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Analysis on Bargaining about Global Climate Change Mitigation in Aviation Sector

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Abstract

Understanding the basic interaction mechanism among nations surrounding the CO_2 emissions is critically important for the policy formulation analysis in aviation sector at present. We performed simulation analysis on the international climate change policy, especially on the effects on pricing of emission allowances by including major players such as China and India into the hypothetical global CO_2 emission trading scheme according to non-cooperative game framework. In the presence of negative public goods, i.e., CO_2, we extended the Lindahl-Bowen-Samuelson condition to include a class of uncertainty into utility. By using the result, we explained, with some numerical examples, the welfare effects caused by the changes of factors, such as level of uncertainty, degree of risk averse, asymmetric utility structure, initial allocation among players, based on our model surrounding the bargaining of CO_2 emissions allocation games.

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CO_2 emission allowance, Lindahl-Bowen-Samuelson Condition, Uncertainty, Air Transport

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1 Introduction

The policy formulation needs for Climate Change mitigation in the aviation sector is very much urgent, since Kyoto Protocol's period ends 2012 and we have to agree about the Post-Kyoto Protocol structure by the end of 2009. Kyoto protocol designated to ICAO the task of mitigating the growth of international aviation to meet the challenge of sustainability under the threat of global climate change[1]. The ICAO is the international organization and their major decisions always entail the negotiation and bargaining among the contracting nations.[2]

Therefore, in order to formulate the effective mitigation policy for international aviation and climate change, we need to know the basic mechanism about the negotiation and bargaining and also need to understand the social welfare effect of such bargainings.

The negotiation is the sequence of the following phases.

i)The negotiation/bargaining on global emission level[3]

ii)The negotiation/bargaining on the rule of distribution of initial emission allowance among countries[4].

iii)The negotiation/bargaining on the allowances distribution among countries[5].

[1]Kyoto Protocol article2 2. "The parties shall pursue limitation or reduction of emissions of greenhouse gases... from aviation ..., working through the International Civil Aviation Organization...."

[2]ICAO, in order to perform the task of mitigation, set up a special high level group (GIACC) among selected counties to discuss globally aspirational goals and other measures. GIACC adopted GIACC Report 2009 [3] in May session. According to the report, the group agreed the goal of 2 % annual fuel consumption efficiency (liter per revenue ton-kilometer) reduction from the 2005 level until 2050.

[3]Global emission level in flow base is , for example, 10 % reduction of the annual global emission. Global emission level in stock base is , for example, 10 % reduction of the global GHG concentration (CO2concentration) in the earth atmosphere.

[4]The negotiation of total amount and that of rule of distribution could be interchangeable, i.e., first negotiate on the rule of distribution and then negotiate on the total amount.

[5]In the same sense, the negotiation/bargaining of rule of distribution and that of the allowance distribution among countries could be interchangeable, i.e., first negotiate on the rule of distribution and then negotiate on the allowance distribution among countries.

iv)The negotiation on the emission allowance among individual and cooperation within each country.

Here we use the recent method of game theory and bargaining theory to simulate the negotiation process and extend the pareto optimal condition with the presence of public goods, which is in this case, CO_2 emission into the earth atmosphere. With the new optimal condition with the uncertainty, we did some welfare effect analysis by comparative statics using numerical examples.

2 Emission Allowance Allocation Process Simulation

2.1 Past Literature

A lot of analyses were performed about the emission allowance allocation process, like that of the Kyoto Protocol Negotiation process. Okada (2004) [10], for example, analyzed the bargaining mechanism for the initial allowance and reduction costs by the method of non-cooperative game theory based on the empirical work done by the Nordhaus(1991) [14] and Bohm=Larsen(1994) [1].

Also a lot of cooperative game theoretic analyses were done about the allowance distribution bargaining. The work done by Tadenuma in Imai and Okada (2005) [4] shows there is a stable coalition set (von Neumann-Morgenstern solution) even though, according to Okada [9], the core of the voting game on distributions of a fixed total amount of emission allowances is empty.

Here we focus on non-cooperative game framework and make some simulation of including new members into the bargaining.

2.2 Base Model

According to the work by Nordhaus(1991) [14]: Bohm=Larsen(1994) [1]: Okada(2004) [10], we have the formula for the competitive price of carbon emission allowances under the framework of non-cooperative game approach, indicated below.

