東京大学 公共政策大学院
ワーキング・ペーパーシリーズ
GraSPP Working Paper Series
The University of Tokyo

GraSPP-DP-E-07-004 and ITPU-DP-E-07-004

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April 2007

GraSPP Discussion Paper E-07-004
ITPU Discussion Paper E-07-004

GRADUATE SCHOOL OF PUBLIC POLICY
THE UNIVERSITY OF TOKYO
HONGO, BUNKYO-KU, JAPAN

GraSPP-DP-E-07-004

Sequential Peak-Load Pricing in a Vertical Setting: The Case of Airports and Airlines

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1. INTRODUCTION

During the last several years airlines and passengers have been suffering from congestion and delays at busy airports, and airport delays have become a major public policy issue. Since the early work of Levine (1969), Carlin and Park (1970) and Borins (1978), economists have approached airport runway congestion by calling for the use of price mechanism, under which landing fees are based on a flight's contribution to congestion. While congestion pricing is economically desirable, it has not really been practiced. The existing landing fees depend on aircraft weight, and the fee rates are based on the accountancy principle of cost recovery required usually for a public enterprise.[1] Airports have traditionally been owned by governments, national or local. This is changing, however. Starting with the privatization of seven airports in the UK to BAA plc. in 1987, many airports around the world have been, or are in the process of being, privatized.[2] One of the leading arguments for airport privatization is that privatised airports might well shift toward peak-load congestion pricing of runway services they provide to airlines, thus reducing delays in peak travel times (Poole, 1990; Gillen, 1994; Vasigh and Haririan, 1996). For example, Gillen (1994) argues that privatization does a better job of producing efficient runway pricing mechanisms compared to public ownership.

Taken together, today's shortage of airport capacity has revived much of the recent discussions about peak-load congestion pricing and airport privatization. In this paper we carry out an analysis of peak-load congestion pricing for a private, unregulated airport, as well as for a public airport that maximizes social welfare. The comparison of the two cases then allows us to shed some light on their pricing policies and traffic allocations to the peak and-off peak periods. We find that compared to a welfare-maximizing airport, a profit-maximizing airport would charge higher peak and off-peak runway prices, as well as a higher peak/off-peak price differential. As a consequence, privatisation would lead to both fewer total air passengers and fewer passengers using the premium peak hours of the day for their

[1] Airport charges include landing fees, aircraft parking and hangar fees, passenger terminal fees and air traffic control charges (if the service is provided by the airport authority), with landing fees being most dominant. The revenues derived from these charges are referred to as aeronautical revenues. In addition, busy airports derive significant revenues from non-aeronautical business, such as concessions and other commercial activities. As Daniel (2001) pointed out, landing fees in the U.S. traditionally recovered the "residual" costs –those remaining after all other revenue sources are fully exploited– with the fee rate equalling the annual residual costs divided by the weight of all aircraft landing during the year.

[2] A number of major airports in Europe, Australia, New Zealand and Asia were recently privatized, or are in the process of being privatized. In the U.S., on the other hand, the airports that are used by scheduled airlines are virtually all publicly owned facilities run by the local (city) government or by an agency on behalf of the local government. Canada may represent a middle-of-the-road case in which airports recently devolved from direct Federal control to become autonomous entities and major airports, though still government-owned, are now managed by private not-for-profit (but subject to cost recovery) corporations.

travel, both of which reduce social welfare. Hence, the alleged benefits of privatization would seem to have been achieved, because those passengers who still use the peak period indeed face less congestion delays; but overall it is not economically efficient to have such a lower level of peak congestion. This suggests that airport privatization cannot be judged based on its effect on congestion delays alone.

Our analysis also shows that whilst private, profit-maximizing airports will always use peak-load pricing, somewhat surprisingly, a public airport may actually charge a peak price that is *lower* than the off-peak price. Here the public airport, on the surface, is not practicing the peak-load pricing, but such pricing structure is nevertheless socially optimal.

We further investigate a case where a private airport strategically collaborates with the airlines so that it maximizes the joint airport-airline profits, since it has been often argued that greater airlines' *countervailing power* or more strategic collaboration between airports and airlines may improve efficiency of privatized airports by allowing a better alignment of incentives. The analysis shows that while the airport's pricing practices would induce a collusive outcome in the airline market, they would also, owing to the elimination of "double marginalization," induce greater total traffic and greater peak traffic than a pure (no-collaboration) private airport. Nevertheless, assuming no differences in technical efficiency, both figures will still be smaller than those for a public airport.

As indicated above, the present paper investigates airport peak-load pricing (PLP) and analyzes both the price level and price structure (peak vs. off-peak). This is in contrast to the majority of airport pricing studies which did not address inter-temporal pricing across different travel periods. In these *congestion pricing* studies, there is only one demand function (i.e., a single-period model) for the airport, which is obtained by aggregating the demands of many agents –in this case, the airlines. Since airport runways are congestible, when an airline decides to schedule a new flight, it induces extra-delays on every other flight. The airline however would only internalize the delays it imposes on its own flights and not others. Congestion pricing then looks at the price the airport, or a regulatory authority, should charge to the airline for the new flight, in order for the airline to internalize all the congestion it produces (e.g., Morrison, 1987; Zhang and Zhang, 2003, 2006; Pels and Verhoef, 2004; Basso, 2005). Notice that under congestion pricing, since time-varying congestion is absent, there is only one way for either the airport or the airlines to internalize congestion: raising prices to suppress the demands. In a PLP framework, on the other hand, excess demand problems arise because of the variability of demands during the reference times of the day. If the same price was charged throughout the day, there would be peak periods at which the demand would be much higher than at off-peak periods. PLP looks at the optimal time-schedule of prices so as to flatten the demand curve and make better use of existing capacity. As discussed below, both airports and airlines may engage in such demand spreading by using PLP. Note that in this PLP framework, the airport is still a congestible facility, which implies that in the resulting optimal price-schedule, prices at peak periods would still have to correct for uninternalized congestion: peak-load prices will have a congestion pricing component.[3]

Another major feature of our analysis lies in the basic model structure used, which has strong implications for peak-load pricing. Here an airport, as an input provider, makes its price decisions prior to the airlines' output decisions. This vertical structure gives rise to *sequential* PLP: The PLP schemes implemented by the downstream airlines induce a different periodic demand for the upstream airport, with the shape of that demand depending on the number of downstream carriers and the type of competition they exert. The airport then would have an incentive to use PLP as well, which in turn affects the way the downstream firms use PLP. Although several very useful models of airport peak-load congestion pricing have been developed (e.g., Morrison, 1983; Morrison and Winston, 1989; Oum and Zhang, 1990; Arnott, De Palma and Lindsey, 1993; Daniel, 1995, 2001), these studies considered PLP primarily at the airport level. Brueckner (2002, 2005), on the other hand, investigated PLP primarily at the airline level. Most of these studies considered only a public airport that maximizes social welfare, making no assessments about the effects of privatization on airport price structures.

There is an extensive body of literature on peak-load pricing. The classical papers (Boiteaux 1949; Steiner 1957; Hirschleifer 1958; Williamson 1966) focused on normative rules for pricing a public utility's non-storable service subject to periodic demands. Some of the usual assumptions were: (i) demand is constant within each pricing period; (ii) demand in one period is independent of demand in other periods; (iii) constant marginal costs; (iv) the length of pricing periods is fixed and exogenous; (v) the number of pricing periods is exogenous; and (vi) peak time is known. Many authors have since contributed to the generalization of PLP results by relaxing one or a group of these

[3] As demonstrated in the text, the PLP-congestion pricing distinction is also important in that a single-period congestion toll is not optimal unless it is charged on top of the optimal charge in the off-peak period, *which may not be the marginal cost*. In other words, restricting the analysis to the toll that should be charged during the peak hours offers only a partial view of the problem.

assumptions, including Pressman (1970), Panzar (1976), Dansby (1978), Craven (1971, 1985), Crew and Kleindorfer (1986, 1991), Gersten (1986), De Palma and Lindsey (1998), Dana (1999), Laffont and Tirole (2000), Shy (2001) and Calzada (2003).[4] However, the case of sequential peak-load pricing, be it for public or private utilities, has yet been analyzed. In the telecommunications research, for instance, Laffont and Tirole looked at PLP only at the upstream level (the network access charge) whilst Calzada considered PLP only at the downstream level. Because of this, we think our paper could be a contribution to the general peak-load pricing literature as well.

The paper is organized as follows. In the next section we set out the basic model. In Section 3 we analyze and characterize the output-market equilibrium, paying particular attention to the peak and off-peak derived demands for airport services. Section 4 examines the airport's decisions and discusses how the airport ownership influences the peak and off-peak runway prices, traffic volumes, delays and welfare. Section 5 examines the case where a private airport maximizes the joint airport-airline profits. Section 6 contains concluding remarks.

2. THE MODEL

We consider a two-stage model of airport and airline behavior, in which N air carriers service a congestible airport. In the first stage the airport decides on its runway charges on airlines, and in the second stage each carrier chooses its output in terms of the number of flights.

