:

$$\Pi_t = AN_t - w_t N_t$$

The budget constraint for the government is given as following:

$$\mu_t c_t = \Gamma_t$$

In equilibrium, $n_t = N_t$ and $c_t = C_t$

2.2 The epidemic

The dynamics of epidemic

We follow the Susceptible-Infected-Recovered Model from ERT. In our assumption, we divide the population into four groups: "susceptible" people of S_t, who do not acquire immunity and have not infected the disease but are facing the risk, "infected" people of I_t, who are currently infected by the virus, "recovered" people of R_t, who recovered from the virus and are immune, "dead" people of D_t, who died from the virus. The total newly infected people is denoted by T_t. Extending ERT, we assume that the probability τ_t for a susceptible agent getting infected not only depends on consumption and working and events not concerned with not involved in economy activities, but also depends on cautiousness. Since people can choose cautiousness, when they are involved in these economic activities and interact with others.

First, the total newly infected people that get infected from consumption activities is denoted by $\pi_1(S_tC_t^s)(I_tC_t^i)$, where$S_tC_t^s$ represents the aggregate consumption of susceptible people and $I_tC_t^i$ represents the aggregate consumption of infected people. Also the parameter π_1 represents the probability of getting infected caused by consumption.

Second, the total newly infected people that get infected from interactions at work is denoted by $\pi_2(S_tN_t^s)(I_tN_t^i)$. The term $S_tN_t^s$ represents the aggregate hours worked by susceptible people and $I_tN_t^i$ represents the aggregate labor supply by infected people. The parameter π_2 reflects the probability of getting infected caused by work interactions.

Third, susceptible and infected individuals are able to interact with each other in events not concerned with not involved in economy activities. The number of random meetings is related to the number of infected and susceptible people S_tI_t . These interactions lead to $\pi_3S_tI_t$ newly infected people. The parameter π_3 represents the probability of getting infected caused by random interactions.

Forth, as in Toxvaerd(2020), we assume that susceptible people are able to affect the infection rate by deciding the level of which they expose themselves to the risk of infection, which means cautiousness is an effective method of infection control. Susceptible people choose cautiousness $\varepsilon_t^s \in [0,1]$, while exposure level is $1 - a\varepsilon_t^s$, where The parameter a represents the effectiveness of cautiousness. Since we assume the susceptible people can control the exposure rate perfectly, $a = 1$. Effectively, this reduces the infection rate for susceptible people to $1 - \varepsilon_t^s$ proportion of the original. Besides, infected and recovered people cannot gain any benefit from cautiousness and not engage in any preventive efforts since they are assumed to be atomistic.

The total number of newly infected people is denoted by:

$$T_t = (1 - a\varepsilon_t^s)[\pi_1(S_t C_t^s)(I_t C_t^i) + \pi_2(S_t N_t^s)(I_t N_t^i) + \pi_3 S_t I_t]. \tag{1}$$

The dynamics of the population in the epidemic period is illustrated by the following equations,

$$S_{t+1} = S_t - T_t, \tag{2}$$

$$I_{t+1} = I_t + T_t - (\pi_r + \pi_d)I_t, \tag{3}$$

$$R_{t+1} = R_t + \pi_r I_t, \tag{4}$$

$$D_{t+1} = D_t + \pi_d I_t, \tag{5}$$

$$Pop_{t+1} = Pop_t - \pi_d I_t, \tag{6}$$

with initial conditions,

$$I_0 = \delta,$$

$$S_0 = 1 - \delta,$$

$$R_0 = 0,$$

$$D_0 = 0.$$

where π_r and π_d are the recovery rate and mortality rate for infected people, respectively. The total population in period t is denoted by Pop_t. As ERT do, we assume that the epidemic outbreaks with a pandemic shock $I_0 = \delta$ and $S_0 = 1 - \delta$.

The SIR-Macro model for epidemic

In contrast to ERT, we consider the utility function of susceptible People including disutility from social distancing as Toxvaerd (2020) and Getachew (2020) do. The term $\frac{\chi}{2}\varepsilon_t^{s2}$ is the cost of cautiousness. The parameter χ reflects how costly the cautiousness is.

Current utility function for susceptible people is given by

$$u^s(c_t^s, n_t^s, \varepsilon_t^s) = \ln c_t^s - \frac{\theta}{2} n_t^{s2} - \frac{\chi}{2} \varepsilon_t^{s2}.$$

Current utility function for infected or recovered people is given by

$$u^j(c_t^j, n_t^j) = \ln c_t^j - \frac{\theta}{2} n_t^{j2},\ j = i, r.$$

Like ERT, budget constrains for different types of people are revised as following

$$(1 + \mu_t) c_t^j = w_t \phi^j n_t^j, \quad j = i, s, r. \tag{7}$$

The parameter ϕ denotes the labor productivity, and it is equal to 1 for susceptible and recovered people ($\phi^s = \phi_r = 1$). The labor productivity of infected people is smaller than one ($\phi_i < 1$).

Susceptible People The lifetime utility of individuals who are currently susceptible from period t on is denoted as U_t^s and the lifetime utility of currently infected individuals from period t on is denoted as U_t^i. As in ERT, the recursive form for the lifetime utility U_t^s is

$$U_t^s = u^s\left(c_t^s, n_t^s, \varepsilon_t^s\right) + \beta\left[\left(1 - \tau_t\right) U_{t+1}^s + \tau_t U_{t+1}^i\right], \tag{8}$$

where τ_t is the infection rate that a susceptible person becomes infected.

$$\tau_t = \left(1 - a\varepsilon_t^s\right)\left[\pi_1 c_t^s \left(I_t C_t^I\right) + \pi_2 n_t^S \left(I_t N_t^I\right) + \pi_3 I_t\right]. \tag{9}$$

τ_t increases in the consumption expenditure, working hours, random meeting and and decreases in cautiousness. susceptible people maximize the lifetime utility U_t^s subject to the budget constraint (7), and the infection probability constraint (9), by choosing consumption c_t^s hours worked n_t^s, cautiousness ε_t^s and infection probability τ_t.

The first order conditions of the susceptible people's problem for consumption and hours worked are

$$u_1\left(c_t^s, n_t^s, \varepsilon_t^s\right) - (1+\mu_t)\lambda_{bt}^s - \lambda_{\tau t}^s(1-a\varepsilon_t^s)\pi_1\left(I_t C_t^I\right) = 0, \tag{10}$$

$$u_2(c_t^s, n_t^s, \varepsilon_t^s) + w_t\lambda_{bt}^s - \lambda_{\tau t}^s(1-a\varepsilon_t^s)\pi_2\left(I_t N_t^I\right) = 0, \tag{11}$$

where λ_{bt}^s and $\lambda_{\tau t}^s$ are the Lagrange multipliers associated with the constraints (7) and (9).

The first order conditions for cautiousness is

$$u_3(c_t^s, n_t^s, \varepsilon_t^s) + \lambda_{\tau t}^s a\left[\pi_1 c_t^s\left(I_t C_t^I\right) + \pi_2 n_t^S\left(I_t N_t^I\right) + \pi_3 I_t\right] = 0, \tag{12}$$

The first order conditions for infection rate is

$$\beta(U_{t+1}^s - U_{t+1}^i) - \lambda_{\tau t}^s = 0. \tag{13}$$

The firs order conditions (12) and (13) imply that the marginal cost (disutility) of cautiousness is equal to the marginal benefit marginal reduction of infection rate multiplied by the discounted welfare gap between a susceptible individual and an infected individual.

Infected People As in ERT, the lifetime utility of individuals who are currently infected, U_t^i, is

$$U_t^i = u(c_t^i, n_t^i) + \beta\left[(1-\pi_r-\pi_d)\,U_{t+1}^i + \pi_r U_{t+1}^r + \pi_d \times 0\right]. \tag{14}$$

The expression of U_i^i imply a common assumption in macro and health economics, utility of a dead person is null.

The first order conditions of the infected individual's problem with respect to consumption expenditure and hours worked are

$$u_1\left(c_t^i, n_t^i\right) - \lambda_{bt}^i(1+\mu_t) = 0, \tag{15}$$

$$u_2\left(c_t^i, n_t^i\right) + \lambda_{bt}^i w_t h^i = 0, \tag{16}$$

where λ_{bt}^i is the Lagrange multiplier associated with budget constraint (7).

Recovered people As in ERT, the lifetime utility of individuals who are currently recovered, U_t^r, is

$$U_t^r = u\left(c_t^r, n_t^r\right) + \beta U_{t+1}^r. \tag{17}$$

The first order conditions of the recovered individual's problem with respect to consumption expenditure and hours worked are

$$u_1\left(c_t^r, n_t^r\right) - \lambda_{bt}^r(1+\mu_t) = 0, \tag{18}$$

$$u_2\left(c_t^r, n_t^r\right) + \lambda_{bt}^r w_t = 0, \tag{19}$$

where λ_{bt}^r is the Lagrange multiplier associated with budget constraint (7).

Government budget constraint

The budget constraint of government is given by

$$\mu_t\left(S_t c_t^s + I_t c_t^i + R_t c_t^r\right) = \Gamma_t\left(S_t + I_t + R_t\right). \tag{20}$$

Equilibrium In equilibrium, every individual maximizes their lifetime utility satisfying the government budget constraint. Also, the goods markets and labor markets clearing conditions are:

$$S_t C_t^s + I_t C_t^i + R_t C_t^r = A N_t, \tag{21}$$

$$S_t N_t^s + I_t N_t^i \phi^i + R_t N_t^r = N_t. \tag{22}$$

Our algorithm for computing the equilibrium is illustrated in the appendix A.

3 Treatments and vaccines

In this section, we apply two extensions to the SIR-macro model[2]. First, we consider the probability of developing a treatment that can cure the disease. Second, we consider the probability of developing a vaccine that can immunize people who are susceptible.

3.1 The SIR-macro model with treatment

Our basic SIR-macro model does not consider the probabilistic development of an effective treatment against the virus. We assume that the probability of discovering an effective treatment that can cure infected people is δ_c. Once the treatment is discovered, all infected people can receive treatment in the discovery period and they are transformed into recovered people in all subsequent periods, which implies that the number of newly dead people becomes zero.