Let $N = 1, ..., n$ be the set of counties. For every $i \in N$, we denote by E_i country i's current level of carbon emission. The total level of carbon emitted by n countries is given by $E = \sum_{i \in N} E_i$. x_i denotes the country i's reduction of carbon emission. ω_i is the emission allowance allocation for country i. $\bar{\omega} = \sum_{i \in N} \omega_i$

$$p^* = -185.2 \ln(1 - \frac{E - \bar{\omega}}{\sum_{i \in n} E_i(1 - r_i)}) \tag{1}$$

$$x_i^* = \frac{E(1 - r_i)}{\sum_{i \in N} E_i(1 - r_i)}(E - \bar{\omega}) \tag{2}$$

$$c_i^e = 185.2 x_i^* + p^*(E_i r_i - \omega_i) \tag{3}$$

$$r_l = \begin{cases} 1 - \frac{e_i}{e_{USA}} \ (e_i \leq e_{USA}) \\ \frac{e_{USA}}{e_i} - 1 \ (e_{USA} \leq e_i) \end{cases} \tag{4}$$

$$e_i = \frac{E_i}{GDP_i} \tag{5}$$

e_i is the carbon intensity of country i, which is emission lebel E_i over GDP. p^* is the competitive equilibrium price of this negotiation. x_i^* is the equilibrium reduction of CO_2 emissions for country i. c_i^e is the cost of country i with the initial allocation ω_i, competitive equilibrium price p^* and reduction amount x_i^*.

This model is about the mechanism of negotiation phase iii) in the Introduction based on the relative reduction cost of each nation with that of U.S. By the equations (1) through (3), we can simulate any number of countries with any type of initial allowances, or any type of emission intensity.

However, since these numbers are available, based on empirical works, only for 1990, we can only perform the simulation about 1990, and cannot do about other years like 2008, for example.

2.3 Simulation

What would have happened if major players, like China and India were included in the Kyoto Protocol and the hypothetical global emission trading in 1990 ?

Table 1 below is the basic data for the carbon emission level(million ton carbon, GDP in US dollars, carbon intensity for EU 15 nations, Former Soviet Union nations, Japan, U.S., China (including Hong Kong and Macau), Korea (not including DPRK) and India.

(Table 1 is about here.)

If the hypothetical emission trade happens in 1990 among the original contracting countries of EU 15 nations, Former Soviet Union nations, Japan, U.S., what would happen? The simulation result is indicated in Table 2. The equilibrium price is about 9.65 US dollars per ton Carbon[6].

(Table 2 is about here.)

If the major emission countries, like China, Korea and India also participated in the hypothetical emission trading with the obligation of zero reduction like Former Soviet Union nations, the results are in Table 3.

(Table 3 is about here.)

The equilibrium price is about 6.65 US dollars, since the new entrants are not obliged to reduce the emission level and they can sell emission allowances to U.S., EU15 and Japan.

What happens if the reduction obligation increases among the three countries from zero percent to higher percentages is in Figure 1.

(Figure 1 is about here.)

According to our basic model, 1 percent increase of reduction obligation among these three nations lead to about 33 cents increase of the emission

[6]Notice that the price here is US$ per ton Carbon, not US$ per ton CO_2. External cost from car gasoline consumption is about 5,000 yen to 50,000 yen per ton Carbon, and 30,000 yen per ton Carbon is the medium estimate according to Kanemoto et.al.(2006) [7]. According to IPCC Assessment Report 4 (Synthesis Report p69) [6] , the average social cost of CO_2 based on 100 estimates is about 12 US$ per ton of CO_2 for 2005, although the estimation range is from -3 to 95 US$.

allowance price, which is the slope of our estimated line in Figure 1.

What happens if carbon intensity of these three countries improves by 1 percent, the emission allowance price would go up by about 2 cent. This mechanism is depicted in Figure 2. The slope of the estimated line is about 0.02.

(Figure 2 is about here.)

With these simulations, more players' participation can reduce the price of emission allowance but the situation depends on the factors like the reduction obligation of the new players and carbon intensity variations.

3 Welfare Analysis with a Negative Public Goods

3.1 Past Literature

Emission charge on airlines is studied by several authors on the effect on airfares, service quality, and aircraft design, like Brueckner and Zhang(2009) [2].

In more general context, including other sectors than transportation, a lot of literature on the Climate Change Policy is published. For example, Guesnerie and Tulkens(2008) [11] and Yang(2008) [15] are among the most recent publications.

But a limited number of literatures are on the welfare impact analysis using the characteristics of CO_2 as negative public goods and further limited number of literatures are based on the game theory or bargaining theory analysis in addition to the characteristic as negative public goods. This study explores the welfare implication analysis, using the characteristic of negative public goods, by game theory and bargaining theory.

3.2 Lindahl-Bowen-Samuelson condition

【Lindahl-Bowen-Samuelson condition】
The condition for the allocation in the economy with a public good to be

pareto efficient is that the sum of all members' marginal rate of substitution of private goods for the public good is equal to the marginal rate of transformation of private goods for the public good.

$$\sum_{i=1}^{n} \frac{u_{s+j}^{i}}{u_{r}^{i}} = \frac{F_{s+j}}{F_{r}} \; (i = 1, 2, ..., n; \; r = 1, 2, ..., s; \; j = 1, 2, ...m) \tag{6}$$

n is the number of the players. s is the number of private goods and r is their index. m is the number of public goods and j is their index. u is the utility function. F is the production function.