We shall consider a discrete choice model in which the consumer chooses between three mutually exclusive alternatives, namely: $h=p$, travel during peak hours of a day; $h=o$, off-peak period travel; and $h=n$, not traveling. There is a continuum of consumers labelled by θ. Denoting $B_h(\theta)$ the gross benefit for consumer θ from traveling in period h, consumers' utility function may be written as:

$$U(x, B_h(\theta), -D_h) \tag{1}$$

In (1), x is consumption expenditure and D_h denotes the flight delay associated with travel in period h. We assume that for any given θ,

[4] See Crew, Fernando and Kleindorfer (1995) for an excellent survey of the peak-load pricing literature.

$$B_p(\theta) > B_o(\theta) > B_n(\theta) = 0 \tag{2}$$

where $B_n(\theta)$ is, for convenience, normalized to be zero. The inequalities say that if travel were free and without congestion, the consumer would always prefer traveling to non-traveling. Furthermore, with identical airfares and delays, consumers would always prefer traveling in the peak period to off-peak traveling. Thus, peak travel and off-peak travel are *vertically differentiated*: Controlling for fares and delay costs, passengers regard a peak flight as a better product than an off-peak flight. This vertical-differentiation feature of air travel can arise if the peak period represents the day's more desirable travel times. Since people want to travel in those "popular" hours, the (unfettered) demand approaches or exceeds the capacity of the existing infrastructure, thereby resulting in (potential) congestion during the peak hours. Note that our formulation (2) is different from the one used in Brueckner (2002, 2005), in which he assumed a "single crossing property" in that the benefit functions intersect at an intermediate value of θ, thus indicating that $B_p(\theta) > B_o(\theta)$ for large values of θ but $B_p(\theta) < B_o(\theta)$ for small values of θ. The single-crossing condition was imposed to avoid a degenerate (corner) equilibrium in his analysis.

For simplicity we assume that $B_h(\theta)$ takes the simple linear form of $B_h \cdot \theta$, with B_h being constant. Condition (2) then implies $B_p > B_o > 0$, indicating that consumers differ in their travel benefits, with small θ indexing consumers with low benefit values. For simplicity, we assume θ is distributed uniformly on $\Theta = [\underline{\theta}; \overline{\theta}]$ and normalize the number of total consumers to $\overline{\theta} - \underline{\theta}$, so the number of passengers with type belonging to $[\theta_1, \theta_2]$ is directly given by $\theta_2 - \theta_1$. We further assume that the utility function U is linear, and that consumers maximize it by choosing x and $h \in \{p, o, n\}$ subject to the budget constraint $x + t_h \leq I(\theta)$, where t_h is the ticket price (airfare) of traveling in period h, and $I(\theta)$ is consumer θ's income. We can then focus on the part of the utility function that determines the discrete choice. This *conditional indirect utility function* is given by:

$$\overline{V}_h(\theta) = B_h\theta - \alpha D_h - t_h \tag{3}$$

In (3), while θ indexes consumers according to their gross travel benefits, the positive parameter α represents the subjective value of time savings and so αD_h represents monetary costs of delays to passengers. Note that our demand problem is identical to the one that will result if we fix θ but allow the value of time α to have a distribution among consumers (in simple models with endogenous hours of work, the consumers' "opportunity cost" of time lost in delays is proportional to their wages). One could also argue that θ and α are related (Yuen and Zhang, 2005), but we do not do this here.

As for the delay itself, the flight delay at period *h*, for *h=p,o*, may be given by $D_h = D(Q_h; L_h, K)$, where Q_h is the total number of flights in the period, L_h is the length (duration) of the pricing period, and *K* is the airport's runway capacity (measured in terms of the maximum number of flights that the airport's runways can handle per hour). In this paper we consider that *K* and L_h are exogenously given.[5] We further assume L_o is sufficiently long so that $D(Q_o; L_{o,} K) = 0$ throughout the relevant range of our analysis. In other words, whilst the narrow peak period is congestible, congestion never arises in the broader off-peak period.[6] For the peak delay function, we make the standard assumption that $D_p = D(Q_p)$ is differentiable in Q_p and

$$D_p^{'} = \frac{dD}{dQ_p} > 0, \qquad D_p^{''} = \frac{d^2 D}{dQ_p^2} \geq 0 \tag{4}$$

This assumption is quite general, requiring only that for given airport capacity, increasing peak traffic will increase congestion of the peak period and the effect is more pronounced when there is more congestion; that is, for given capacity, the peak delay is convex in traffic volume. The assumption is certainly satisfied under a linear delay function, $D(Q_p; L_p, K) = \delta \cdot Q_p /(L_p K)$ –which has been used by, e.g., Pels and Verhoef (2004)– or under the functional form suggested by Lave and de Salvo (1968), that is, $D(Q_p; L_p, K) = Q_p \left[L_p K(K - (Q_p / L_p))\right]^{-1}$.[7]

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[5] The case of variable and endogenous capacity is examined in Basso (2005) and Zhang and Zhang (2006) in a congestion-pricing framework.

[6] This is similar to the two-period (peak/off-peak) formulation developed in Brueckner (2002).

[7] This functional form was previously estimated from steady-state queuing theory and is further discussed in U.S. Federal Aviation Administration (1969) and Horonjeff and McKelvey (1983). It has been used by, e.g., Morrison (1987), Zhang and Zhang (2003), and Basso (2005).

To obtain the consumer demands for peak and off-peak travel, we first note the following characteristics about the allocation of consumers: (i) if consumer θ_1 flies, then consumers $\theta \geq \theta_1$ fly; (ii) if consumer θ_1 does not fly, then consumers $\theta < \theta_1$ do not fly; and (iii) if θ^* denotes the consumer who is indifferent between traveling in the peak and off-peak periods, then passengers $\theta \geq \theta^*$ choose peak travel whereas passengers $\theta < \theta^*$ choose off-peak travel or non-travel.[8] Hence, if we denote θ^f the consumer who is indifferent between flying and not flying, (i), (ii) and (iii) above imply, in the case of an interior solution, that $\underline{\theta} < \theta^f < \theta^* < \overline{\theta}$. We assume for now the allocation is interior, but later shall find conditions on the parameters for this to hold. Using q_h to denote the total number of passengers in period h (h=p,o), then $q_p = \overline{\theta} - \theta^*$ and $q_o = \theta^* - \theta^f$.

Since runway charges are imposed on aircraft (flights), we need to transform the passenger-based demands q_p and q_o into per-flight demand functions. As in Brueckner (2002), Pels and Verhoef (2004), Basso (2005) and Zhang and Zhang (2006), we make a "fixed proportions" assumption, i.e., S = Aircraft Size × Load Factor, is constant and the same across carriers.[9] It then follows immediately that $q_p = Q_p S = \overline{\theta} - \theta^*$ and $q_o = Q_o S = \theta^* - \theta^f$, or equivalently,

$$\theta^* = \overline{\theta} - Q_p S, \qquad \theta^f = \theta^* - Q_o S \tag{5}$$

From (3), the indifferent flyer θ^* is determined by $\theta^*(B_p - B_o) - \alpha D_p = t_p - t_o$ (recall that $D_o = 0$). This says that a passenger's gain $\theta^*(B_p - B_o)$, net of the delay cost αD_p when shifting from the off-peak to peak

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[8] *Proof:* (i) If θ_1 flies, then $\theta_1 B_h - \alpha D_h - t_h \geq 0$ for h=p,o. If $\theta \geq \theta_1$, $\theta B_h - \alpha D_h - t_h \geq \theta_1 B_h - \alpha D_h - t_h \geq 0$ and so θ flies. (ii) is analogous. (iii) Let $\Delta\overline{V}(\theta) \equiv \overline{V}_p(\theta) - \overline{V}_o(\theta) = (B_p - B_o)\theta - \alpha(D_p - D_o) - (t_p - t_o)$ and suppose θ flies. Then if $\Delta\overline{V}(\theta) \geq 0$, θ chooses to fly in the peak period. If $\Delta\overline{V}(\theta) < 0$, θ chooses to fly in the off-peak period. Now, suppose that there exists θ^* such that $\Delta\overline{V}(\theta^*) = 0$ (interior solution). Then it follows, since $\Delta\overline{V}'(\theta) > 0$, that if $\theta \geq \theta^*$, θ chooses peak travel and if $\theta < \theta^*$, θ chooses off-peak travel or non-travel. ■

[9] That is, the number of passengers in each flight is constant. This assumption also allows us to abstract away from the issue of weight-based pricing as aircraft here have the same weight, and thereby focus on the main issue of peak-load congestion pricing.

periods, is balanced by the fare differential $t_p - t_o$. The final flyer θ^f is determined by $\theta^f B_o = t_o$. Replacing θ^* and θ^f in (5) we obtain:

$$t_o(Q_o, Q_p) = B_o\overline{\theta} - B_o SQ_o - B_o SQ_p \tag{6}$$

$$t_p(Q_o, Q_p) = B_p\overline{\theta} - B_o SQ_o - B_p SQ_p - \alpha D(Q_p) \tag{7}$$

Equation (6) is the (inverse) consumer demand function faced by the airlines for the off-peak period, whereas (7) is the consumer demand function for the peak period. Note that this demand system is not linear if D is not. Further, the peak and off-peak flights are substitutes for the final passengers, which gives the room for airlines to "spread the demand" across the peak and off-peak travel periods by using peak-load airfares. The analytical expressions of the cross elasticity of demand between peak travel and off-peak travel can be obtained from (6) and (7).

We now turn to the airlines. They have identical cost functions, given by:

$$c_A^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) = \sum_{h \in p,o} [c + P_h + \beta D(Q_h)] Q_h^i \tag{8}$$

where Q_h^i is the number of airline i's flights in period h, $\mathbf{Q}_h^{-i}$ denotes the vector of flights of airlines other than i, c is the airline's operating cost per flight, and P_h is the airport landing fee in period h.[10] Further, parameter β (>0) measures the delay costs to an airline per flight, which may include wasted fuel burned while taxiing in line or holding/circling in the air, extra wear and tear on the aircraft, and salaries of flight crews. Airlines' profit functions can then be written as:

$$\phi^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) = \sum_{h \in p,o} t_h(Q_o, Q_p) Q_h^i S - c_A^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) \tag{9}$$

With these functions at hand, we shall investigate the subgame perfect equilibrium of our two-stage airport-airlines game.