As a result, like in ERT, the lifetime utility of people are currently infected before the treatment is discovered becomes

$$U_t^i = u(c_t^i, n_t^i) + (1-\delta_c)\left[(1-\pi_r-\pi_d)\,\beta U_{t+1}^i + \pi_r \beta U_{t+1}^r\right] + \beta\delta_c U_{t+1}^r. \quad (23)$$

The equation implies that an infected individual in period t remains being infected in period $t+1$ with probability $1-\delta_c$. This individual receives treatment and gets recovered with probability δ_c.

Until the treatment is introduced, the dynamics of population evolution follows equations (1),(2),(3),(4) and (5). If the treatment is not available until period t^*, and all infected people become recovered since they are cured. The dynamics of population changed once the treatment is available and the

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[2] As in ERT, we include treatments and vaccines in our model, but in contrast to ERT, we shut medical preparedness preparedness. For comparison and robustness, we also provide a version of results with higher mortality.

number of dead people stabilizes as

$$D_t = D_t^* \text{ for } t \geq t^*.$$

And since an effective treatment can cure any infected individual immediately, we normalize the number of susceptible people and infected people to zero for $t > t^*$. Then the total number of recovered people becomes

$$R_t = 1 - D_t.$$

3.2 The SIR-macro model with vaccination

The basic SIR-macro model does not consider the probabilistic development of an effective treatment against the virus. We assume that the probability of discovering a vaccine that can cure infected people is δ_v. Once the vaccine is discovered, it is available to all susceptible individuals from the discovery period.

As a result, like in ERT, the lifetime utility of susceptible people before the vaccine is discovered is

$$U_t^s = u(c_t^s, n_t^s) + (1 - \delta_v)[(1 - \tau_t)\beta U_{t+1}^s + \tau_t \beta U_{t+1}^i] + \beta \delta_v U_{t+1}^r. \tag{24}$$

The expression implies that an susceptible individual in period t remains being susceptible in period $t + 1$ with probability $1 - \delta_v$. This individual is vaccinated and which means becoming recovered with probability δ_v. The vaccine has no influence on infected and recovered people.

Until the vaccine is introduced, the dynamics of population evolution follows equations (1),(2),(3),(4) and (5). If the vaccine is not available until period t^*, and all susceptible people become immune since they can get vaccinated. All susceptible people become recovered once the vaccine is available and dynamics of population changed.

The number of people who are susceptible and recovered shortly since a vaccine is discovered at time t^* are denoted by S'_{t*} and R'_{t*}, as in ERT. These variables are given by

$$\begin{aligned} S'_{t*} &= 0, \\ R'_{t*} &= R_{t*} + S_{t*}. \end{aligned}$$

For $t \geq t^*$ we have

$$R_{t+1} = \begin{cases} R'_t + \pi_r I_t & \text{for } t = t^*, \\ R_t + \pi_r I_t & \text{for } t > t^*. \end{cases}$$

The evolution of infected people I_t and dead people D_t are given by (3) and (5).

TABLE 1: Parameter Values

Parameter	**Value**	**Description**
Parameter	Value	Description
π_1	7.8408×10^{-8}	Infection intensity on consumption
π_2	1.2442×10^{-4}	Infection intensity on work
π_3	0.3901	Autonomous Infection Intensity
π_d	1.944×10^{-3}	Mortality rate
π_r	0.387	Recovered rate
χ	1	Cost of cautiousness
a	1	Effectiveness of cautiousness
θ	1.275×10^{-3}	Cost of hours work
A	39.835	Technology
β	$0.96^{1/52}$	Discounted factor
ϕ_s	1	Productivity of susceptible people
ϕ_i	0.8	Productivity of infected people
ϕ_r	1	Productivity of recovered people

4 Competitive equilibrium

We analyze the competitive equilibrium with several numerical exercises in this section. First, we describe our parameter values. Second, we discuss the responses of people to the epidemic in the SIR-macro model and contrast it to SIR model and the SIR-macro model in ERT. Third, we analyze probabilistic development of the effects of treatments and vaccines. Finally, we display the robustness of our results.

4.1 Parameterization

The parameters we choose refer to Eichenbaum, Rebelo, and Trabandt, 2020 as Table 1.

4.2 The SIR-macro model and cautiousness

The equilibrium population dynamics of our SIR model are the as same as ERT, as the black dashed lines in Figure 1 show. The infection peak is 6.8

percent of the initial population in week 31. Until 60 percent of the initial population has got infected, the population reaches "herd immunity". This shows that about 200 million Americans will get infected if we assume a U.S. population of 300 million people. And about one million people in the U.S. die of the virus which has a mortality rate of 0.5 percent.

The solid blue lines and red-dashed-dotted lines in Figure 1 display the dynamics of the epidemic in our model, and the results of ERT, respectively. In contracts to ERT, our SIR-macro model include cautiousness which is chosen by susceptible people and is effective against infection. In our SIR-model, susceptible people are able to decrease the infection rate by decreasing consumption expenditure, labor supply, and adopting cautiousness while cautiousness is not included in ERT.

The infection peak occurs at 1.389 percent in week 30. The peak is substantially lower although it occurs earlier while the corresponding peak is 5.3 percent in week 33 in the SIR-macro model in ERT. Cautiousness is an influential factor that substantially lowers the infection to roughly and 26 percent of the corresponding peak in the SIR-macro model in ERT. In the end, 35.1 percent of the original population has got infected. Hence, in the case of U.S., about 116 million will get infected and 580 thousand people will die. Death toll becomes 0.175 percent while it is 0.268 percent in SIR-macro model in ERT and 0.3 percent in the SIR model.

Adopting cautiousness, eventually, the share of the initial population get infected is 35.1 percent while it is 53.5 percent in the SIR-macro model in ERT. Figure 1 displays that the severity of the epidemic is less in our SIR-macro model including cautiousness than in the SIR model and the SIR-macro model in ERT. Since in our SIR-macro model, susceptible people apply voluntary cautiousness that can significantly lower infection rate and also reduce consumption and hours worked. Figure 2 displays that infected and recovered people behave the same as in the SIR model.

The evolution of cautiousness is displayed in Figure 4. First, susceptible people escalate cautiousness levels gradually over time. The cautiousness in the competitive equilibrium increases from 1.5 percent in period zero to a peak of 23.5 percent in period 32. The increase in cautiousness approximately parallels the evolution of the rates of infection. We can obtain intuitions as follows. Susceptible people face the trade-offs among consumption, hours worked, and cautiousness. When the number of infected people rises, though the discounted welfare gap between a susceptible and an infected individual is invariant, it is optimal for susceptible individuals to raise cautiousness level. Because the marginal reduction of infection rate provided by marginal cautiousness rises.

Although, as in ERT, the severity of the recession becomes higher in the SIR-macro model. The first-year decline on average aggregate consumption since the epidemic outbreaks is 2.0 percent, 2.9 times larger than the corresponding decline in the SIR model. But, it is 42.9 percent of the fall in the SIR-macro model in ERT, which means cautiousness can curb the recession. Similarly, the dynamics and magnitude of the reduction in hours worked in our SIR-macro model are different from the model in ERT. The peak of the decline of 2.8 percent appears in week 30. After that period, aggregate hours increase and converge to a new steady state. The economic decisions of susceptible individuals drive these dynamics. Also, the long-run decline in aggregate hours is 0.2 percent in our model, which is lower than 0.27 percent in ERT. Because fewer people die of the virus, the remaining population is more in our SIR-macro model than in ERT.

Figure 1 and Figure 2 show that cautiousness adopted by susceptible people allows them to consume and work more safely, so that it lowers the fall of consumption. Facing the threat of virus, assuming cautiousness is effective, susceptible people choose voluntary cautiousness level and slightly decrease their consumption expenditure and labor supply to reduce the infection rate, rather than just severely reduce the consumption and hours worked. The infection peak occurs 3 weeks earlier than the corresponding peak in ERT, but the magnitude is weaker, and cautiousness flattens the curve of infection and reduces the death toll significantly.

Figure 3 and Figure 4 imply the competitive equilibrium in the SIR-macro model. And they indicate the optimal containment policy.

4.3 The treatments and vaccines models

The treatments model

Figure 5 displays, as in ERT, people would like to participate more in economic activities since the expectation of loss caused by getting infected becomes smaller. The first-year decline on average aggregate consumption since the epidemic outbreaks is 1.969 percent with treatment, while it is 1.996 percent without the treatment. The difference in consumption is slight. The peak of infection rises to 1.44 percent from 1.389 percent in our basic SIR-macro model. The death toll is increased by 4 thousand people, in the case of the U.S.. Cautiousness level becomes lower than in our basic SIR-macro model since the expected cost of being infected is smaller. These results implies that the expectation of an effective treatment encourages susceptible to run the risk of infection.

The vaccines model

As in ERT, vaccines immunize susceptible people from becoming infected but can not cure infected people. In contrast, the treatment is a remedy for infected people but does not help susceptible people acquire immunity. In contrast to ERT, these differences are not only important for the design of optimal policy but also important for the competitive equilibrium. People become less willing to engage in economic activities than in the basic SIR-macro model. Because the lifetime utility of remaining susceptible increases which means the expected utility associated with being infected is smaller.

Figure 5 and Figure 6 show that the impacts of vaccines and treatments are opposite on the competitive equilibrium. With vaccines as a possibility, susceptible people reduce consumption and hours worked while they increase the intensity of cautiousness. These actions lower the peak of infection and the death toll, and flatten the curve of infection, accompanied by a more severe recession. The impact of the vaccine on the competitive equilibrium has a larger magnitude than the impact of treatment.

The model combining treatments and vaccines

Figure 9 and Figure 10 display that by combining treatments and vaccines, the net results follow the pattern of vaccines. These imply that the impact of vaccines is dominant in the SIR-macro model with treatments and vaccines except for the death toll. The impact of vaccines is dominant on the death toll before period 185. The anticipation of vaccines also reduces the importance of building "herd immunity". The impact of vaccines is larger in the early period than in the late period when herd immunity is almost provided.

4.4 Robustness

Table 2 displays the results of the robustness test where we change crucial parameters of the basic SIR-macro model. We veer from our baseline parameters, such that as 60 percent of long-run infection rate in the SIR model to 50 and 70 percent. Intuitively these results show that a larger value of long-run infection rate leads to a more severe decline in consumption, higher crest infection rate, total death rate, and cautiousness.