This is the condition of Lindahl-Bowen-Samuelson (=LBS condition) based on Samuelson [12].The CO_2 emission quantity is the negative public good. So in the case of the CO_2 emission, we could directly use the LBS condition for the pareto efficient allocation of the CO_2 emission.

3.3 Extension of LBS condition to consumption externalities

In the presence of external effect from the consumption as well as that from production, we can use the extension of the Lindahl-Bowen-Samuelson according to Tadenuma [13]. This is also the negotiation/bargaining phase of iii) depicted in the Introduction.

There are n countries, $N = 1, ..., n$. Let $y_i \in R_+$ denote the gross domestic product(GDP) of country $i \in N$, $c_i \in R_+$ the consumption of country i. Both production and consumption are accompanied by emissions of greenhouse gases. Let $x_i^p \in R_+$ denote the emission of greenhouse gases from production.
The relation of x_i^p and y_i is represented by the function $x_i^p = f_i(y_i)$, where $f_i' > 0$, $f_i'' > 0$.

Let $x_i^c \in R_+$ denote the emission of greenhouse gases from consumption. The relation of x_i^c and c_i is represented by the function $x_i^c = g_i(c_i)$, where $g_i' > 0$, $g_i'' \geq 0$.

Let $x_i \equiv x_i^p + x_i^c$ be the total emission of greenhouse gases of country i, and let $X \equiv \sum_{i \in N} x_i$ be the global emission of greenhouse gases.

We assume that there is an amount $\hat{X}$ of global emission of greenhouse gases such that the human beings cannot survive if the emission exceeds

$\hat{X}$. Each country i has the preferences over pairs $(c_i, X) \in R_+ \times [0, \hat{X}]$ of its own consumption and a global amount of emissions of greenhouse gases. The preferences are represented by a continuously differentiable and strictly quasi-concave function $V_i : R_+ \times [0, \hat{X}] \to R$. We call the function V_i **t**he welfare function of country i.

Problem A

$$\max_{(y,c,x)\in R_+^{3n}} V_i(c_i, \sum_{h\in N} x_h) \tag{7}$$

subject to

$$x_j = f(y_j) + g(c_j) \ (\forall j \in N) \tag{8}$$

$$\sum_{h\in N} y_h = \sum_{h\in N} c_h \tag{9}$$

$$V_i(c_i, \sum_{h\in N} x_h) = \check{V}_j \ (\forall j \in N, j \neq i) \tag{10}$$

The condition (10) could be the Individual Rationality condition that guarantees participation is rational for the players, since $\check{V}_j$ is the minimum utility the player can get elsewhere.

Solving this problem, we can get the extended condition of LBS to include the external effect from consumption. The Lagrangean for the problem A is as follows;

$$\begin{aligned} L((c_h)_{h\in N}, (x_h)_{h\in N}, (y_h)_{h\in N}, (\lambda_h)_{h\in N}, (\gamma_h)_{h\in N}, \delta) \\ \equiv V_i(c_i, \sum_{i\in N} x_h) - \sum_{j\neq i} \lambda_j (V_j(c_j, \sum_{h\in N} x_h) - \check{V}_j) \\ - \sum_{j\in N} \gamma_j (x_j - f_j(y_j) - g_j(c_j)) - \delta(\sum_{h\in N} y_h - \sum_{h\in N} c_h) \end{aligned} \tag{11}$$

From the first order condition,

$$\frac{\partial V_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial c_i} + \gamma_i g_i'(c_i^*) + \delta = 0 \quad (12)$$

$$\frac{\partial V_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{j\neq i} \lambda_j \frac{\partial V_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_i = 0 \quad (13)$$

$$\gamma_i f_i'(y_i^*) - \delta = 0 \quad (14)$$

and for each $j \neq i$,

$$-\lambda_j \frac{\partial V_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial c_j} + \gamma_j g_j'(c_j^*) + \delta = 0 \quad (15)$$

$$\frac{\partial V_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{j\neq i} \lambda_j \frac{\partial V_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_j = 0 \quad (16)$$

$$\gamma_j f_j'(y_j^*) - \delta = 0 \quad (17)$$

Solving these equations, we get the following Extended condition of LBS including the consumption externality, shown in Tadenuma [13].