[10] As indicated earlier, airport charges usually include landing and terminal charges (charges for aircraft parking are minor). While landing fees are based on aircraft movements, terminal charges are typically per-passenger based. Since the present paper is concerned with runway congestion, we shall focus on landing fees.

3. ANALYSIS OF OUTPUT-MARKET EQUILIBRIUM

To solve for the subgame perfect equilibrium we start with the analysis of the second-stage airline competition. Given the airport's runway charges P_p and P_o, the N carriers choose their quantities to maximize profits, and the Cournot equilibrium is characterized by the first-order conditions,

$$\partial\phi^i / \partial Q_h^i = 0 \ , \qquad h=p,o$$

(note the second-order conditions are satisfied).[11] Imposing symmetry $Q_h^i = Q_h / N$ and re-arranging, the first-order conditions can be expressed as:

$$\Omega^o(Q_o, Q_p, P_o, N) \equiv (B_o\overline{\theta}S - c - P_o) - Q_o \frac{B_o S^2(N+1)}{N} - Q_p \frac{B_o S^2(N+1)}{N} = 0 \tag{10}$$

$$\begin{aligned}\Omega^p(Q_o, Q_p, P_p, N) \equiv (B_p\overline{\theta}S - c - P_p) - Q_o \frac{B_o S^2(N+1)}{N} - Q_p \frac{B_p S^2(N+1)}{N} \\ - (\alpha S + \beta)\left(D(Q_p) + \frac{Q_p}{N} D'(Q_p) \right) = 0\end{aligned} \tag{11}$$

As demonstrated in the Appendix (Proposition A.1), there exist conditions on the parameters that guarantee interior solutions, that is, $\underline{\theta} < \theta^f < \theta^* < \overline{\theta}$ or equivalently, $Q_p, Q_o, Q_n > 0$. For example, the peak period is used if the per-passenger airport peak/off-peak price differential is smaller than the incremental gross benefit, for the highest consumer type $\overline{\theta}$, of shifting from off-peak travel to peak travel. In particular, when the airport does not practice peak-load pricing (so $P_p = P_o$), the peak period is always used. The proof also reveals that a smaller airport peak/off-peak price differential increases the likelihood of both the peak and off-peak periods being used, and that the off-peak period is always used if $\underline{\theta}$ is large enough.[12] In the remainder of the paper we shall restrict our attention to interior allocations.

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[11] We have assumed a Cournot game in the output-market competition. Brander and Zhang (1990, 1993), for example, find some empirical evidence that rivalry between duopoly airlines is consistent with Cournot behaviour.

[12] These results suggest that Brueckner (2002, 2005)'s single crossing property, which was introduced to guarantee the existence of a non-empty peak/off-peak interior solution, may not be needed. This may be desirable because the

A useful equation obtained from (10) and (11) is:

$$-\Omega^p + \Omega^o = Q_p \frac{(B_p - B_o)S^2(N+1)}{N} + (\alpha S + \beta)\left(D(Q_p) + \frac{Q_p}{N} D'(Q_p) \right) + (P_p - P_o) - \bar{\theta} S(B_p - B_o) = 0 \tag{12}$$

Since equation (12) depends on Q_p but not on Q_o, it implicitly defines Q_p as a function of P_o, P_p and N. Substituting this function into (10), equation (10) then implicitly defines Q_o as a function of P_o, P_p and N, leading to:

$$Q_p = Q_p(P_o, P_p; N), \qquad Q_o = Q_o(P_o, P_p; N) \tag{13}$$

Equations (13) are the *airport*'s demands for the use of its peak and off-peak periods, respectively. Here it is worth stressing that, while $t_o(Q_o, Q_p)$ and $t_p(Q_o, Q_p)$ –defined by equations (6) and (7)– capture the *final* consumer (inverse) demands for air travel, $Q_o(P_o, P_p; N)$ and $Q_p(P_o, P_p; N)$ are the *derived* demands faced by the airport.

We now characterize the airport's demands $Q_p(P_o, P_p; N)$ and $Q_o(P_o, P_p; N)$. Totally differentiating (10) and (12) with respect to P_p yields:

$$\begin{aligned} \frac{\partial Q_p}{\partial P_p} &= -\frac{\partial(-\Omega^p + \Omega^o)/\partial P_p}{\partial(-\Omega^p + \Omega^o)/\partial Q_p} \\ &= -\frac{N}{(B_p - B_o)S^2(N+1) + (\alpha S + \beta)\big((N+1)D'(Q_p) + Q_p D''(Q_p)\big)} < 0 \end{aligned} \tag{14}$$

where the inequality follows from conditions (2) and (4). So the airport's demand for the peak period is, as expected, downward-sloping in the peak charge. Similarly, we can obtain:

property implies, if using Brueckner's interpretation of θ as an index of a passenger's tendency to travel on business (as opposed to leisure travel), that peak benefits are higher than off-peak benefits for business travelers, but are lower than off-peak benefits for leisure travelers. This appears contradictory with the idea that the peak and off-peak periods are vertically differentiated.

$$\frac{\partial Q_p}{\partial P_p} < 0, \qquad \frac{\partial Q_p}{\partial P_o} = -\frac{\partial Q_p}{\partial P_p} > 0,$$
$$\frac{\partial Q_o}{\partial P_p} = -\frac{\partial Q_p}{\partial P_p} = \frac{\partial Q_p}{\partial P_o} > 0, \quad \frac{\partial Q_o}{\partial P_o} = -\frac{\partial Q_p}{\partial P_o} - \frac{N}{B_o S^2 (N+1)} < 0, \tag{15}$$
$$\frac{\partial (Q_o + Q_p)}{\partial P_p} = 0, \qquad \frac{\partial (Q_o + Q_p)}{\partial P_o} = -\frac{N}{B_o S^2 (N+1)} < 0$$

We can see that, *ceteris paribus*, the airport peak charge does not influence total traffic but only the allocation of traffic to the peak and off-peak periods. Furthermore, from (12) and (14) we get:

$$\frac{\partial Q_p}{\partial \Delta P_{p-o}} = \frac{\partial Q_p}{\partial P_p} < 0 \tag{16}$$

where $\Delta P_{p-o} \equiv P_p - P_o$. The above results (15) and (16) lead to:

Proposition 1: The airport's demands $Q_p(P_o, P_p; N)$ and $Q_o(P_o, P_p; N)$ have the following properties:

(i) They are downward-sloping in own prices;

(ii) The peak and off-peak periods are gross substitutes;

(iii) The off-peak runway charge (P_o) determines the amount of total traffic, while the difference between the peak and off-peak charges (ΔP_{p-o}) determines the partition of that traffic into the two periods, with peak traffic declining with the charge differential.

Notice that Part (ii) of Proposition 1 shows that the airport has the room to "spread the flights" across the peak and off-peak periods by using peak-load landing fees. Together with the discussion following equations (6) and (7), therefore, our vertical airport-airline structure gives rise to a possible *sequential* PLP: the PLP schemes implemented by the downstream airlines (higher peak airfare) induce a different periodic demand for the upstream airport. As shown in Proposition 2 below, the shape of that demand depends on the number of downstream carriers and the type of competition they exert. Our analysis conducted later in Section 4 shows that indeed, the airport then would have an incentive to use PLP as well, which in turn affects the way the downstream firms use PLP.

Next we examine the airport demands change with the number of airlines, *N*. We obtain the following comparative static results:

Proposition 2: At the sub-game Cournot equilibrium,

(i) $0 < \partial Q_p / \partial N < Q_p / \big(N(N+1)\big)$, so the number of passengers traveling in the peak period increases with *N*;

(ii) $\partial(Q_o + Q_p) / \partial N = (Q_o + Q_p) / \big(N(N+1)\big) > 0$, so the total number of traveling passengers increases with *N*;

(iii) $\partial Q_o / \partial N > Q_o / \big(N(N+1)\big)$, so if the off-peak period is used ($Q_o > 0$) then the number of passengers traveling in the off-peak period increase with *N*.

The proof of Proposition 2 is provided in the Appendix. Given that we consider interior solutions, conditional on runway fees P_p and P_o, both the peak and off-peak traffic volumes increase with the number of firms in the output market. The proposition also shows that the (positive) elasticity of total traffic with respect to *N*, is equal to $1/(N+1)$, whereas the elasticities of the peak traffic and off-peak traffic (with respect to *N*) are, respectively, smaller and larger than $1/(N+1)$. All three elasticities become smaller as *N* increases.

The final ingredient to characterize the Cournot equilibrium in the output market is related to the important issue of airfares: For given airport charges, how do the peak and off-peak airfares compare with each other? From (6) and (7) it follows that

$$\Delta t_{p-o} \equiv t_p - t_o = \bar{\theta}(B_p - B_o) - Q_p S(B_p - B_o) - \alpha D(Q_p) \tag{17}$$

From the equilibrium condition (12) we obtain an expression for $\bar{\theta}(B_p - B_o)$. Replacing that expression in (17) gives rise to the following airfare-differential formula, evaluated at the Cournot equilibrium:

$$\Delta t_{p-o}\Big|_{\text{Cournot eq}} = \frac{P_p - P_o}{S} + \frac{\beta}{S} D(Q_p) + \frac{\beta}{S}\frac{Q_p}{N} D'(Q_p) + \alpha \frac{Q_p}{N} D'(Q_p) + Q_p \frac{(B_p - B_o)S}{N} \tag{18}$$

It is clear from (18) that if $P_p \geq P_o$, then $\Delta t_{p-o} > 0$, that is, if the airport uses peak-load pricing (in the sense that it charges a higher landing fee in the peak than in the off-peak), airlines will also use PLP (i.e., higher peak airfares) in equilibrium. More interesting perhaps is the fact that, even if the airport price the periods *backwards*, i.e., $P_p < P_o$, airlines may still use peak-load pricing in equilibrium, because the remaining four terms in (18) are all positive.