Then we vary the parameter ϕ^i that changes the production ability of infected people. A smaller ϕ^i leads to the less severe average consumption drop, and less crest infection rate, total death rate, the death toll of U.S., and cautiousness.

The results for changing parameters of the infection transmission function are shown as follows. We choose our baseline parameters so that economic activities are responsible for 1/3 of initial the infection rate. Table 2 displays results for the scenario where economic activities are responsible for 1/6 of the initial infection rate. In this case, the decline in consumption is less. This change raises the peak infection rate, the total death rate, the total number of U.S. deaths, and the cautiousness. These results reflect that people realize that economic activities have less of an impact on the infection rate and they are willing to consume more. In addition, table 2 displays the scenario in which economic activities are responsible for 2/3 of the initial infection rate. In this case, the decline in consumption is more severe. However, the crest infection rate, the total death rate, and cautiousness are less. Intuitively, people reduce more on consumption and hours worked since the impacts of these activities on the risk of infection become larger.

Also, we discuss the case that the mortality rate rises from 0.5 to 1 percent. This variation intensifies the severity of the recession since people reduce their consumption expenditure and labor supply to lower the risks of infection. Although crest infection rates decrease, the total death rate, the death toll, and cautiousness increase.

Table 2 indicates the influence of a variation in the medical-preparedness parameter, κ in ERT. When κ decreases, the degree of medical preparedness increases. We choose a value of $\kappa = 0.9$ as ERT do. Table 2 displays that this higher value of κ leads to a more severe recession. Since individuals reduce consumption and hours worked to reply for the higher mortality rates. While the peak level of infections decreases, the death toll of U.S. deaths increases.

But the overall quantitative sensitivity is small.

Considering that reduce the discount factor from $0.96^{1/52}$ to $0.94^{1/52}$, the value of life decreases from 9.3 million to 6.1 million 2019 dollars. Since people become more impatient, consumption expenditure decreases less in the pandemic period, crest infection rates increase and cautiousness falls.

Also, we report the results for different parameters of the cost of cautiousness. Recall that the cost of cautiousness is 1 in the benchmark model. Table 2 shows results for the case where economic decisions respond to 0.5 of the cost of cautiousness. In this scenario, the cautiousness is larger. The fall in consumption, the crest infection rate, and the death toll is smaller. In this scenario that 2 of the cost of cautiousness, the cautiousness is smaller. The reduction in consumption, the peak infection rates, and the total death rate become larger.

Finally, we reduce the effectiveness of cautiousness from 1 to 0.5. People adopt less cautiousness. This variation increase leads to more severe recession since people decrease their consumption and labor supply to reduce the risk of infection. And it also increases the peak infection rate the total death rate, the number of U.S deaths.

From the above, Table 2 reports that the qualitative results of the benchmark model are very robust. These results imply that the intensity of cautiousness is more sensitive to the cost and mortality rate than the effectiveness.

5 Economic policy

5.1 Ramsey problem

The competitive equilibrium in the SIR-macro model is not Pareto optimal since there are externalities related with the behavior of infected people. Assuming people are atomistic, they do not consider the impacts of their action on infection when they choose consumption expenditure, hours worked and cautiousness. In order to deal with this externality, we set up a Ramsey problem with method of consumption tax[3], as ERT do, and compare our results to theirs. We consider this tax as the containment rate, like ERT.

We firstly solve the model for the optimal sequence of 250 containment rates $\{\mu_t\}_{t=0}^{249}$ that maximize social welfare, U_0. U_0 is defined as a weighted average of the lifetime utility of the different types of individuals. Since in period zero $R_0 = D_0 = 0$, the value of U_0 is

$$U_0 = S_0 U_0^s + I_0 U_0^i.$$

After the sequence of containment rates is obtained, we calculate the competitive equilibrium and the value of the social welfare function. We apply iteration to obtain find the optimum sequence, as ERT do.

Our optimal policies are displayed in Figure 3. In contrast to ERT, the increase in containment rates is different from the evolution of the infection rate itself, while cautiousness parallels that dynamics. The optimal containment rate increases from 2.6 percent to a peak value of 16.0 percent, from period 0 to period 104. But it only increases from 2.6 percent in period zero to 3.8 percent in period 33, which is roughly the week of infection peak, and it

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[3] As mentioned in ERT, in reality, governments are able to decrease social interactions in many ways. We model these measures as consumption tax, and the proceed is rebated lump sum to every individual.

increases from 3.8 percent in period 33 to 16.0 percent in period 104. The cautiousness under the optimal containment policy roughly parallels the cautiousness in the decentralized economy, and the peak decreases from 23.5 percent to 22.9 percent.

In ERT, as infected people increases, the policymaker should introduce stringent containment measures. But in our model, as the number of infected people rises from the initial state to the peak value, the optimal containment rate only rises from 2.6 percent to 3.8 percent. The increase accelerates right after the peak of cautiousness. These imply that the intensity of voluntary cautiousness chosen by susceptible people in the competitive equilibrium is analogous to maximizing social welfare. Since cautiousness is a contributing factor to the decline in infection rate, and cautiousness roughly parallels the pattern of the number of infections, when the intensity of cautiousness is relatively high, it is optimal to keep the containment rate low and strengthen containment when the intensity of cautiousness falls.

These results imply that containment policies internalize the externalities which are associated with the activities of infected people. For example, in period zero only a few infected people, which means the externality is relatively less important. Stringent containment rate in period zero takes a larger social cost compare to the benefit. When the infection becomes severe, the externality is more crucial and the intensity of the voluntary cautiousness adopted by susceptible people also increases to the peak keeping pace with infection, the optimal containment rate increases in relatively small magnitude. The cautiousness weakens externality to some extent since it effectively lowers the infection rate when a susceptible individual interacts with infected people. When the intensity of cautiousness decrease following infection, externality becomes contributing and it is optimal to increase the containment rate.

The optimal containment policies decrease the peak infection rates from

1.389 to 1.324 percent. Also, these containment measures lower the death roll rate from 0.176 to 0.165 percent. In the case of U.S., about 36,000 lives saved. This beneficial result is associated with a much more severe recession. The decline in average aggregate consumption in the first year of the epidemic period is roughly 1.8 times, raising from about 2.00 percent to about 3.65 percent with containment measures. The implication of this result is intuitive: higher containment rates increase the cost of consumption, so that people reduce their consumption and labor supply.

Figure 3, compares our result to the one in ERT under optimal containment. Infection peak falls from 3.2 to 1.3 percent and the decline in average aggregate consumption in the first year of the epidemic falls from 17 to 3.7 percent. Also, the death roll falls from 0.21 to 0.17 percent.

Our optimal containment policy is different from the one in ERT. The reason is susceptible people adopt cautiousness to prevent themselves from getting infected when they are involved in economic activities. In contrast, susceptible people in the model of ERT can only lower the infection rate by less consumption and hours worked. Hence, containment policy is effective in ERT for the reduction of infection and death toll, accompanied by a more severe recession. The main purpose of the containment policy in our model is to reduce the death toll. In order to avoid a reappearance of the epidemic without a vaccine, it is necessary for enough proportion of the population to obtain immunity by becoming infected and recovered. Voluntary cautiousness effectively lowers the peak of infection and flatten the curve of infection, which postpones the time of acquiring "herd immunity". These explain why the optimal containment policy in our model persists longer.

Comparing to the basic SIR-macro model, the intensity of cautiousness decrease since containment policy lowers the consumption of infected people. These results imply the mechanism between optimal cautiousness and optimal containment policy.

5.2 Externality

The externality associated with the behavior of infected people caused the Pareto inefficiency in the competitive equilibrium. We consider a social planner's problem in which the planner can choose the same consumption and labor supply for people regardless of health status.

The planner's problem is

$$\begin{aligned}\max_{C_t,N_t} S_tU_t^s + I_tU_t^i + R_tU_t^r = {} & S_t\{u^s\left(C_t,N_t,\varepsilon_t^s\right) + \beta\left[\left(1-\tau_t\right)U_{t+1}^s + \tau_tU_{t+1}^i\right]\} \\ & + I_t\{u(C_t,N_t) + \beta\left[\left(1-\pi_r-\pi_d\right)U_{t+1}^i + \pi_rU_{t+1}^r + \pi_d\times 0\right]\} \\ & + R_t\{u\left(C_t,N_t\right) + \beta U_{t+1}^r\}.\end{aligned}$$

The first order conditions of the planner's problem with respect to consumption expenditure and hours worked are

$$(S_t+I_t+R_t)u_1 - \lambda_{bt}(S_t+I_t+R_t) - 2\lambda_{\varepsilon t}(1-a\varepsilon_t^s)\pi_1c_tI_t = 0, \quad (25)$$

$$(S_t+I_t+R_t)u_2 + \lambda_{bt}A(S_t+\phi^iI_t+R_t) - 2\lambda_{\varepsilon t}(1-a\varepsilon_t^s)\pi_2n_tI_t = 0, \quad (26)$$

where λ_{bt} and $\lambda_{\varepsilon t}$ are the Lagrange multipliers associated with the constraints of market clearing condition and (9).

The first order conditions of the planner's problem with respect to infection rate is τ_t:

$$\lambda_{\varepsilon t} - S_t\beta({U_{t+1}}^S - {U_{t+1}}^I) = 0$$

We derive the marginal substitution rate for the infected people

$$MRS_{SP}^i = -\frac{u_1}{u_2}$$

$$= \frac{1}{A\phi^i} + \frac{\lambda_{bt}A(\phi^i-1)(S_t+R_t) + 2\lambda_{\varepsilon t}(1-a\varepsilon_t^s)(A\pi_1C_tI_t\phi^i + \pi_2I_tN_t)}{A\phi^i(\lambda_{bt}A(\phi^i+S_t+R_t) - 2\lambda_{\varepsilon t}(1-a\varepsilon_t^s)\pi_2I_tN_t)}. \quad (27)$$

The second term in the right hand side of the last expression is referred as externality caused by the behavior of infected people.

Compare to the marginal substitution rate for the infected people in the competitive equilibrium

$$MRS^i_{CE} = -\frac{u_1}{u_2} = (1+\mu_t)\frac{1}{A\phi^i}. \tag{28}$$

In the Ramsey problem we discuss in the last subsection, the planner can choose consumption tax μ_t to control the marginal substitution rate for the infected people and maximize the social welfare, dealing with the externality. Actually, the allocations decided by the planner are similar to the containment policy showed in Figure 3. We can show the results upon request in an additional appendix.