[Extended Condition of LBS for Consumption Externalities]
The allocation (y^*, c^*, x^*) is pareto efficient, if the following is satisfied.

$$\sum_{i\in N} \nu_i\left(c_i^*, \sum_{h\in N} x_h^*\right)(f_i'(y_i^*) + g_i'(c_i^*)) = 1 \quad (18)$$

$$\nu_i(c_i, X) \equiv \left| \frac{\frac{\partial V_i(c_i,X)}{\partial X}}{\frac{\partial V_i(c_i,X)}{\partial c_i}} \right|$$

This is the extension of Lindahl-Bowen-Samuelson condition in a sense that the impact of CO_2 emission from consumption as well as that of production are considered. Specifically, at Pareto optimal allocation, the weighted sum of the marginal rate of substitution of consumption for global emission of GHGs over all the countries is equal to one, where the each weight is the sum of the marginal emission from production and the marginal emission from consumption in each country.

3.4 Extension of LBS condition to Utility with Uncertainty

So far we deal with the pareto optimal condition with a negative public goods. This setting has no uncertainty. In other words, the setting is based on perfect information.

Now we try to extend the LBS condition further to uncertain world. If we add the uncertainty to utility function, then the **Problem A** becomes the new maximization problem with the uncertainty. But in order to track the utility function and constraints, we need to set the structure of the relationship between the uncertainty and the other factors in utility function, namely consumption c_i,production y_i, and their emission levels, $f_i(c_i)$, $g_i(c_i)$.

In order to set the structure, we introduce the following assumption.

【Assumption 1】
The uncertainty is linearly separable from the economic activities. In other words, the uncertainty does not exist in y_i, consumption c_i, their emission function, $f(y_i)$, or $g(y_i)$, or their relation ship in the utility function.

This means that the uncertainty is not about observation accuracy or accounting consistency, but the uncertainty is purely the remaining category other than human economic activities, i.e., consumption expenditures, or the emission amount of CO_2 from each country's human activities.

So this uncertainty could be the remaining uncertainty after we know the exact amount of CO_2 emission. This could be, for example, such uncertainty about the net ultimate effect of CO_2 emission through the earth ecology system on our utility level even if we know the exact level of consumption expenditures and the entailing exact emission level of CO_2 in each country.[7]

[7] According to "6 Potential Climate Change from Aviation" in IPCC Aviation Report(1999) [5], CO_2 is, unlike ozone and water vapor perturbations, one of well-mixed gases, and there is small uncertainty in calculating radiative forcing (RF, a single measure of climate change defined by IPCC, which calculates the global annual average of radiative imbalance (W/m^2) to the atmosphere-land-ocean system caused by anthropogenic perturbations and sets the RF of pre-industrial atmosphere to be zero). Still the RF for aviation CO_2 in 1992 based on NASA-1992 aviation scenario, for example, is estimated to be $+0.018(W/m^2)$ with a likely range of $\pm30\%$ that includes uncertainties in the carbon cycle and in radiative calculations for a fixed amount of fuel burn ($160.3(million\ tons/year)$) and a fixed CO_2 concentration level (1.0 ppmv).

By **Assumption 1**, our uncertainty is linearly separated from production y_i, consumption c_i or their emission function, $f(y_i)$, and $g(y_i)$.

With this assumption of the linear separability of uncertainty, **Problem A** now becomes **Problem B** below. By solving this problem B, Lindahl-Bowen-Samuelson condition can be extended further to the situation where uncertainty exists.

Problem B

$$\max_{(y,c,x)\in R_+^{3n}} E[V_i(c_i, \sum_{h\in N} x_h, \varepsilon)] \tag{19}$$

subject to

$$x_j = f(y_j) + g(c_j) \ (\forall j \in N) \tag{20}$$

$$\sum_{h\in N} y_h = \sum_{h\in N} c_h \tag{21}$$

$$E[V_i(c_i, \sum_{h\in N} x_h, \varepsilon)] = \check{V}_j \ (\forall j \in N, j \neq i) \tag{22}$$

ε could be a random variable according to any probability distribution. In this sense, since we do not know the probability distribution of ε yet, what ε represents is not "risk" but "uncertainty" in the meaning of Frank Knight's terminology.

Because of **Assumption 1**, we can describe the equality constrains without quoting ε [8].

But solving **Problem B** is not simple. Here we introduce additional assumption.

First, we define $\bar{V}_i$ as the expected value of $V_i(c_i, \sum_{h\in N} x_h, \varepsilon)$;

$$\bar{V}_i(c_i, \sum_{h\in N} x_h) \equiv E[V_i(c_i, \sum_{h\in N} x_h, \varepsilon)] \tag{23}$$

[8] The uncertainty here is only one dimension. The structure, however, can be extended to multi-dimension uncertainties without loss of generality.