To further interpret (18), first note that holding P_p and P_o constant, $\partial \Delta t_{p-o} / \partial N$ is negative, which can be seen by differentiating (17) and recalling, from Proposition 2, that sub-game equilibrium Q_p and Q_o increase in N. This implies that a monopoly airline would have the largest airfare differential. Since, from (6), $\partial t_o / \partial N$ is also negative, the lower the N, the larger the off-peak fare. These two observations are consistent with what we have already shown in Proposition 2 with respect to total peak and off-peak traffic. Next, it can be seen that for very large N, the airfare differential approaches to the difference between an airline's peak and off-peak per-passenger average costs, i.e., the first and second terms on the right-hand side (RHS) of (18). When there is an oligopoly, however, three extra terms are added. Specifically, the third term on the RHS of (18) is the cost of extra congestion on an airline's own flights and caused by an additional passenger flying in the peak period. Thus, the first three terms on the RHS of (18) represent the difference between an airline's peak and off-peak marginal costs. The fourth term represents the money value of extra congestion to an airline's passengers when a new passenger chooses to fly in the peak period, whereas the fifth term is the mark-up term that arises from the oligopoly airlines' exploitation of market power. Hence, as it is now known, the airlines in oligopoly only internalize (charge for) the congestion they impose on their own flights, which has two cost components: extra operating costs for the airline, and extra delay costs for its passengers (Brueckner, 2002). When there is a monopoly airline, congestion is perfectly internalized but exploitation of market power is at its highest degree. When N is large, exploitation of the market power is small but congestion is imperfectly internalized.

These points can be made more clearly if the Cournot case is compared to the case in which a social planner maximizes total surplus in the second-stage game. To do this we first need a measure of consumer surplus (*CS*). Given the linearity of our conditional indirect utility function in (3), *CS* is given by:

$$CS = \int_{\theta^*}^{\bar{\theta}} \left[\theta B_p - \alpha D(Q_p) - t_p(Q_p,Q_o)\right] f(\theta)d\theta + \int_{\theta^f}^{\theta^*} \left[\theta B_o - t_o(Q_p,Q_o)\right] f(\theta)d\theta \qquad (19)$$

where $f(\theta)$ is the density function. Using (6) and (7) for t_o and t_p, solving the integrals and replacing θ^* and θ^f with (5), we finally obtain:

$$CS = \frac{S^2}{2}\left(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2\right) \qquad (20)$$

As said, we now examine the case in which the planner maximizes, for given airport charges, the sum of consumer surplus and airline profits:

$$CS + \Phi \equiv CS + \sum_{i=1}^{N} \phi^i \qquad (21)$$

where Φ denotes the aggregate airline (equilibrium) profits. The first-order conditions of (21) with respect to airline quantities, together with the imposition of symmetry, then lead to two equations, analogous to (10) and (11), which characterize the optimum. Subtracting the two equations from each other yields:

$$Q_p(B_p - B_o)S^2 + (\alpha S + \beta)\left(D(Q_p) + Q_p D'(Q_p)\right) + (P_p - P_o) - \bar{\theta} S(B_p - B_o) = 0 \qquad (22)$$

Using (22) to obtain a new expression for $\bar{\theta}(B_p - B_o)$ and replacing the term in (17), we get:

$$\Delta t_{p-o}\Big|_{\substack{\text{efficient}\\ \text{output}}} = \frac{P_p - P_o}{S} + \frac{\beta}{S} D(Q_p) + \frac{\alpha S + \beta}{S} Q_p D'(Q_p) \qquad (23)$$

Conditional on the airport charges and the airline market structure, (23) gives the socially efficient difference between the peak and off-peak airfares. This fare differential is equal to the difference between an airline's peak and off-peak average costs (the first and second terms on the RHS of (23)), plus all the external costs associated with a new flyer in the peak period, with the latter being the extra congestion cost of *all* the airlines and passengers, not just that of the airline that carries the new peak passenger. Obviously, the last two terms represent the portion of the optimal airfare differential that is not directly affected by the airport's pricing practices.

It is also insightful –and useful later in Section 5– to compare the Cournot case with the "cartel" case in which the airlines choose outputs to maximize their joint profit $\Phi = \sum_{i=1}^{N} \phi^i$. The first-order conditions in the cartel case, together with the imposition of symmetry, lead to two equations, analogous to (10) and (11), which characterize the cartel optimal solution. Subtracting the two equations then yields:

$$2Q_p(B_p - B_o)S^2 + (\alpha S + \beta)\big(D(Q_p) + Q_p D'(Q_p)\big) + (P_p - P_o) - \bar{\theta} S(B_p - B_o) = 0 \qquad (24)$$

Using (24) to obtain a new expression for $\bar{\theta}(B_p - B_o)$ to replace the term in (17), we find the difference between the peak and off-peak airfares in the cartel case:

$$\Delta t_{p-o}\Big|_{\text{cartel output}} = \frac{P_p - P_o}{S} + \frac{\beta}{S} D(Q_p) + \frac{\alpha S + \beta}{S} Q_p D'(Q_p) + Q_p (B_p - B_o) S \qquad (25)$$

Here, the fare differential is equal to the difference between an airline's peak and off-peak average costs (the first and second terms on the RHS of (26)), plus all the external costs associated with a new flyer in the peak period. Comparing (25) with (23) reveals that the cartel, as in the social-planner case, internalizes the congestion costs of all the carriers and passengers. Here however, there is a fourth term which will increase the fare differential. This term is related to the "business stealing" externality: Since oligopoly carriers behave in non-cooperative fashion, they produce too much with respect to the optimum for the airlines as a whole. This is so because they fail to consider the profits lost by the other airlines when they increase own output, depressing airfares: the fare differential of an oligopoly is insufficiently large from the cartel's point of view, and this problem worsens the "looser" the oligopoly is (i.e., the larger N is). Consequently, the cartel, as a monopoly, is interested in having a less used peak period. In effect, the cartel airfare differential is identical to the monopoly's; see (18) by imposing N=1. The cartel and monopoly traffic volumes will differ however, since cost functions are convex and not flat.

4. AIRPORT PRICING, TRAFFIC, DELAY AND WELFARE COMPARISONS

We have shown that the airport decisions, namely, P_p and P_o, can influence the subsequent output-market competition among airlines. When deciding its runway charges in the first stage, therefore, the airport will take the second-stage equilibrium output into account. These decisions may in reality be set by a public airport or a privatized airport. Consequently, the objective of an airport may be to maximize welfare or to maximize profit. In this section, we compare airport charges and consequent airfares for these two airport types.

Consider first a public airport that chooses P_p and P_o to maximize welfare. With three agents –namely, airport, airlines, and passengers– social welfare (*SW*) is the sum of their payoffs:

$$SW(P_o, P_p; N) = \pi(P_o, P_p; N) + CS + \Phi \tag{26}$$

where the airport's profit, π, is given by

$$\pi(P_o, P_p; N) = P_o Q_o + P_p Q_p - C \cdot (Q_o + Q_p) - rK \tag{27}$$

In (27), $Q_o = Q_o(P_o, P_p; N)$ and $Q_p = Q_p(P_o, P_p; N)$ are the airport's demands for the peak and off-peak periods, respectively, and are given by (13). In addition, C is the unit runway operating cost of the airport and r is its unit cost of capital. Note that we have assumed, as is common in the literature, that the operational and capital costs are separable and that the marginal operating cost is constant. For the latter, the estimation of cost function showed that airport runways have relatively constant return to scale (e.g., Morrison, 1983; Pels, Nijkamp and Rietveld, 2003). Consumer surplus, *CS*, is given by (19), whereas the aggregate airlines' profit, $\Phi = \sum_{i=1}^{N} \phi^i$, is introduced in (21). Since the downstream equilibrium is symmetric, $\phi^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) = \phi^1\left(Q_o(P_o, P_p; N), Q_p(P_o, P_p; N), P_h\right)$ is each airline's equilibrium profit. We can then easily calculate Φ as $\Phi(P_o, P_p; N) = N \cdot \phi^1(P_o, P_p; N)$, that is,

$$\begin{aligned}\Phi(P_o, P_p; N) = {} & \bar{\theta} S(B_p Q_p + B_o Q_o) - S^2\left(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2\right) - (\alpha S + \beta) Q_p D(Q_p) \\ & - (c + P_o) Q_o - (c + P_p) Q_p\end{aligned} \tag{28}$$

We do not include a budget constraint in the public airport problem, noting that fixed fees may solve the problem of budget adequacy. If lump-sum transfers are not feasible, then Ramsey-Boiteaux prices should be considered (see Basso, 2005, for more discussion on this).