5.3 The SIR-macro model treatment and vaccines

Figure 3 and Figure 5 display that the optimal containment policies in the model with treatment and basic SIR-macro models are analogous. And it is implied by a worst-case scenario where the treatment is never available.

In Figure 7, the black-dashed lines imply that optimal policies are very different in the basic SIR-macro model and the model with vaccines. With a possible vaccine, the optimal strategy for the policymaker is to introduce severe containment policies as soon as possible to restrain the death roll. Those containment measures lead to a more severe recession. The decline in average consumption in the first year of the epidemic is roughly 6.95 percent. However, this recession deserves acceptance since the anticipation that the vaccination is developed while many people remain susceptible.

The best policy is which can flatten the infection curve in anticipation of available vaccines. Figure 7 shows the reaction of people in the vaccine model under optimal containment policies on a path in which vaccines are not developed. In contracts to the competitive equilibrium which is displayed by red-dashed-dotted lines, the crest infection rates fall from 1.22 to 1.14 percent. In addition, the infection peak occurs in period 35 rather than in period 31. Though vaccines are not developed, the optimal containment policies decrease the death roll from 0.1748 to 0.1742 percent. In the case of U.S., about two thousand lives are saved. We repeat that this reduction is associated with the scenario that the vaccine is never available.

The containment rate decreases from 22.0 percent in period 1 to 0 in period 138. As we discuss in the introduction. Figure 5 and Figure 7 display the difference between results of our model and those in ERT under optimal containment policy with either treatments or vaccines. We discuss the total impact of cautiousness with our benchmark model in Section 6.

6 Results

6.1 Optimal policy in the benchmark model

We discuss the quantitative exercises of our model under different assumptions in previous sections. From our point of view, the model permitting the possibility of vaccines and medical treatment is our most meaningful version.

In Figure 9, the solid blue and black dashed lines in each panel respectively represent the dynamics of the epidemic in the competitive equilibrium and under optimal containment policies. Consistent with earlier discussions, we show an optimal containment policy sequence along which the vaccine and the treatment are not available.

Similar to the model with vaccines, the optimal policy for the policymaker is to immediately introduce stringent containment measures which are 21.0 percent, and gradually weaken the containment to 0 in period 138. The optimal containment policies considerably intensify the severity of the recession. Absent containment, the decline in average consumption in the first year of the epidemic is about 2.05 percent. And it becomes 6.81 percent adopting containment. The gain of the severe recession associated with optimal containment in the conjunct model is a less severe infection. Compared to the competitive equilibrium scenario, the peak infection rate falls from 1.269 to 1.185 percent of the initial population. The optimal policy leads to reduction of death toll as a percent of the initial population from 0.176 percent to 0.175 percent. In the case of U.S., containment policies save about 30 thousand lives. The containment also flattens the curve of cautiousness, where the peak falls from 23.5 percent in period 32 to 21.8 percent in period 38. We repeat that this reduction is associated with the scenario that treatments and vaccines are never available.

Figure 9 shows the significant difference caused by considering cautiousness. We compare the results of optimal containment policy in our SIR-macro model with Vaccines and Treatments with those in ERT. The cautiousness reduces the infection peak from 3.4 to 1.2 percent, while reduces the fall of average consumption in the first year of the epidemic from 16.6 to 6.8 percent. It also reduces the death toll from 0.240 to 0.175 percent. These results imply that the optimal containment policy is flattened by the cautiousness.

6.2 Containment and cautiousness

Above we discuss optimal containment policy in our SIR-macro model with vaccines and treatments, including cautiousness.

In the basic SIR-macro model, the optimal strategy is strengthening containment policy while the intensity of cautiousness decreases. Compare our SIR-macro model to the SIR-macro in ERT, the infection peak falls from 5.3 to 1.4 percent by involving social distancing in the former, while it falls from 5.3 to 3.2 percent by involving containment policy in the latter.

The reason is that the property of cautiousness reduces the magnitude of the externality associated with the activity of infected people and infection rate effectively. The effectiveness of cautiousness, allows policymakers to strengthen containment policy later and avoid severe recession.

With treatments and vaccines as a possibility, our model with cautiousness captures the different responses of people. The optimal containment policy in our SIR-macro model with vaccines and treatments. The effectiveness of cautiousness helps policymakers to choose a relatively smaller magnitude value of containment rate, which can avoid severe recession while a vaccine and a treatment are never available.

It is an unavoidable trade-off between the severity of the short-run economic loss associated with the pandemic and the health consequences of the virus. It is a key challenge confronting policymakers to deal with this tradeoff. Our results show that the main effect of cautiousness is flattening the infection curve and avoiding severe recession. The main effect of optimal containment policy is reducing the death toll. The reason why we need optimal containment policy accomplished by a more severe recession, even if cautiousness is effective, is the social welfare loss caused by death is much larger than the recession. Recession brings temporary economic loss while death brings permanent loss for production. If we introduce a high containment stringency policy without considering cautiousness, it might unnecessarily intensify the recession. The optimal strategy is combining optimal containment policy with cautiousness.

7 Conclusion

We are motivated by the SIR-macroeconomics model of Eichenbaum, Rebelo, and Trabandt, 2020, and the equilibrium social distancing model of F.M.O Toxvaerd, 2020. In our model, we veer from the analysis in ERT and we take cautiousness into consideration, which is effective for reducing the infection rate. We find that the economic outcome differs significantly. We report our results that the economic outcome is significantly different. With cautiousness, about fifty percent of the decline of economic activity is restrained in our benchmark model and the infection "curve" is flattened substantially. We demonstrate that an SIR-macro model embodying cautiousness, which is a contributing factor in preventing infection, implies a different magnitude value of containment. Someone could view our results of voluntary cautiousness as the “Swedish” solution: Sweden avoids government restricting economic activities, and permits people to make their own decisions on consumption, work, and voluntary social distancing. These private incentives and effective voluntary cautiousness may not only end the epidemic on their own but also decelerating the decline in economic activities. Although voluntary cautiousness is more contributing to the reduction of infection than containment policy, in our model. Containment policy is necessary for maximizing social welfare since the social welfare loss caused by death is much larger than the recession.