【Assumption 2】
ε is from normal distribution, $N(\mu, \sigma^2)$ and utility function is CARA (Constant Absolute Risk Aversion) utility function.

Under **Assumption 1** and **Assumption 2**, the equation (23) becomes

$$\begin{aligned}\bar{V}_i(c_i, \sum_{h\in N} x_h) &= E[V_i(c_i, \sum_{h\in N} x_h, \varepsilon)] \\ &= E[-\exp\{-\eta_i(H_i(c_i, \sum_{h\in N} x_h) + \varepsilon)\}] \qquad (24)\end{aligned}$$

where $H_i(c_i, \sum_{h\in N} x_h)$ is the relationship function between c_i and $\sum_{h\in N} x_h$ without any uncertainty. This can be possible because of the linear separability of uncertainty from other economic activities under **Assumption 1** and the CARA utility function's characteristics form **Assumption 2**.

So we get,

$$\begin{aligned}\bar{V}_i(c_i, \sum_{h\in N} x_h) &= -\exp\{-\eta_i(H_i(c_i, \sum_{h\in N} x_h))\}E[-\exp\{-\eta_i(\varepsilon)\}] \\ &= -\exp\{-\eta_i(H_i(c_i, \sum_{h\in N} x_h) - \mu\eta_i + \frac{\eta_i^2\sigma^2}{2}\} \qquad (25)\end{aligned}$$

Notice that under **Assumption 2**, $\varepsilon \sim N(\mu, \sigma^2)$;

$$\begin{aligned}&E[-\exp\{-\eta(\varepsilon)\}] \\ &= \int_{-\infty}^{\infty} e^{-\eta\varepsilon}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(\varepsilon-\mu)^2}{2\sigma^2}}d\varepsilon \\ &= e^{-\eta\mu+\frac{\eta^2\sigma^2}{2}}\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{\frac{-(\varepsilon-(\mu-\sigma^2\eta))^2}{2\sigma^2}}d\varepsilon \\ &= e^{-\eta\mu+\frac{\eta^2\sigma^2}{2}}\cdot 1 \;.\end{aligned}$$

Now we can derive the extension of LBS condition to include the uncertainty described above. The equations (19) through (22) become as follows;

Problem B$\star$

$$\max_{(y,c,x)\in R_+^{3n}} \bar{V}_i(c_i, \sum_{h\in N} x_h) \tag{26}$$

subject to

$$x_j = f(y_j) + g(c_j) \ (\forall j \in N) \tag{27}$$

$$\sum_{h\in N} y_h = \sum_{h\in N} c_h \tag{28}$$

$$\bar{V}_i(c_i, \sum_{h\in N} x_h^*) = \check{V}_j \ (\forall j \in N, j \neq i) \tag{29}$$

By the equation(25), we can treat the equations (26) through (29) without any random variables. These equations in **Problem B$\star$** can be solved by the usual maximization problem just as in Problem A.

Define the Lagrangean as follows;

$$\begin{aligned} L((c_h)_{h\in N}, (x_h)_{h\in N}, (y_h)_{h\in N}, (\lambda_h)_{h\in N}, (\gamma_h)_{h\in N}, \delta) \\ \equiv \bar{V}_i(c_i, \sum_{i\in N} x_h) - \sum_{j\neq i} \lambda_j (\bar{V}_j(c_j, \sum_{h\in N} x_h) - \check{V}_j) \\ - \sum_{j\in N} \gamma_j (x_j - f_j(y_j) - g_j(c_j)) - \delta(\sum_{h\in N} y_h - \sum_{h\in N} c_h) \end{aligned} \tag{30}$$

Notice that the term $V_i(c_i, \sum_{i\in N} x_h)$ in the Lagrangean (11) in **Problem A** now bocomes $\bar{V}_i(c_i, \sum_{i\in N} x_h)$ the Lagrangean (30) in **Problem B$\star$** .

Other than this, the Lagrangean is the same as in solving the Problem A. Thus we can get the following familiar condition.

As the first order condition,

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial c_i} + \gamma_i g_i'(c_i^*) + \delta = 0 \tag{31}$$

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{j\neq i} \lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_i = 0 \tag{32}$$

$$\gamma_i f_i'(y_i^*) - \delta = 0 \tag{33}$$

and for each $j \neq i$,

$$-\lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial c_j} + \gamma_j g_j'(c_j^*) + \delta = 0 \tag{34}$$

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{j\neq i} \lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_j = 0 \tag{35}$$

$$\gamma_j f_j'(y_j^*) - \delta = 0 \tag{36}$$

Notice also that;

$$\tfrac{\partial \bar{V}_i}{\partial X} = \bar{V}_i(-\eta_i)\tfrac{\partial H_i}{\partial X}, \tfrac{\partial \bar{V}_i}{\partial c_i} = \bar{V}_i(-\eta_i)\tfrac{\partial H_i}{\partial c_i}$$

