小テスト 3 解答

- **1**:
  a. True
  b. Interchange μ and $\bar{X}$.
  c. If we quadruple the sample size, ...
  d. True

- **2**:
  a. The 95% confidence interval is

$$[\bar{X} - t_{.025}SE,\ \bar{X} + t_{.025}SE]$$
$$= [99.8 - 2.78(24.4),\ 99.8 + 2.78(24.4)]$$
$$= [99.8 - 67.9,\ 99.8 + 67.9],$$

  where there are 4 d.f. in calculating SE$= \frac{s}{\sqrt{n}}$.
  b. $[5000 - 3400,\ 5000 + 3400]$, which is the confidence interval for 50μ.
  c. Yes.

- **3**:
  a. The 95% confidence interval is

$$[(\bar{X}_1 - \bar{X}_2) - t_{.025}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ (\bar{X}_1 - \bar{X}_2) + t_{.025}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}]$$
$$= [7.0 - 2.31(3.05),\ 7.0 + 2.31(3.05)]$$
$$= [7.0 - 7.04,\ 7.0 + 7.04],$$

  where there are 8 d.f. in calculating SE. That is, the men at the university earn $7.0(\pm 7.04)$ thousand dollars more than the women, on average. Or equivalently, women earn $7.0(\pm 7.04)$ less than the women.
  b. It fails to show discrimination on two counts: (1) This is an observational study, and whatever differences exist may be due to confounding factors such as men having better qualifications, more experience, etc. (2) We are not even sure a difference exists in the population; the confidence interval includes $\mu_1 - \mu_2 = 0$.

- **4**:
  Let $D = X_1 - X_2$. Then, the 95% confidence interval is

$$[\bar{D} - t_{.025}\frac{s_D}{\sqrt{n}},\ \bar{D} - t_{.025}\frac{s_D}{\sqrt{n}}]$$
$$= [-4 - 2.78(2.07),\ -4 + 2.78(2.07)]$$
$$= [-4 - 5.8,\ -4 + 5.8],$$

  where there are 4 d.f. in calculating SE. (Here, $s_D = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(D_i - \bar{D})^2}$.)

- **5 a**. In 1963, the sample proportion $P = 160/200 = 0.8$. Since $n = 200$ is large, the 95% confidence interval is

$$\left[P - z_{.025}\sqrt{\frac{P(1-P)}{n}},\ P + z_{.025}\sqrt{\frac{P(1-P)}{n}}\right]$$
$$= [0.8 - 1.96(0.028),\ 0.8 + 1.96(0.028)]$$
$$= [0.8 - 0.05,\ 0.8 + 0.05]$$
$$= [80\% - 5\%,\ 80\% + 5\%].$$

In the same way, in 1968, we have

$$\left[P - z_{.025}\sqrt{\frac{P(1-P)}{n}},\ P + z_{.025}\sqrt{\frac{P(1-P)}{n}}\right]$$
$$= [0.35 - 1.96(0.034),\ 0.35 + 1.96(0.034)]$$
$$= [0.35 - 0.07,\ 0.35 + 0.07]$$
$$= [35\% - 7\%,\ 35\% + 7\%].$$

b. The 95% confidence interval for the change is

$$\begin{aligned} & [(P_1 - P_2) - z_{.025}\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}, \\ & \qquad (P_1 - P_2) + z_{.025}\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}] \\ = \quad & [-0.45 - 1.96(0.044),\ -0.45 + 1.96(0.044)] \\ & = [-0.45 - 0.086,\ -0.45 + 0.086] \\ & = [-45\% - 9\%,\ -45\% + 9\%] \end{aligned}$$