Figure 1: SIR-Macro Model vs. SIR Model

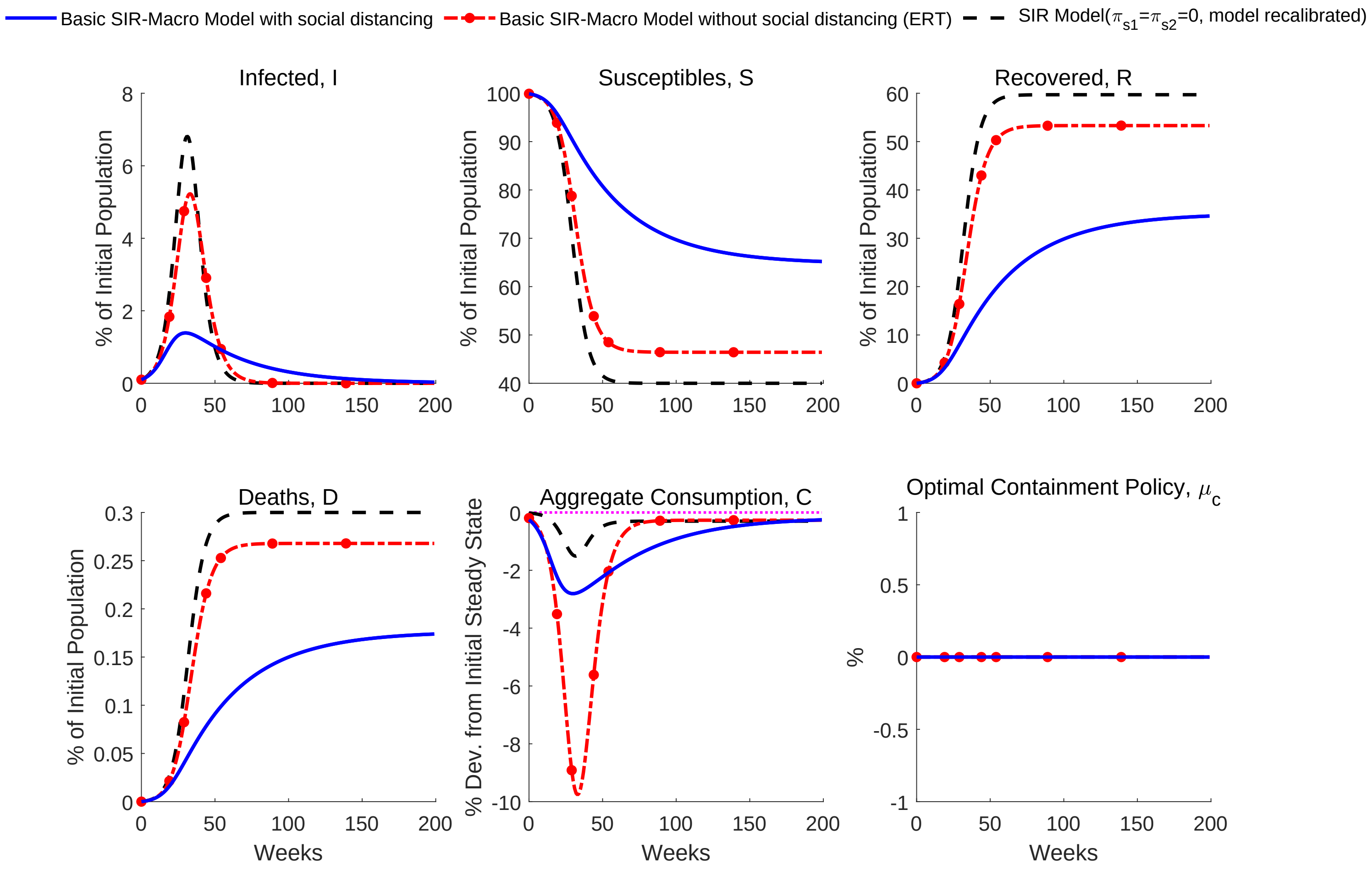


Figure 2: Consumption and Hours by Type in Basic SIR-Macro Model

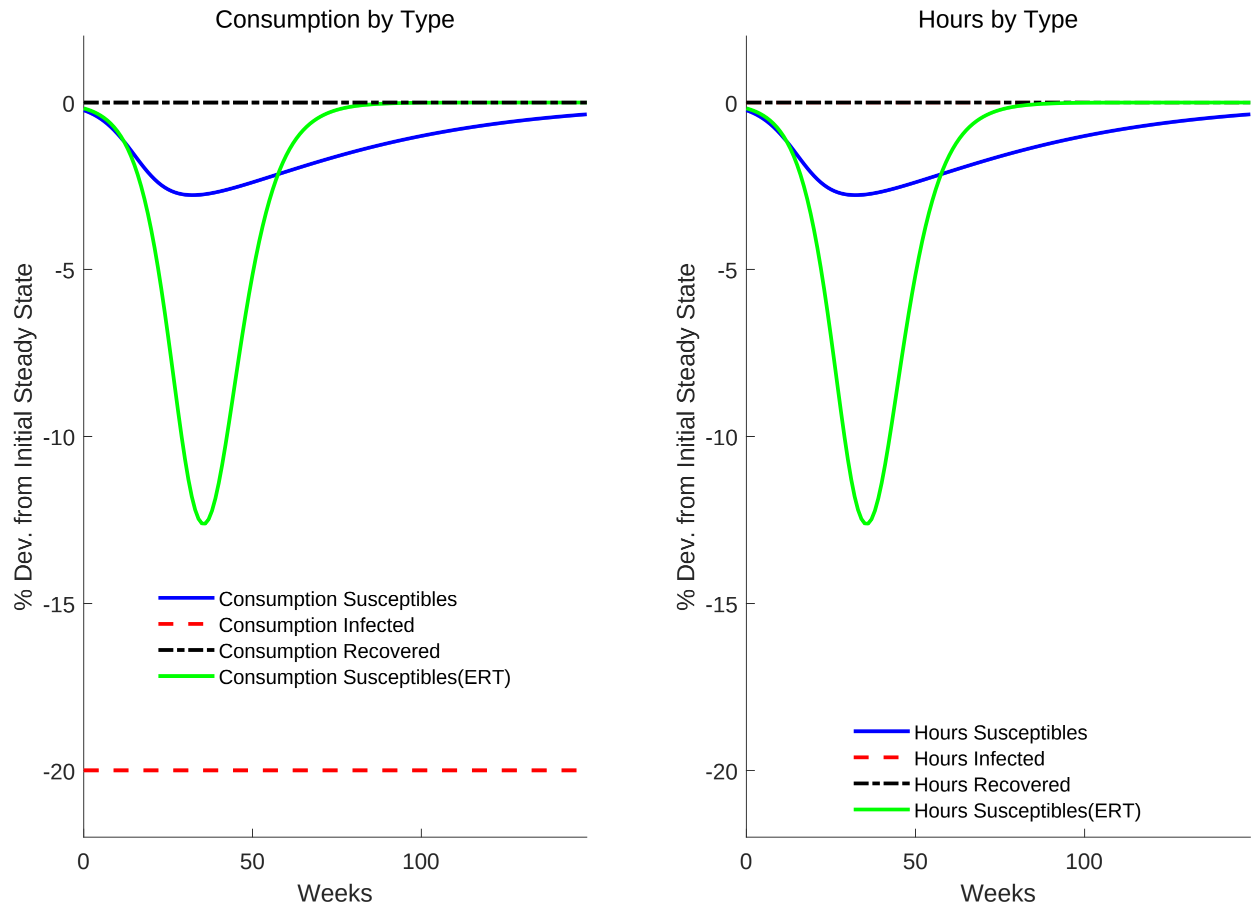


Figure 3: Basic SIR-Macro Model With and Without Containment

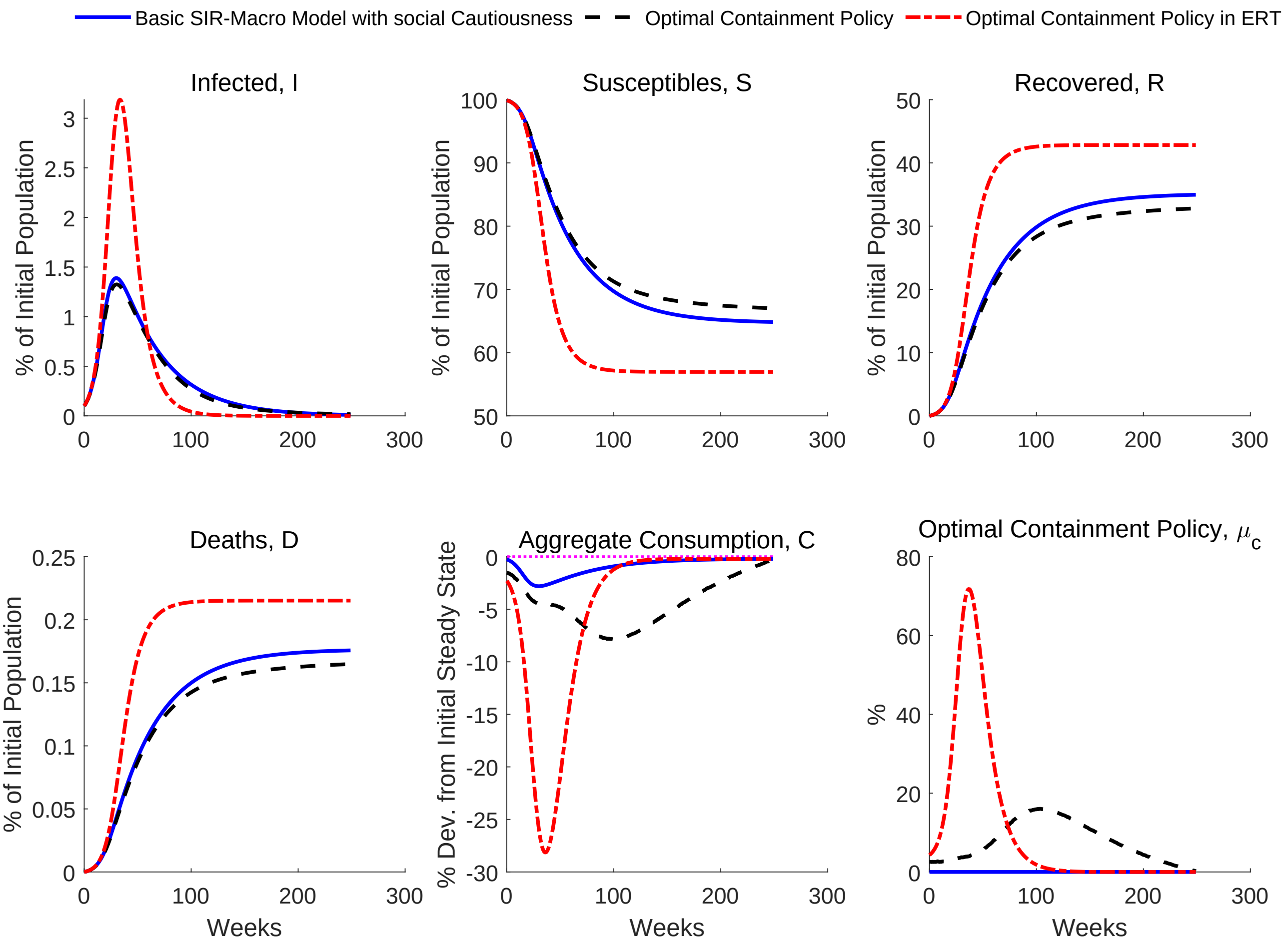