Substituting Φ, π , and *CS* into (26) we obtain:

$$\begin{aligned} SW = \bar{\theta} S(B_p Q_p + B_o Q_o) - c \cdot (Q_p + Q_o) - C \cdot (Q_p + Q_o) - rK \\ - S^2 (B_o Q_o^2 + 2 B_o Q_o Q_p + B_p Q_p^2) / 2 - (\alpha S + \beta) Q_p D(Q_p) \end{aligned} \tag{29}$$

Derivation of the pricing formulas then follows from the first-order conditions. More specifically, we obtain, using equations (10), (12) and (15) (P_o^W, P_p^W denoting the welfare-maximizing runway charges):

$$P_o^W = C - \frac{Q_o S^2 B_o}{N} - \frac{Q_p S^2 B_o}{N} \tag{30}$$

$$\Delta P_{p-o}^W = \frac{N-1}{N} (\alpha S + \beta) Q_p D'(Q_p) - \frac{Q_p S^2 (B_p - B_o)}{N} \tag{31}$$

The above welfare-maximizing airport pricing may be seen as if the fees were determined in two phases. First, choice of an off-peak price P_o^W induces the (socially) right amount of total traffic; as can be seen from (30), P_o^W is below the airport's marginal cost. This is needed because exploitation of market power in the airline market would induce allocative inefficiencies by producing too little output. A welfare-maximizing airport fixes this inefficiency by providing a "subsidy" to the airlines and hence lowering their marginal costs in the off-peak period. The exact amount of the subsidy depends in part on the extent of market power, which here is captured by *N*. Once the total traffic is set to its optimal level, the next phase is concerned with the optimal allocation of this traffic to the peak and off-peak periods, which is, as indicated earlier, determined by ΔP_{p-o}. In particular, the public airport sets the peak/off-peak price differential to ΔP_{p-o}^W that will induce the optimal airfare differential downstream. This is apparent from substituting (31) into (18), which yields

$$\Delta t_{p-o}^W \Big|_{\text{Cournot eq}} = \frac{\beta}{S} D(Q_p) + \frac{\alpha S + \beta}{S} Q_p D'(Q_p) > 0 \tag{32}$$

The RHS of (32) is equal to the optimal airfare differential that is not directly affected by the airport's pricing practices, as discussed in (23). Hence, the outcome is the same as if the airport were to set $P_o = P_p$, which is optimal because there are no differences in costs, and then social welfare is maximized in the airline market.

Brueckner (2002) identified the first term in (31) as the per-flight toll that should be charged by the airport authorities to address the problem of uninternalized congestion (note that when *N*=1, this toll is equal to zero). Pels and Verhoef (2004), Basso (2005) and Zhang and Zhang (2006) pointed out that the optimal toll should also include the second term, the market-power effect;[13] they did this, however, using models of congestion pricing (one period), while Brueckner (2002) and the present paper use a model of peak-load pricing. This distinction is important because a toll equal to the two terms, thereby capturing both the congestion and market power effects, will not be optimal unless it is charged on top of the optimal charge in the off-peak period, *which is not the marginal cost*. In other words, restricting the analysis to the toll that should be charged during the peak hours offers only a partial view of the problem.

But note further that the charge differential, ΔP_{p-o}^{W}, given in (31), is not signed *a priori*. Hence, *it may happen that the airport charge is smaller in the peak period than in the off-peak period*. More specifically, the airport charge differential will be negative for small *N*. This is so because a "tight" airline oligopoly has an airfare differential that is too large due to strong market power, while congestion is reasonably internalized. As a consequence, the airport price differential is driven predominantly by the market-power effect. When *N* is large, on the other hand, the airport price differential will be positive. This is so because a "loose" oligopoly would have an airfare differential that is too small due to uninternalized congestion, whereas market power is relatively weak. The airport charge differential is then driven by the congestion effect. Note from (32) that although P_p^W (the welfare-maximizing peak charge) may be less than P_o^W, final passengers will, nevertheless, always pay higher peak airfare than off-peak airfare.

The above discussion may be summarised in the following proposition:

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[13] To be fair, although Brueckner did not formally consider the second term in the toll to be charged, he did point out that, depending on the size of the market-power term, a pure congestion toll could be detrimental for social welfare.

Proposition 3: For a public, welfare-maximizing airport, (i) the off-peak runway charge is below its marginal cost; (ii) for small N, the off-peak runway charge may be *greater* than its peak runway charge; in this sense, it appears that the airport does not use peak-load pricing; (iii) although the airport's peak charge may be less than its off-peak charge, final passengers will nevertheless always pay higher peak airfare than off-peak airfare.

Note that if lump-sum transfers (two-part tariffs) are unfeasible, the pricing rules previously discussed may lead to airport's budget inadequacy. If budget adequacy has to be ensured but lump sums are not feasible, then the first best may not be attainable: Marginal prices would have to do both –namely, aligning incentives and transferring surplus– making the airport fall short of "control instruments" (Mathewson and Winter, 1984).

Next, consider a private, unregulated airport. The airport's profit is given by (27). The airport will choose P_p and P_o to maximize its profit, and the first-order conditions lead to (P_o^π, P_p^π denoting the profit-maximizing airport charges):

$$P_o^\pi - C = \frac{P_o^\pi}{\varepsilon_{oo}} + \frac{(P_p^\pi - C)Q_p\varepsilon_{po}}{Q_o\varepsilon_{oo}} \tag{33}$$

$$P_p^\pi - C = \frac{P_p^\pi}{\varepsilon_{pp}} + \frac{(P_o^\pi - C)Q_o\varepsilon_{op}}{Q_p\varepsilon_{pp}} \tag{34}$$

where $\varepsilon_{oo} \equiv -(\partial Q_o / \partial P_o)(P_o / Q_o)$ is the (positive) price elasticity of off-peak airport demand, $\varepsilon_{po} \equiv (\partial Q_p / \partial P_o)(P_o / Q_p)$ is a cross-price elasticity, and ε_{pp} and ε_{op} are defined analogously. Since $\partial Q_p / \partial P_o > 0$ and $\partial Q_o / \partial P_p > 0$ –see (15) or Proposition 1– both ε_{op} and ε_{po} are positive, implying that the airport charges are higher than would be if the peak and off-peak charges were chosen independently (in which case the mark-ups would be proportional to the inverse of own-price demand elasticities only). This is a well-known result for multi-product monopolies that produce substitutes.

We can simplify the pricing equations and show that $P_p^\pi > P_o^\pi$. To do this, replace the elasticities' definitions and simplify, using the fact that $\partial Q_o / \partial P_p = -\partial Q_p / \partial P_p$ in (15) and then using equation (14). We obtain the following charging formulas:

$$P_o^\pi = C + \frac{Q_o S^2 B_o (N+1)}{N} + \frac{Q_p S^2 B_o (N+1)}{N} \tag{35}$$

$$\Delta P_{p-o}^\pi = \frac{\alpha S + \beta}{N} Q_p \left[(N+1) D'(Q_p) + Q_p D''(Q_p) \right] + \frac{Q_p (B_p - B_o) S^2 (N+1)}{N} \tag{36}$$

The RHS of (36) is, by (2) and (4), positive and hence $P_p^\pi > P_o^\pi$: The private airport charges higher runway fees in the peak period than in the off-peak period, and this is true for any *N*. Thus, a profit-maximizing airport has an incentive to use peak-load pricing. Further, note that, since $(\alpha S + \beta) Q_p (N-1)/N$ is the extra cost each airline induces by not fully internalizing congestion, the first term on the RHS of (36) shows that the private airport will overcharge for congestion. Moreover, notice from (35) that the off-peak charge, which determines the amount of total traffic, is above marginal cost. This is a result of monopoly power on the part of the airport. There is, therefore, a "double marginalization" problem, which is typical of an uncoordinated vertical structure. The discussion leads to the following proposition:

Proposition 4: A private, profit-maximizing airport would use peak-load pricing but would charge more than the cost of uninternalized congestion. Further, it would charge an off-peak runway fee that is above its marginal cost.

Having derived and characterized the pricing structures for both the public and private airports, we now want to compare them. To have a clearer picture about their performance differences, we shall compare not only the off-peak runway fees and the peak/off-peak fee differentials, but also the induced traffic levels, delays and total surplus levels. Moreover, we want to assess how these differences (if any) change with the number of airlines, *N*, which is exogenously given and may be considered as a proxy for airline market structure. We summarize our findings in the following proposition (the proof is provided in the Appendix):

Proposition 5: Comparisons of airport pricing, traffic, delay and welfare between the private and public airports are as follows:

(i) $P_o^W < P_o^\pi$ and $\dfrac{dP_o^W}{dN} > \dfrac{dP_o^\pi}{dN} = 0$;

(ii) $\Delta P_{p-o}^W < \Delta P_{p-o}^\pi$ and $\dfrac{d\Delta P_{p-o}^W}{dN} > 0$. If the delay function is linear, then $\dfrac{d\Delta P_{p-o}^\pi}{dN} = 0$;

(iii) $Q_p^W > Q_p^\pi$ and $\dfrac{dQ_p^\pi}{dN} > \dfrac{dQ_p^W}{dN} = 0$;

(iv) $Q_t^W > Q_t^\pi$ and $\dfrac{dQ_t^\pi}{dN} > \dfrac{dQ_t^W}{dN} = 0$, where $Q_t \equiv Q_p + Q_o$ is total traffic volume;

(v) $D_p^W > D_p^\pi$ and $\dfrac{dD_p^\pi}{dN} > \dfrac{dD_p^W}{dN} = 0$;

(vi) $SW^W > SW^\pi$ and $\dfrac{dSW^\pi}{dN} > \dfrac{dSW^W}{dN} = 0$.

From Proposition 5 we see that a private, profit-maximizing airport would induce too small total traffic as compared to the first-best outcome, thereby resulting in allocative inefficiencies. Additionally, a private airport has a greater peak/off-peak runway charge differential than a public airport. Hence, with a private airport, the peak period would be underused not only because the airport has smaller total traffic, but also because its charge differential is too large. This reduction in peak traffic volume is welfare-reducing since passengers view, other things being equal, traveling in the peak times as a higher quality product than traveling in the off-peak times. Less peak traffic then means fewer consumers will enjoy a premium product. And although those passengers who still use the peak period benefit from less delays as part (v) enounces, overall it is not economically efficient to have such a lower level of peak congestion because total welfare is in fact reduced as shown in part (vi) of Proposition 5. To help better understand this proposition, we also offer a schematic representation of the findings in Figure 1.