We get the following result;

$$\begin{aligned} \nu_i(c_i, X) \equiv \left| \frac{\frac{\partial \bar{V}_i(c_i,X)}{\partial X}}{\frac{\partial \bar{V}_i(c_i,X)}{\partial c_i}} \right| &= \left| \frac{\bar{V}_i(\eta_i)\frac{\partial H_i}{\partial X}}{\bar{V}_i(\eta_i)\frac{\partial H_i}{\partial c_i}} \right| \\ &= \left| \frac{\frac{\partial H_i(c_i,X)}{\partial X}}{\frac{\partial H_i(c_i,X)}{\partial c_i}} \right| \end{aligned} \tag{37}$$

By solving these equations (31) $\sim$ (36) and the equation (37), we can describe the extended condition of LBS for uncertainty.

[Proposition 1: Extended Condition of LBS for Uncertainty]
Under **Assumption 1** and **Assumption2**, at Pareto optimal allocation, the weighted sum, over all the countries, of the marginal rate of substitution of consumption for global emission of GHGs, which is composed only of the

certain part of the utility function with uncertainty, is equal to one, where the each weight is the sum of the marginal emission from production and the marginal emission from consumption in each country.

$$\sum_{i \in N} \nu_i \left(c_i^*, \sum_{h \in N} x_h^* \right) (f_i'(y_i^*) + g_i'(c_i^*)) = 1 \tag{38}$$

$$\nu_i(c_i^*, X) = \left| \frac{\frac{\partial H_i(c_i^*, X)}{\partial X}}{\frac{\partial H_i(c_i^*, X)}{\partial c_i}} \right|$$

This is the extended condition of LBS to include the uncertainty under **Assumption 1** and **Assumption 2**. Notice that in $\nu_i(c_i^*, X)$, we have $H_i(c_i^*, X)$, instead of $V_i(c_i^*, X)$. $H_i(c_i, X)$ is the certain (= without any uncertainty) part of the utility function $V_i(c_i, \sum_{h \in N} x_h, \varepsilon)$ with uncertainty ε under **Assumption 1** and **Assumption 2**. The derivation of Proposition 1 is in the appendix.

4 Welfare implication with numerical examples

4.1 Basic Model

Let N=1,2. The emission functions are defined as follows;
for every $i \in N$,

$$f_i(y_i) = y_i^2 \tag{39}$$

$$g_i(c_i) = c_i \tag{40}$$

For each $i \in N$, the welfare function $V_i :\in R_+^3 \rightarrow R$ is defined as follows;

$$E[V_i(c_i, X, \varepsilon)] = E[-\exp\{-\eta_i(H_i + \varepsilon)\}] \tag{41}$$

$$H_i = c_i^{a_i}(10 - X)^{1-a_i} \ (0 \leq a_i \leq 1) \tag{42}$$

Under the assumption of CARA utility function and the normal distribution for ε, we can get the follows;

$$\bar{V_i}(c_i, X) = -\exp\{-\eta_i(c_i^{a_i}(10 - X)^{1-a_i}) - \mu\eta_i + \frac{\eta_i^2\sigma^2}{2}\} \quad (43)$$

If we pick the specific numbers for parameters $a_1 = 0.8, a_2 = 0.2$, then we have the following equations;

$$\bar{V_1}(c_1, X) = -\exp\{-\eta_1(c_1^{0.8}(10 - X)^{0.2}) - \mu\eta_1 + \frac{\eta_1^2\sigma^2}{2}\} \quad (44)$$

$$\bar{V_2}(c_2, X) = -\exp\{-\eta_2(c_2^{0.2}(10 - X)^{0.8}) - \mu\eta_2 + \frac{\eta_2^2\sigma^2}{2}\} \quad (45)$$

With these equations (44) and (45) as well as Extended Condition of LBS for uncertainty, we can derive the pareto frontier, bargaining frontier, disagreement point, Nash product[9] based on Tadenuma(2003) [13], which is the model for no uncertainty.

Here we do not go into the details for these frontiers and points, which are explained in Tadenuma(2003) [13]. But the gist of them is as follows;

Pareto frontier is the locus of the welfare vectors of the two countries under our study that can be attained by being satisfying the extended condition of LBS depicted in the equation (18) or (38). Namely, the vectors (V_1, V_2) must satisfy the following relationships in our numerical example;

$$\begin{aligned}
V_1 &= \bar{V_1}(c_1(c_2), X(c_2)) \\
V_2 &= \bar{V_2}(c_2, X(c_2)) \\
(c_1 + c_2 + 1)(\frac{c_1}{4} + 4c_2) - 10 + \frac{(c_1 + c_2)^2}{2} + c_1 + c_2 &= 0 \\
X(c_2) &= \frac{(c_1 + c_2)^2}{2} + c_1(c_2) + c_2
\end{aligned}$$

The last two equations can be derived from the extended condition of LBS

[9] The bargaining model is theoretically founded in Nash(1950) [8]

in (18) or (38).The locus welfare vectors (V_1, V_2) are drawn for the relevant value for c_2.