Figure 4: Basic SIR-Macro Model With and Without Containment

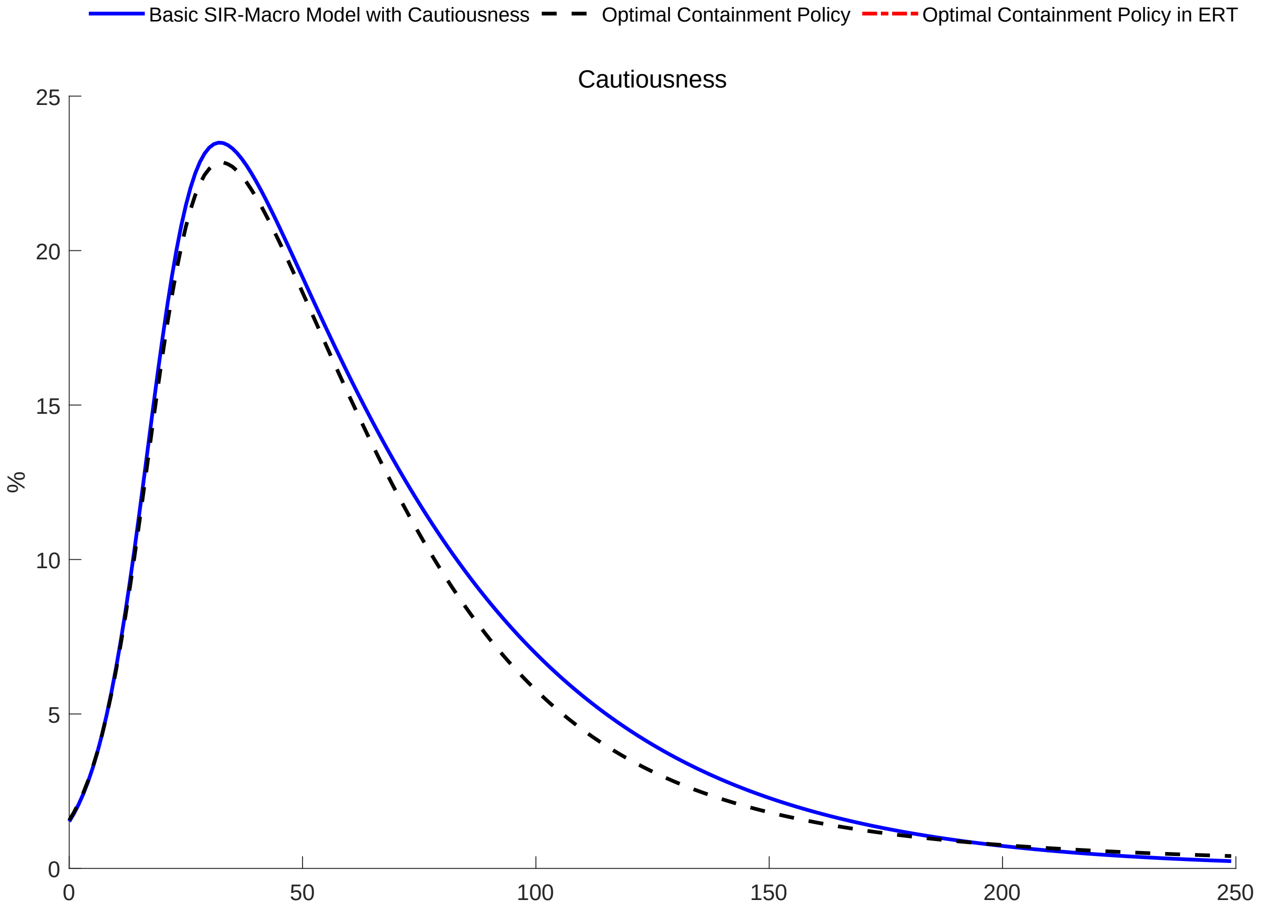


Figure 5: SIR-Macro Model With Treatments

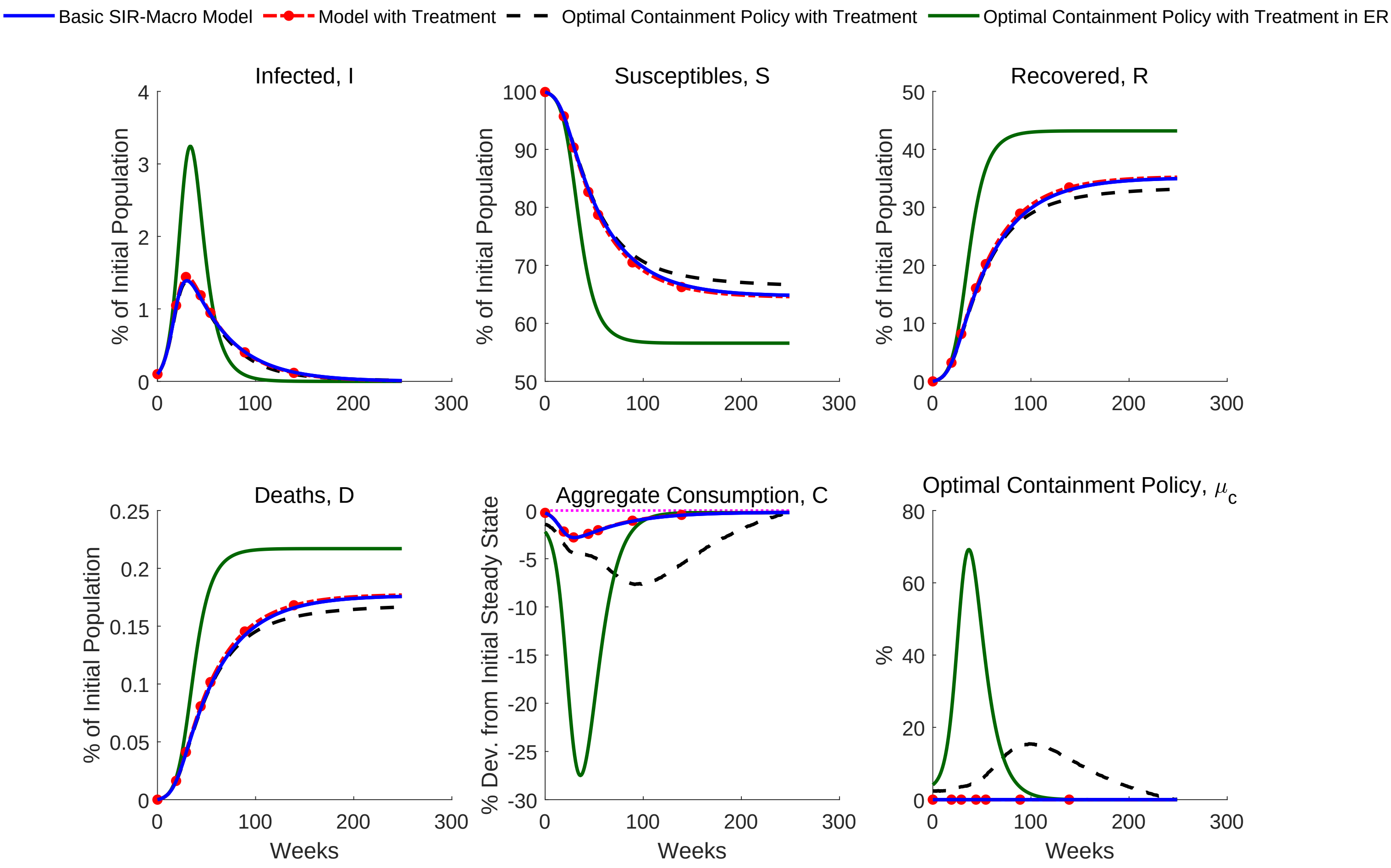


Figure 6: SIR-Macro Model With Treatments

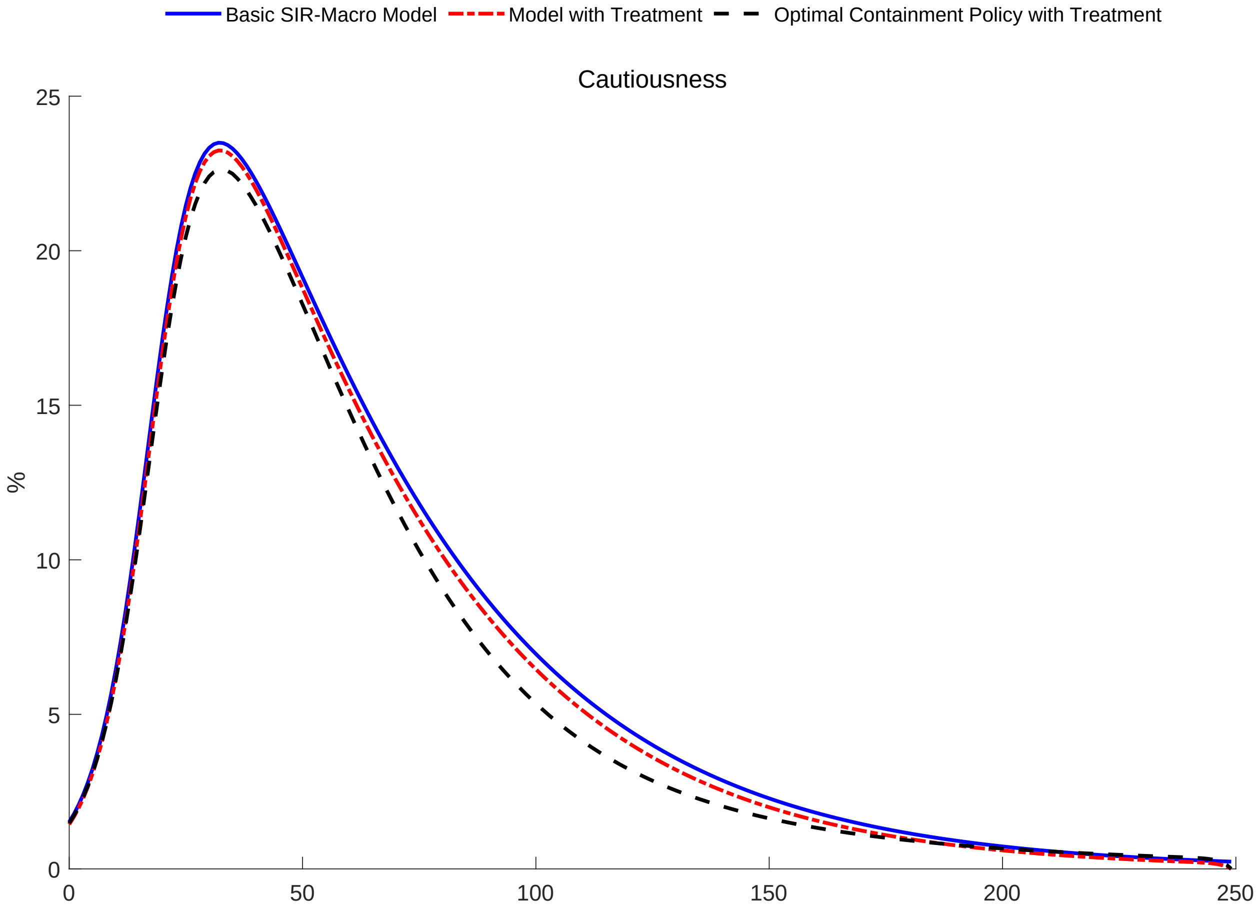