A numerical simulation helps to better see these deadweight losses.[14] In Table 1, we show total welfare for a private airport as a percentage of the total welfare obtained by a public airport (as can be seen from part (vi), SW^W does not depend on *N*), and for different values of *N*. Because lump-sum transfers may not be feasible, we also provide the social welfare levels attained by a public airport that must achieve budget adequacy and therefore uses Ramsey-Boiteaux prices.

Table 1: Percentage of maximum social welfare attainable

	$N=1$	$N=3$	$N=5$	$N=10$
Private airport	49	65	69	74
Budget-constrained public airport	81	96	98	99

This discussion and the results in Table 1 highlight an important issue: one of the main ideas behind airport privatization has been that it would allow airports to use peak-load pricing and thus help solve the congestion problems. But if privatization is measured solely by its effect on congestion delays, it may be seen as a better idea than it actually is, and important deadweight losses may be overlooked. This result, which holds here for a fixed capacity–peak-load pricing model, was also found by Basso (2005) in a congestion pricing model with variable (endogenous) capacity.

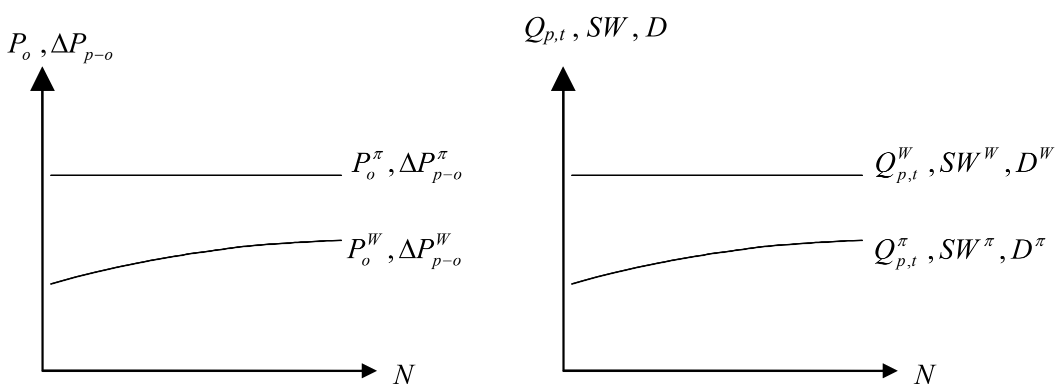

Figure 1: Schematic representation of the results in Proposition 5

[14] Details of the simulation are available upon request.

We have seen that the public airport is indifferent between values of N –although Basso (2005) showed that this may not be the case if airlines are not homogenous or if passengers are affected by schedule delay cost. Given that the (welfare) performance of a private airport improves as the number of airlines rises (see Proposition 5 and Figure 1), it seems important to know what would be the preferred N of a private airport itself. From (34) we have:

$$\left|\frac{d\pi}{dN}\right|_{P_o^\pi, P_p^\pi} = \left|\frac{\partial \pi}{\partial N}\right|_{P_o^\pi, P_p^\pi} = \left(P_o^\pi - C\right)\frac{\partial Q_o}{\partial N} + \left(P_p^\pi - C\right)\frac{\partial Q_p}{\partial N} > 0$$

where the first equality follows from the envelope theorem and the inequality follows from Proposition 2 and the first-order conditions (33) and (34), which indicate that prices are above marginal costs. Thus, the private airport prefers a large N, which is a desirable property, given the findings of Proposition 5.

5. Airport-Airline Joint Profit Maximization

In this section, we shall consider an airport that has some sort of strategic agreement with the airlines using it. The reasons why it is interesting to look at this case are two-fold: on one hand, a simple pricing mechanism, two-part tariff, may be enough for the outcome of joint profit maximization to arise. On the other hand, it has been often argued that greater airlines' *countervailing power* or more strategic collaboration between airlines and airports may improve efficiency of privatized airports by allowing a better alignment of incentives, and even may make price regulation unnecessary (see, e.g., Beesley, 1999; Condie 2000; Forsyth, 1997; Starkie, 2001; Productivity Commission, 2002; Civil Aviation Authority UK, 2004). The analysis of joint profit maximization may then serve as a benchmark case.

The objective faced by this airport is to maximize the sum of the airport's profit and airlines' profits. Using π in (27) and Φ in (28), the problem can be re-written as:

$$\begin{aligned}\max_{P_o, P_p} \pi + \Phi = \bar{\theta} S(B_p Q_p + B_o Q_o) - c\cdot(Q_p + Q_o) - C\cdot(Q_p + Q_o) - rK \\ - S^2(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2) - (\alpha S + \beta) Q_p D(Q_p)\end{aligned}$$

Derivation of the pricing formulas follows from the first-order conditions of the above problem, using equations (10), (12) and (15) and rearranging (P_o^{JP}, P_p^{JP} denoting the joint profit-maximizing airport charges),

$$P_o^{JP} = C + \frac{Q_o S^2 B_o (N-1)}{N} + \frac{Q_p S^2 B_o (N-1)}{N} \tag{37}$$

$$\Delta P_{p-o}^{JP} = \frac{N-1}{N}(\alpha S + \beta) Q_p D'(Q_p) + \frac{Q_p S^2 (B_p - B_o)(N-1)}{N} \tag{38}$$

The interpretation of the JP airport's pricing rules (37) and (38) is as follows. As before, this airport may be seen as deciding its runway fees in two phases; first, it induces a contraction of total traffic by choosing an off-peak price P_o^{JP} above its marginal cost. It does so because the failure of coordination among the airlines results in them producing too much with respect to what would be best for them as a whole. The amount of excess production depends on how tight the oligopoly is, which is why the off-peak mark-up decreases with *N*. In particular, when the airline market is monopolized, (37) shows that $P_o^{JP} = C$ so the airport does not need the mark-up at all. Comparison of (37) and (35), however, shows that the total traffic contraction in the JP case is smaller than that in the pure private case; indeed, coordination of the vertical chain resolves the double-marginalization problem.

In the second phase, the airport chooses the (non-negative) price differential ΔP_{p-o}^{JP} that will induce the airlines' cartel outcome, destroying airline competition downstream. This is apparent from substituting (38) into (18), and noting that the result is equal to the cartel's airfare differential not directly affected by the airport pricing practice. Hence, the outcome is the same as if the airport were to set $P_o = P_p$ and a cartel were running the airline market. This result, which was obtained by Basso (2005) in a congestion pricing setting, has different intuitions depending on why the maximization of joint profits is the relevant case. With two-part tariffs, the private airport use the variable prices, peak and off-peak, to destroy competition downstream in order to maximize the profits of airlines, which are later captured by the airport through the fixed fee. When the joint profit maximization arises because of collaboration between airlines and the airport, what happens is that the airlines would like to collude in order to increase profits, but are unable to do so themselves because of the incentives to defect on any possible agreement. What they manage to do, however, is to "capture" an input provider to run the cartel for them. By altering the prices of the inputs (runway services) and hence the downstream marginal costs in both the peak and the off-peak periods, the input provider (airport) induces both the collusive total output and the "right" (to the airlines) allocation of passengers to the peak and off-peak periods. The upstream firm is then rewarded with part of the collusive profits, which is where bargaining power enters the picture.

Note also that the airport pricing rules (37) and (38) take into account both the congestion externality and the business-stealing externality, at both pricing phases: the airport's price differential has two parts.[15] When N=1, there is no business-stealing effect and congestion is perfectly internalized by the monopolist. Consequently, both terms vanish: *with a monopoly airline, the airport will not use peak-load pricing*.

Now, despite the fact that the result is as if airlines were colluding, this case is not worse, in terms of social welfare, than a private airport charging linear prices as before. This is because, here, the two other harmful externalities, namely, the vertical double marginalization and the congestion externality, have been dealt with. In effect, we can show that the JP case (where the airport has strategic agreements with airlines) represents a middle-of-the-road case: in Proposition 5, the runway fees, traffic volumes, delays and social welfare will be in between those of the private and public airport cases. And in Figure 1, the curves pertaining to the JP case would be parallel displacements of the public airport curves, lying in between the two existing public and private curves. Strategic collaboration between the airport and the airlines smoothes the airport-charge problem. But recall that the downstream airfares would be as if the airlines were engaging in collusion, so we cannot expect that the JP ends up being very close to the first best. In effect, a numerical simulation has shown that the JP would correspond to 78% of the maximum social welfare attainable.[16] The performance of the JP airport gets closer the performance of a budget-constrained public airport (see Table 1), yet it is still worse. The difference is smallest when there is a monopoly airline because, in that case, the subsidies required for the first best are higher, and hence the social welfare attained by the second-best public airport is the lowest. But a slight increase in the number of airlines is enough to increase the gap in an important way.

[15] This idea of an upstream firm running the cartel for the downstream firms has been discussed in the vertical control literature and, particularly, in the input joint-venture case. For example, Chen and Ross (2003) formalized the conjecture that input joint-ventures can facilitate collusion and push a market toward the monopoly outcome. If airport provision was seen as an input joint-venture by the airlines, our results show three things in addition to what Chen and Ross have found. First, the results hold even in a peak-load pricing setting, i.e., when demand is periodic. Second, if there are externalities, the input prices are, additionally, used to force their internalization by downstream competitors. Third, when marginal costs downstream are not constant, the outcome is not as in a monopoly or a downstream merger, but as in a cartel.