Bargaining frontier is the locus of the welfare vectors that are attained at various levels of the total emission X with a given proportional rule, i.e., the share of initial emission allowances, (θ_1, θ_2) in our case, and emission allowance trading, entailing the trade price $q(X)$. Namely, the vectors (V_1, V_2) in the locus are those that satisfy the following relationships.

$$V_i = \bar{V_i}(c_i(X), X))$$
$$c_i = \frac{1}{4q(X)(1+q(X))} + \frac{q(X)\theta_i X}{1+q(X)}$$
$$X = \frac{1}{4q^2} + \frac{1}{4q^2} + \frac{1}{q}$$

In this settings, the $q(X)$ is the inverse function of the last equation. Over the relevant range of X, we have the locus of (V_1, V_2), which is the bargaining frontier. Notice that the bargaining frontier depends on the initial allocation,(θ_1, θ_2) The bargaining frontier changes its shape according to the value of the initial allocation.

Disagreement point of the Nash bargaining theory is the Nash equilibrium welfare levels of the two countries in the emission game without any regulation. Specifically, we have the two players who do not bargain with each other and take the other's emission level as given. Their utility function is as follows;

$$V_1 = \bar{V_1} = -\exp\{-\eta_1(c_1^{0.8}(10 - x_2 - c_1^2 - c_1)^{0.2}) - \eta_1\mu + \frac{\eta_1^2\sigma^2}{2}\}$$
$$V_2 = \bar{V_2} = -\exp\{-\eta_2(c_2^{0.2}(10 - x_1 - c_2^2 - c_2)^{0.8}) - \eta_2\mu + \frac{\eta_2^2\sigma^2}{2}\}$$

The best response functions are obtained by differentiating the functions with respect to c_i and setting the value to zero. Then, by solving these response function equations, the Nash equilibrium consumption, emissions, and welfare levels are derived. This vector of the derived welfare, which could be thought as what you get if you fail to bargain, is the disagreement point (d_1, d_2).

Nash product is defined by;

$$(V_1 - d_1)(V_2 - d_2)$$

Before we go further, we first set the parameters as follows;

$\mu = 0$
$\sigma = 1$
$\eta_1 = \eta_2 = 0.2$

Also the initial allocation for the emission allowances for player 1 and player2 are θ_1 and θ_2 respectively. We set these as follows ;

$\theta_1 = 0.925$
$\theta_2 = 0.075$

Figure 3 is the result loci for these frontiers and points.

(Figure 3 is about here.)

As in Tadenuma(2003) [13], pareto frontier and bargaining frontier touches at one point, where each player's emission levels are equal, i.e., $x_1^* = x_2^*$. In this settings, disagreement point is within the bargaining frontier. So it could be possible that starting from disagreement point both player can reach higher utility level by bargaining. But as Figure 3 shows, limit of bargaining is not as high as the pareto frontier, of which is the point where each player's emission is the same, namely $x_1^* = x_2^*$.

4.2 Symmetric Structure World

If the both players' H_i function is the same function (their degree of risk averse was already set to be the same, i.e.$\eta_1 = \eta_2 = 0.2$), then the both players' V_i functions are as follows;

$$\bar{V_1}(c_1, X) = -\exp\{-\eta(c_1(10 - X)) - \mu\eta + \frac{\eta^2\sigma^2}{2}\} \tag{46}$$

$$\bar{V_2}(c_2, X) = -\exp\{-\eta(c_2(10 - X)) - \mu\eta + \frac{\eta^2\sigma^2}{2}\} \tag{47}$$

Furthermore, the initial allocation is even, namely,

$\theta_1 = 0.5$
$\theta_2 = 0.5$

Under the setting, the bargaining frontier collapses as in Figure 4. In this world, everything is symmetric. So the both players can start with the disagreement point(DP), moving along the bargaining frontier, now a line, and the two can reach the pareto frontier and realize the social efficiency.

(Figure 4 is about here.)

If the real world is symmetric as in figure 4, then the bargaining could lead to the social efficiency.

4.3 Impact of Uncertainty Increase

Back in the case of basic case, which is in Figure 3. If the uncertainty increases, the σ would increase. In Figure 5, we depict the world as σ increases, namely, $\sigma = 1, 2, 3, 4, 5, 6$. As the uncertainty, namely σ, increases, the world shrink to the down left corner.