Figure 7: SIR-Macro Model With Vaccines

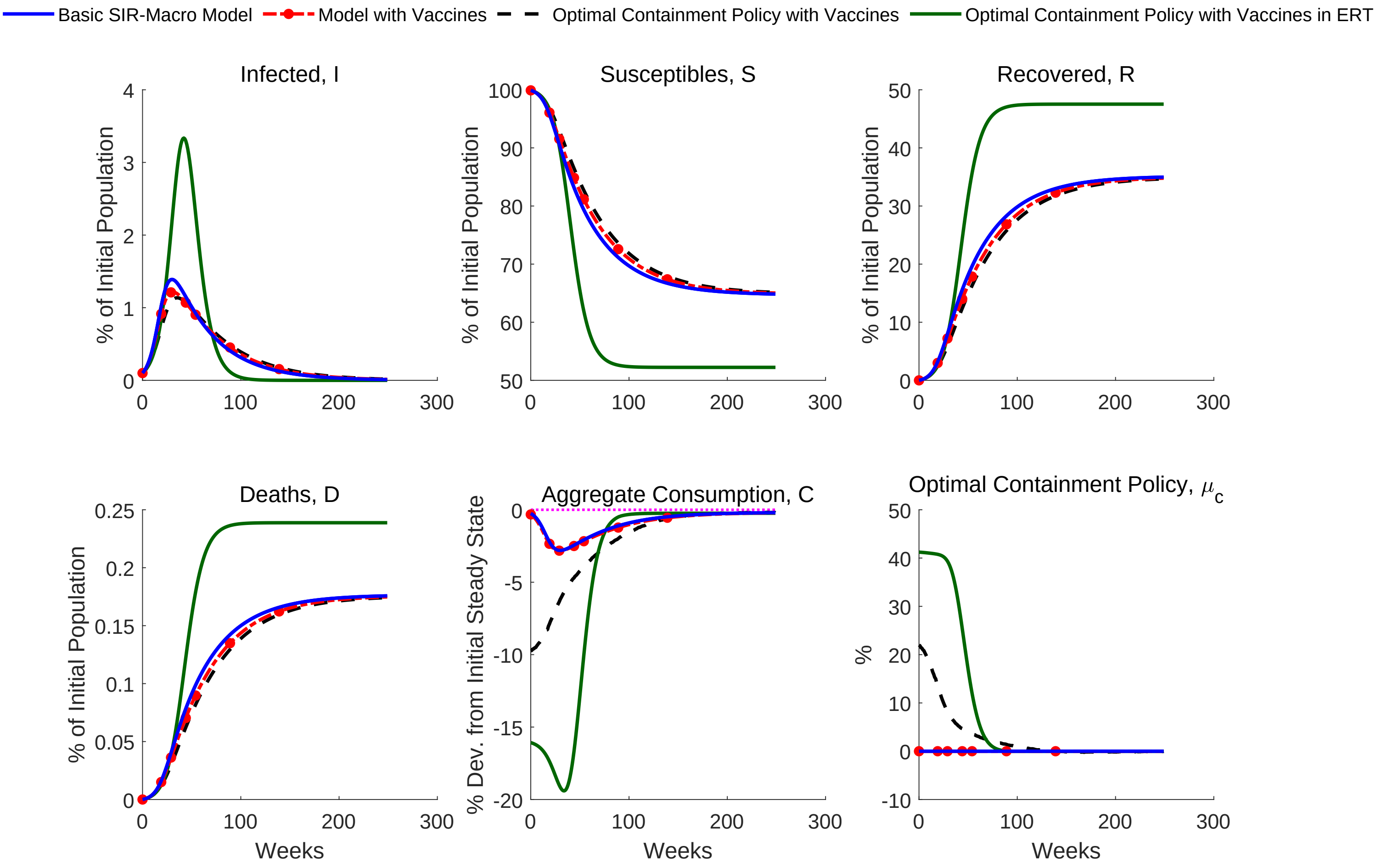


Figure 8: SIR-Macro Model With Vaccines

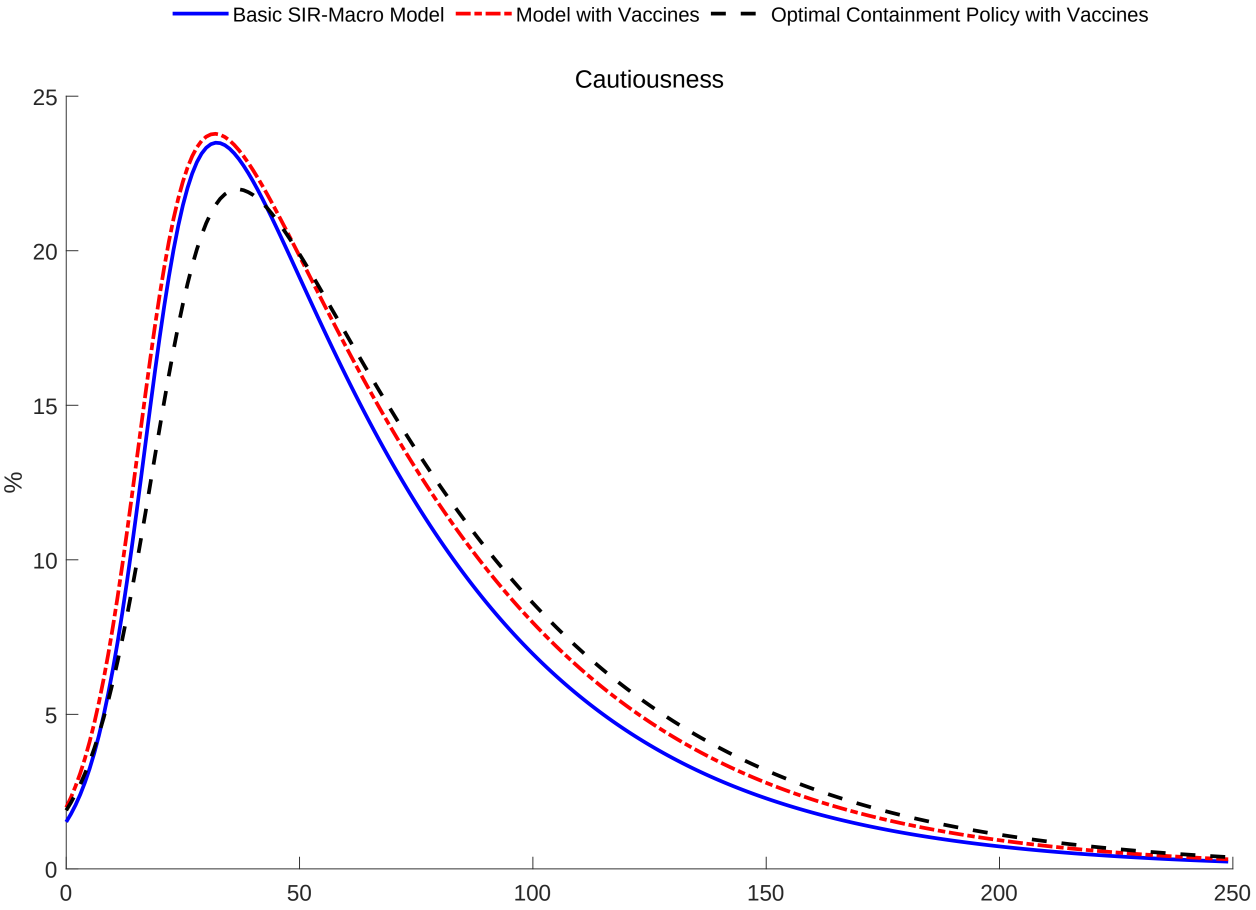


Figure 9: Benchmark SIR-Macro Model (Vaccines and Treatments)

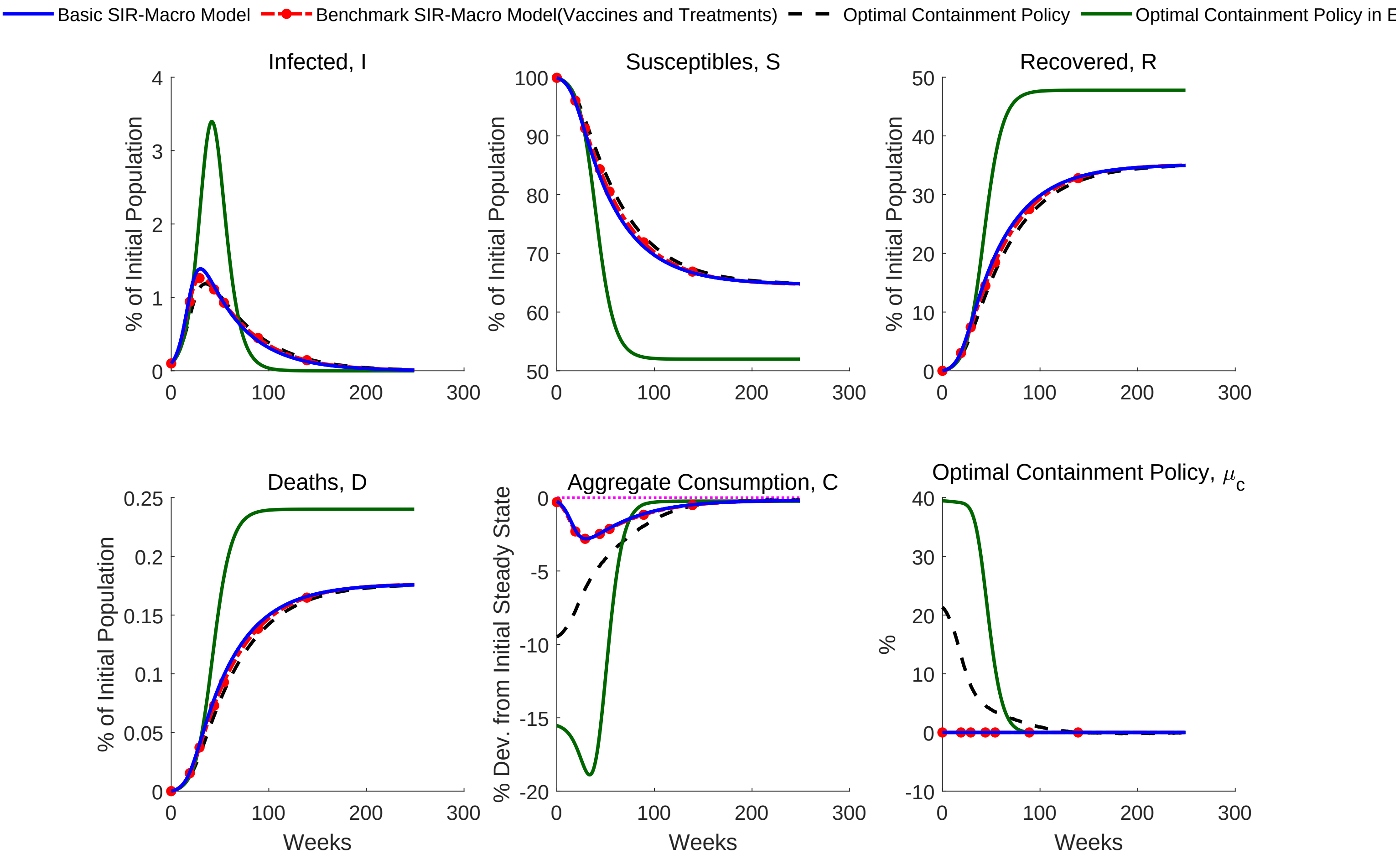
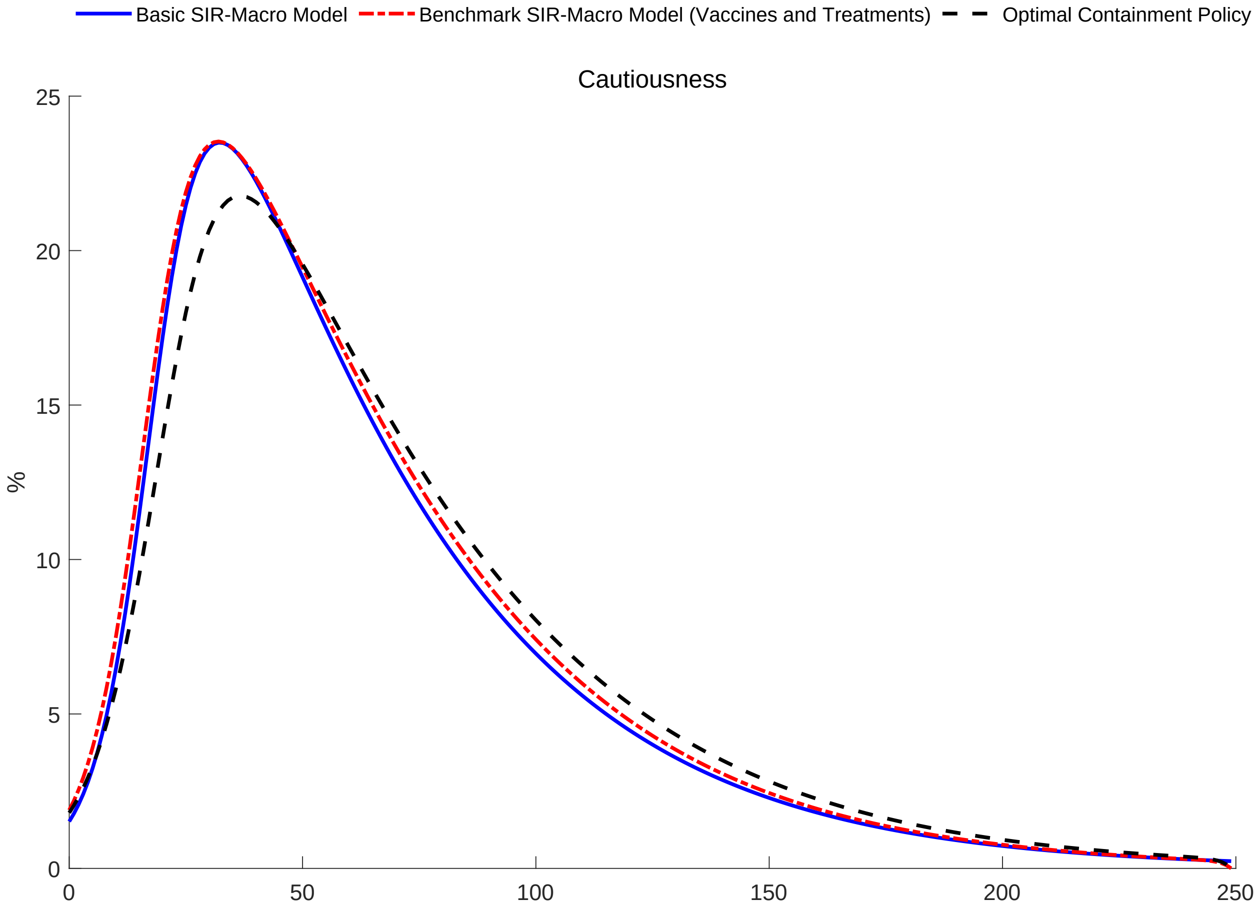