[16] The proofs of the results discussed in this paragraph, and details of the simulation are available upon request.

6. CONCLUDING REMARKS

In this paper, we have analyzed the sequential peak-load pricing (PLP) problem that arises when airports are recognized as input providers for final consumer markets facing periodic demands. We have analyzed this PLP problem for a private unregulated airport, for a public airport that maximizes social welfare, and for an airport that strategically collaborates with the airlines and hence maximizes their joint profits. We found that privatization would not induce efficient peak-load pricing schemes as it has been argued in some studies. While a private airport always has an incentive to use PLP –higher runway fees in the peak than off-peak periods, even when the airlines have used PLP themselves and irrespective of the number of airlines servicing the airport– its pricing structure would induce insufficient total traffic and insufficient peak traffic. Private airports will overcharge for congestion. Somewhat surprisingly, depending on the degree of market power (captured here by the number of carriers at the airport), a public airport may choose a peak runway charge that is lower than the off-peak charge, so as to offset the market power downstream at the airline level. Here, the public airport, on the surface, is not practicing the peak-load pricing, but such pricing structure is nevertheless socially optimal. Finally, a private airport that strategically collaborates with the airlines would induce greater total traffic and greater peak traffic than a pure private airport, but both figures will still be smaller than those for a public airport. If the airport collaborates with a monopoly airline, it would not use peak-load pricing.

Although the airline industry is chosen for analysis, our basic model structure, in which airports, as input providers, make their pricing decisions prior to airlines' strategic interactions in the final output market, is highly relevant to several other industries including electricity, telecommunications, and transport terminals (e.g., the vertical chain of ports-sea carriers-shippers). In telecommunications, for example, at the upstream level there are the network owners, while downstream there are carriers who must use the network in order to produce the final good (telephone calls). Like airports, these industries are undergoing privatization in a number of countries. We note that the sequential PLP method used in the present paper may be useful in examining similar issues in those sectors as well.

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APPENDIX

- **Proposition A.1**: Conditions for an interior allocation of consumers

(i) If $(P_p - P_o)/S < \overline{\theta}(B_p - B_o)$, then the peak period is used, that is $\theta^* < \overline{\theta}$.

(ii) If $\underline{\theta}B_o < (c + P_o)/S$, then some consumers will not fly, that is $\theta^f > \underline{\theta}$.

(iii) If $\overline{\theta}$ is large enough, then the off-peak period is used, that is $\theta^* > \theta^f$.

Proof:

First, equivalent conditions for interior allocations, but in terms of Q_p and Q_o are:

The peak is used: $\theta^* < \overline{\theta} \Leftrightarrow (\overline{\theta} - \theta^*)/S > 0 \Leftrightarrow Q_o > 0$

Some consumers do not fly: $\theta^f > \underline{\theta} \Leftrightarrow (\overline{\theta} - \theta^f)/S < (\overline{\theta} - \underline{\theta})/S \Leftrightarrow Q_o + Q_p < (\overline{\theta} - \underline{\theta})/S$

The off-peak is used: $\theta^* > \theta^f \Leftrightarrow (\theta^* - \theta^f)/S > 0 \Leftrightarrow Q_o > 0$

With this, the proofs of each part are:

(i) Note that $(-\Omega^p + \Omega^o)$ in (12) is strictly increasing in Q_p, and $\left(-\Omega^p + \Omega^o\right)_{Q_p \to \infty} > 0$. Also, $\left(-\Omega^p + \Omega^o\right)_{Q_p = o} = (P_p - P_o) - \overline{\theta}S(B_p - B_o)$. Hence, if $P_p - P_o < \overline{\theta}S(B_p - B_o)$, then $\left(-\Omega^p + \Omega^o\right)_{Q_p = o} < 0$ and $Q_p > 0$. ■

(ii) From Ω^o=0 in (10) we get that $(Q_o + Q_p)B_o S^2(N+1)/N = B_o\overline{\theta}S - c - P_o$, This imply that $Q_o + Q_p < (B_o\overline{\theta}S - c - P_o)/(B_o S^2)$. Hence, a sufficient condition for $Q_o + Q_p < \dfrac{\overline{\theta} - \underline{\theta}}{S}$ is: $(B_o\overline{\theta}S - c - P_o)/(B_o S^2) < \overline{\theta} - \underline{\theta}/S$, which leads to $\underline{\theta}B_o < (c + P_o)/S$. ■

(iii) From Ω^o=0 we know that $Q_o + Q_p = \dfrac{(B_o\overline{\theta}S - c - P_o)N}{B_o S^2(N+1)}$. Hence, $Q_o > 0$ is equivalent to $Q_p < \dfrac{(B_o\overline{\theta}S - c - P_o)N}{B_o S^2(N+1)} \equiv \tilde{Q}_p$. In order to ensure $Q_p < \tilde{Q}_p$, we need that $\left(-\Omega^p + \Omega^o\right)_{Q_p=\tilde{Q}_p} > 0$ (see proof of part i). Straightforward algebra gives us $(-\Omega^p + \Omega^o)(\tilde{Q}_p) = (\alpha S + \beta)\left[D(\tilde{Q}_p) + \dfrac{\tilde{Q}_p}{N}D'(\tilde{Q}_p) + \dfrac{(P_p - P_o)}{(\alpha S + \beta)} - \dfrac{(B_p - B_o)(c + P_o)}{B_o(\alpha S + \beta)}\right]$, so that a sufficient condition for $\left(-\Omega^p + \Omega^o\right)_{\tilde{Q}_p} > 0$ is

$D(\tilde{Q}_p) > \dfrac{(B_p - B_o)(c + P_o)}{B_o(\alpha S + \beta)} - \dfrac{(P_p - P_o)}{(\alpha S + \beta)}$. And since $\partial\tilde{Q}_p/\partial\overline{\theta} > 0$, the condition is always fulfilled for $\overline{\theta}$ large enough. ■

Part (i) says that the peak period is used if the airport price differential between peak and off-peak is not too large. Specifically, the per-passenger airport price differential has to be smaller than the incremental benefit, for the highest consumer type, of changing from the off-peak to the peak. Clearly, when the airport does not practice PLP, the peak is always used. Part (ii) says that if $\underline{\theta}$ is low enough, then some consumers will not fly. In particular, the lowest consumer type must have a willingness to pay for off-peak travel that is smaller than the airlines' per-passenger marginal cost for an off-peak flight. Finally, part (iii) implies that Brueckner (2002, 2005)'s *single crossing property*, which imposes that $B_p(\theta) < B_o(\theta)$ for small θ values, may not be needed to have a non-empty off-peak, and that a smaller airport price differential between peak and off-peak increases the likelihood of the off-peak been used. The lower bound for $\overline{\theta}$ cannot be made explicit because of the non-linearity of the delay function. For a linear delay function $D(Q_p, K) = \delta Q_p/K$, the lower bound on $\overline{\theta}$ is given by $\tilde{\theta} = \dfrac{SK(N+1)}{\delta B_o(\alpha S + \beta)N}\left((B_p - B_o)(c + P_o) - B_o(P_p - P_o)\right) + c + P_o$, while a lower bound not depending on *N*, would be $2\tilde{\theta}N/(N+1)$.

- **Proof of Proposition 2**

(i) Differentiating (12) with respect to N we get: $\frac{\partial Q_p}{\partial N} = -\frac{\partial(-\Omega^p + \Omega^o)/\partial N}{\partial(-\Omega^p + \Omega^o)/\partial Q_p}$. This leads to

$$\frac{\partial Q_p}{\partial N} = \frac{\frac{Q_p S^2 (B_p - B_o)}{N^2} + \frac{(\alpha S + \beta) Q_p D_Q(Q_p)}{N^2}}{\frac{S^2 (N+1)(B_p - B_o)}{N} + (\alpha S + \beta)\left(\frac{N+1}{N} D'(Q_p) + \frac{Q_p}{N} D''(Q_p)\right)}$$, which can be written as

$$\frac{\partial Q_p}{\partial N} = \frac{Q_p}{N(N+1)}\left(\frac{S^2 (B_p - B_o) + (\alpha S + \beta) D_Q(Q_p)}{S^2 (B_p - B_o) + (\alpha S + \beta) D'(Q_p) + \frac{(\alpha S + \beta)}{N+1} Q_p D''(Q_p)}\right)$$, from where

$0 < \partial Q_p / \partial N < Q_p / \left(N(N+1)\right)$ follows. ■

(ii) From Ω^o=0 in (10) we know that $Q_o + Q_p = \frac{(B_o \bar{\theta} S - c - P_o) N}{B_o S^2 (N+1)}$, from where it is direct that

$$\frac{\partial (Q_o + Q_p)}{\partial N} = \frac{(B_o \bar{\theta} S - c - P_o)}{B_o S^2 (N+1)^2} = \frac{Q_o + Q_p}{N(N+1)} > 0$$ ■

(iii) From parts (i) and (ii) we know that $\partial Q_o / \partial N > Q_o / \left(N(N+1)\right)$. If the off-peak is used for all N, then $Q_o > 0$ and therefore $\partial Q_o / \partial N > 0$. ■

- **Proof of Proposition 5**

To prove parts (i) and (ii), it is useful to first state the following Lemma:

Lemma A.2: If two prices P_1 and P_2 are given by the fixed points $P_1 = f(Q(P_1))$ and $P_2 = g(Q(P_2))$ respectively, where f is continuously differentiable in $\left(Q(P_1); Q(P_2)\right)$, Q is continuously differentiable in $(P_1; P_2)$, $f(Q(P_2)) > g(Q(P_2))$, and either $Q(\cdot)$ is non-increasing and $f(\cdot)$ is non-decreasing, or $Q(\cdot)$ is non-decreasing and $f(\cdot)$ is non-increasing, then $P_1 > P_2$.

Proof: We prove this by contradiction. Suppose that $P_1 \le P_2$. Denote $\widetilde{P}_2 = f(Q(P_2))$. Applying the mean-value theorem to $P_1 = f(Q(P_1))$ and $\widetilde{P}_2 = f(Q(P_2))$ yields:

$$P_1 - \widetilde{P}_2 = f'(\overline{Q})(Q(P_1) - Q(P_2))$$

where $\overline{Q}$ is some point between $Q(P_1)$ and $Q(P_2)$. Further applying the mean-value theorem to $Q(P_1)$ and $Q(P_2)$, the above equation becomes:

$$P_1 - \widetilde{P}_2 = f'(\overline{Q})(Q(P_1) - Q(P_2)) = f'(\overline{Q})Q'(\overline{P})(P_1 - P_2) \geq 0$$

where the inequality arises because $f'(\overline{Q})Q'(\overline{P}) \leq 0$ and the assumption that $P_1 \leq P_2$. Thus, $P_1 \geq \widetilde{P}_2$. But since by assumption $f(Q(P_2)) > g(Q(P_2))$ or, equivalently, $\widetilde{P}_2 > P_2$, we obtain $P_1 > P_2$, thus resulting in a contradiction. ■

Now, we can prove Proposition 5.

(i) That $P_o^W < P_o^\pi$ follows from writing the pricing rules (30) and (35) as $P_o^\pi = f_o(Q_o + Q_p)$ and $P_o^W = g_o(Q_o + Q_p)$. Since total traffic $Q_0 + Q_p$ is, by (15), downward sloping in P_0, $f_o(\cdot)$ is increasing and $f_o(\cdot) > g_o(\cdot)$, then $P_o^W < P_o^\pi$ by Lemma A.2.

Next, taking derivative of (34) and (29) with respect to N gives us:

$$\frac{dP_o^W}{dN} = -S^2 B_o \left[-\frac{Q_o + Q_p}{N^2} + \frac{\partial(Q_o + Q_p)}{\partial N} \frac{1}{N} \right] \text{ and}$$

$$\frac{dP_o^\pi}{dN} = S^2 B_o \left[-\frac{Q_o + Q_p}{N^2} + \frac{\partial(Q_o + Q_p)}{\partial N} \left(\frac{N+1}{N} \right) \right].$$

But, from Proposition 2.2 we know that $\partial(Q_o + Q_p)/\partial N = (Q_o + Q_p)/(N(N+1))$. Therefore

$$\frac{dP_o^W}{dN} = \frac{S^2 B_o (Q_o + Q_p)}{N(N+1)} > 0 \text{ and } dP_o^\pi / dN = 0. \quad ■$$

(ii) That $\Delta P_{p-o}^W < \Delta P_{p-o}^\pi$ follows from writing the pricing rules (31) and (36) as $\Delta P_{p-o}^\pi = f_{p-o}(Q_p)$ and $\Delta P_{p-o}^W = g_{p-o}(Q_p)$. Since peak traffic Q_p is, by (16), downward sloping in ΔP_{p-o}, $f_{p-o}(\cdot)$ is non-decreasing as long as the (unsigned) term $D'''(\cdot)$ is non-negative (or if is negative, its magnitude is not too large) and $f_{p-o}(\cdot) > g_{g-o}(\cdot)$, then $\Delta P_{p-o}^W < \Delta P_{p-o}^\pi$ by Lemma A.2.

$\dfrac{d\Delta P_{p-o}^W}{dN} > 0$ follows from differentiation of (31) with respect to N. We get:

$$\frac{d\Delta P^W_{p-o}}{dN} = (\alpha S + \beta)\left[\frac{Q_p D'(Q_p)}{N^2} + \frac{\partial Q_p}{\partial N}\frac{(N-1)}{N}D'(Q_p) + Q_p D''(Q_p)\frac{(N-1)}{N}\right]$$
$$+ S^2(B_p - B_o)\left[\frac{Q_p}{N^2} - \frac{\partial Q_p}{\partial N}\frac{1}{N}\right]$$

Since $\partial Q_p / \partial N > 0$ by Proposition 2.1, the first term in the RHS is positive. Since $\partial Q_p / \partial N < Q_p / \left(N(N+1)\right)$ by Proposition 2.1, $\frac{Q_p}{N^2} > \frac{\partial Q_p}{\partial N}\frac{1}{N}$ making the second term in the RHS positive as well. Therefore, $d\Delta P^W_{p-o} / dN > 0$.

$\frac{d\Delta P^\pi_{p-o}}{dN} = 0$ if the delay function is linear, follows from differentiating (36) and then imposing $D''(Q_p) = 0$, $D'''(Q_p) = 0$. We get:

$\frac{d\Delta P^\pi_{p-o}}{dN} = \left((\alpha S + \beta)D'(Q_p) + (B_p - B_o)S^2\right)\left[-\frac{Q_p}{N^2} + \frac{\partial Q_p}{\partial N}\frac{(N+1)}{N}\right]$, where $D'(Q_p)$ is a constant.

With a linear delay function Proposition (2.1) changes to $\partial Q_p / \partial N = Q_p / \left(N(N+1)\right)$, making $d\Delta P^\pi_{p-o} / dN = 0$. If we consider $D''(Q_p) > 0$, the sign is then undetermined and depends on the values of, for example, B_p, B_o and $D'''(Q_p)$. ■

(iii) $Q^W_p > Q^\pi_p$ flows from part (ii), and the comparative statics in (16) or Proposition (1.3). $\frac{dQ^\pi_p}{dN} > \frac{dQ^W_p}{dN} = 0$ follows from replacing ΔP^π_{p-o} in the sub-game equilibrium equation (12); we get:

$$\left(-\Omega^p + \Omega^o\right)\left(\Delta P^\pi_{p-o}\right) = \frac{2Q^\pi_p (B_p - B_o)S^2(N+1)}{N} - \bar{\theta}S(B_p - B_o)$$
$$+ (\alpha S + \beta)\left(D(Q^\pi_p) + \frac{N+2}{N}Q^\pi_p D'(Q^\pi_p) + \left(Q^\pi_p\right)^2 D''(Q^\pi_p)\right) = 0$$

As in the proof of Proposition 2.1, use this to calculate

$\frac{dQ^\pi_p}{dN} = -\frac{\partial(-\Omega^p + \Omega^o)(\Delta P^\pi_{p-o}) / \partial N}{\partial(-\Omega^p + \Omega^o)(\Delta P^\pi_{p-o}) / \partial Q_p}$ and to prove that $0 < dQ^\pi_p / dN < Q^\pi_p / \left(N(N+1)\right)$, which shows that peak traffic increases with N.

Similarly, replacing ΔP^W_{p-o} in the sub-game equilibrium equation (12), we get:

$$\left(-\Omega^p + \Omega^o\right)\left(\Delta P^W_{p-o}\right) = Q^W_p (B_p - B_o)S^2 - \overline{\theta}S(B_p - B_o) + (\alpha S + \beta)\left(D(Q^W_p) - Q^W_p D'(Q^W_p)\right) = 0$$

Which does not depend on *N*, hence $dQ^W_p / dN = 0$. ■

(iv) $Q^W_t > Q^\pi_t$ flows from part (i) and the comparative statics in (15) or Proposition (1.1). To prove $\frac{dQ^\pi_t}{dN} > \frac{dQ^W_t}{dN} = 0$, use the sub-game equilibrium equation (10) to prove $\frac{dQ^\pi_o}{dN} > \frac{dQ^W_o}{dN} = 0$ in an analogous way as in part (iii). The result then follows directly from this and part (iii).

(v) Direct from part (iii) and $D'(Q_p) > 0$ (equation 4). ■

(vi) Consider the SW function in (29). We can rewrite it in terms of total traffic, Q_t, and peak-traffic, by replacing $Q_o = Q_t - Q_p$. This gives us:

$$SW(Q_t, Q_p) = \overline{\theta}S(B_p Q_p + B_o(Q_t - Q_p)) - c(Q_t) - C(Q_t) - Kr - \frac{S^2}{2}(B_o(Q_t - Q_p)^2 + 2B_o(Q_t - Q_p)Q_p + B_p Q_p^2) - (\alpha S + \beta)Q_p D(Q_p)$$

Now, *SW* is globally concave in (Q_t, Q_p) because

$\frac{\partial^2 SW}{\partial Q_t^{\,2}} = -B_o S^2 < 0$, $\frac{\partial^2 SW}{\partial Q_p^{\,2}} = -(B_p - B_o)S^2 - (\alpha S + \beta)\left(2D'(Q_p) + Q_p D''(Q_p)\right) < 0$ and

$$\frac{\partial^2 SW}{\partial Q_t^{\,2}} \cdot \frac{\partial^2 SW}{\partial Q_p^{\,2}} - \left(\frac{\partial^2 SW}{\partial Q_t \partial Q_p}\right)^2 = B_o S^2\left[(B_p - B_o)S^2 + (\alpha S + \beta)\left(2D'(Q_p) + Q_p D''(Q_p)\right)\right] > 0.$$

Since (Q^W_t, Q^W_p) maximizes *SW*, and from parts (iii) and (iv), $(Q^W_t, Q^W_p) > (Q^\pi_t, Q^\pi_p)$, then $SW^W > SW^\pi$.

Finally, since (Q^π_t, Q^π_p) increases with *N*, SW^π increases with *N*, while SW^W does not change with *N* because (Q^W_t, Q^W_p) does not. ■