Figure 10: Benchmark SIR-Macro Model (Vaccines and Treatments)



Appendix A

Computing the competitive equilibrium and robustness

1 Competitive equilibrium

As a sequence of containment rates, $\{\mu_t\}_{t=0}^{H-1}$, is given, for some large horizon, H, guess sequences for $\{n_t^s, n_t^i, n_t^r, \varepsilon_t^s\}_{t=0}^{H-1}$. In practice, we compute the solution for $H = 250$ weeks. Compute the sequence of the unknowns using following first order conditions and equations:

$$\begin{aligned}
\theta n_t^r &= A\lambda_{bt}^r, \\
\left(c_t^r\right)^{-1} &= \left(1+\mu_t\right)\lambda_{bt}^r, \\
u_t^r &= \ln c_t^r - \tfrac{\theta}{2}\left(n_t^r\right)^2.
\end{aligned}$$

Iterate backwards to solve the values of U_t^r :

$$U_t^r = u\left(c_t^r, n_t^r\right) + \beta U_{t+1}^r.$$

Apply the following first order conditions and equations to solve the sequence for remaining unknown variables:

$$\begin{gathered}
(1+\mu_t)\, c_t^r = A n_t^r + \Gamma_t, \quad \left(\lambda_{bt}^r\right) \\
\theta n_t^i = \phi^i A \lambda_{bt}^i, \\
\left(c_t^i\right)^{-1} = \lambda_{bt}^i, \\
u_t^i = \ln c_t^i - \tfrac{\theta}{2}\left(n_t^i\right)^2, \\
(1+\mu_t)\, c_t^s = A n_t^s + \Gamma_t, \quad \left(\lambda_{bt}^s\right) \\
u_t^s = \ln c_t^s - \tfrac{\theta}{2}\left(n_t^s\right)^2 - \tfrac{\chi}{2}\varepsilon_t^{s2}.
\end{gathered}$$

As Pop_0, S_0, I_0, R_0 and D_0, are given, we iterate forward using the following six equations for $t = 0, \ldots, H-1$:

$$\begin{gathered}
T_t = (1 - a\varepsilon_t^s)[\pi_1\left(S_t c_t^s\right)\left(I_t c_t^i\right) + \pi_2\left(S_t n_t^s\right)\left(I_t n_t^i\right) + \pi_3 S_t I_t], \\
Pop_{t+1} = Pop_t - \pi_d I_t, \\
S_{t+1} = S_t - T_t, \\
I_{t+1} = I_t + T_t - (\pi_r + \pi_d)\, I_t, \\
R_{t+1} = R_t + \pi_r I_t, \\
D_{t+1} = D_t + \pi_d I_t.
\end{gathered}$$

Iterate backwards to obtain the values of U_t^s and U_t^i :

$$\begin{gathered}
U_t^i = u\left(c_t^i, n_t^i\right) + \beta\left[(1-\pi_r-\pi_d)\, U_{t+1}^i + \pi_r U_{t+1}^r\right], \\
\tau_t = \tfrac{T_t}{S_t}, \\
U_t^s = u\left(c_t^s, n_t^s, \varepsilon_t^s\right) + \beta\left[(1-\tau_t)\, U_{t+1}^s + \tau_t U_{t+1}^i\right].
\end{gathered}$$

Calculate the sequence of the remaining unknown variables in the following equations:

$$\begin{gathered}
\beta\left(U_{t+1}^i - U_{t+1}^s\right) - \lambda_{\varepsilon t}^s = 0, \\
\left(c_t^s\right)^{-1} - \lambda_{bt}^s\,(1+\mu_t) + \lambda_{\varepsilon t}^s(1-a\varepsilon_t^s)\pi_1\left(I_t C_t^I\right) = 0.
\end{gathered}$$

Finally, use a gradient-based method to adjust the guesses $\left\{n^s_t, n^i_t, n^r_t, \varepsilon^s_t\right\}_{t=0}^{H-1}$ so that the following four equations hold with arbitrary precision:

$$
\begin{gathered}
(1+\mu_t)\, c^i_t = \phi^i A n^i_t + \Gamma_t, \quad \left(\lambda^i_{bt}\right) \\
\mu_t \left(S_t c^s_t + I_t c^i_t + R_t c^r_t\right) = \Gamma_t \left(S_t + I_t + R_t\right), \\
-\theta n^s_t + A\lambda^s_{bt} + \lambda^s_{\varepsilon t}\pi_2(1 - a\varepsilon^s_t)\left(I_t n^I_t\right) = 0, \\
\chi\varepsilon^s_t - \lambda^s_{\tau t} a \left[\pi_1 c^s_t \left(I_t C'_t\right) + \pi_2 n^S_t \left(I_t N'_t\right) + \pi_3 I_t\right] = 0.
\end{gathered}
$$

2 Robustness

TABLE 1: Robustness in Basic SIR-Macro Model without Containment[a]

	Consumption %[b]	**Infection Rate** %[c]	**Death Rate** %[d]	**U.S. Deaths** Millions[e]	**Social Distancing** %[f]
Percent of population eventually infected in canonical SIR model					
50	-1.5333	1.0008	0.1446	0.4771	11.1558
60(baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
70	-2.4028	1.8583	0.2093	0.6907	21.3880
Productivity of infected people, ϕ^i					
0.7	-1.9450	1.3087	0.1699	0.5607	15.1205
0.8(baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
Share of initial infections due to consumption, work and general contacts					
1/12, 1/12, 5/6	-1.1494	1.4816	0.1818	0.6001	17.0926
1/6, 1/6, 2/3 (baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
1/3, 1/3, 1/3	-3.4386	1.1626	0.1614	0.5327	13.1489
Mortality rate, π_d					
$0.005 \times 7/18$ (baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
$0.01 \times 7/18$	-2.3921	0.7725	0.3096	1.0218	19.9855
Limited healthcare capacity parameter,κ[g] (slope of endogenous mortality rate)					
0(baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
0.9	-2.0232	1.3054	0.1817	0.5997	16.2626
Household discount factor, β					
$0.96^{1/52}$(baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
$0.94^{1/52}$	-1.7718	1.8772	0.1865	0.6156	13.4750
Cost of social distancing, χ					
0.5	-1.2676	0.7747	0.1552	0.5120	20.2470
1(baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471
2	-2.8545	2.2409	0.1946	0.6421	11.4934
Effectiveness of social distancing, a					
0.5	-2.0627	3.2986	0.2204	0.7273	15.1105
1(baseline)	-1.9958	1.3892	0.1757	0.5799	15.9471

---

[a] See section 4.4 for a discussion of the results provided in this table.

[b] Average drop of consumption in first year relative to pre-infection steady state.

[c] Peak infection rate relative to pre-infection population.

[d] Death rate at the end of the epidemic relative to pre-infection population.

[e] Total number of deaths in the U.S. at the end of the epidemic

[f] Average cautiousness in the first year.

[g] In ERT they assume that the mortality rate depends on the number of infected people, I_t, in the medical preparedness model as, $\pi_{dt} = \pi_d + \kappa I_t^2$.

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