(Figure 5 is about here.)

This means you have to settle for smaller level of utility. Under uncertainty increase settings, bargaining is almost surely more difficult than otherwise.

4.4 Asymmetric Risk Aversion

If the player 1's risk aversion level is more than that of the player 2, the player 1's utility function changes like in Figure 6.

(Figure 6 is about here.)

The utility function is getting more skewed into the top upper left as the risk aversion parameter η increases, namely $\eta = 0.2, 0.4, ..., 1.8, 2.0$.

Under these settings, the player 1 evaluate more in the upper side of income, namely higher x in Figure 6, to compensate the lower evaluation for lower side of income. This is the content of risk aversion. As a result, under our parameter setting the player 1 needs more than the player 2. So the world in Figure 3 skewed to the right to allocate more to the player 1 just as in Figure 7.

(Figure 7 is about here.)

All things equal, if the risk aversion of one of the player is more than the other, the player could ask for more in bargaining and this could lead to more difficult bargaining process since in the real bargaining process, you cannot see the other players' risk aversion parameter.

4.5 Initial Allocation Perturbation

To make the comparison easier, we go back to the even world in Figure 4. The initial allocation for each player is even in Figure 4, namely,

$\theta_1 = 0.5$
$\theta_2 = 0.5$

the disagreement point is on the bargaining frontier.

If the allocation is off from the even about 0.1 in our setting in Figure 4, the bargaining frontier dislocate from the original collapsed bargaining frontier (line) to the upper side or lower side depending on the plus or minus of the deviation from the original allocation as in Figure 8.

(Figure 8 is about here.)

In these cases, the disagreement point, which does not move, is out of bargaining frontier in either upper or lower case. Getting to the higher welfare level through bargaining starting from the disagreement point is not feasible, let alone attaining the social efficiency frontier.

5 Conclusion

We performed simulation analysis on the international climate change policy, especially on the effects on pricing of CO_2 emission allowances by including major players such as China and India into the hypothetical trading scheme in the aviation sector according to non-cooperative game theoretic framework.

In the presence of negative public goods, i.e., CO_2 emission into the earth atmosphere, we extended the Lindahl-Bowen-Samuelson condition so as to include, at least some class of the uncertainty about the utility. That is, under **Assumption 1** and **Assumption 2**, we introduce the CARA utility function and linearly separable uncertainty from consumption, production and their emission function.

If uncertainty increases, then both Pareto Frontier and Bargaining Frontier shrink and make the negotiation harder, since the players have to settle for less than before. If the risk preferences are different between the players, then the Pareto Frontier and Bargaining Frontier skews. So the simple allocation rule, like the same percentage reduction for different countries, could be difficult to be agreed upon. More over, under the condition of different risk aversion levels and utility structure, with un-even initial allocations of CO_2 allowances, it is shown that reaching the bargaining frontier, let alone pareto frontier by bargainings could be extremely difficult.

The next step is to explore the price effect of hypothetical global emission trading in later year than 1990 in this study. The marginal cost data must be investigated into the main counties in 2005 or other year, which could be potentially the base year in the international negotiations. The other step is to go into the other class of uncertainty than **Assumption 1**. In this study, the uncertainty is linearly separated from consumption, production or their emission functions. In reality the uncertainty could be attached to any of these aspects as well as the uncertainty linearly separable from them. But the mathematical requirement to treat the more realistic model enhances dramatically as more and more restrictions are being lessened.

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A Appendix Derivation of Proposition 1

As the first order condition of the Lagrangean (30) of **Problem B⋆**, we have the equations (31) $\sim$ (36).

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h \in N} x_h^*)}{\partial c_i} + \gamma_i g_i'(c_i^*) + \delta = 0 \quad (31)$$

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h \in N} x_h^*)}{\partial X} - \sum_{j \neq i} \lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h \in N} x_h^*)}{\partial X} - \gamma_i = 0 \quad (32)$$

$$\gamma_i f_i'(y_i^*) - \delta = 0 \quad (33)$$

and for each $j \neq i$,

$$-\lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h \in N} x_h^*)}{\partial c_j} + \gamma_j g_j'(c_j^*) + \delta = 0 \quad (34)$$

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h \in N} x_h^*)}{\partial X} - \sum_{j \neq i} \lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h \in N} x_h^*)}{\partial X} - \gamma_j = 0 \quad (35)$$

$$\gamma_j f_j'(y_j^*) - \delta = 0 \quad (36)$$

By the equations (32) and (35),

$\gamma_i = \gamma_j$.

So with this result and the equations (33) and (36),
$f_i'(y_i^*) = f_j'(y_j^*)$.

The **Problem B$\star$** can be formulated for j instead of i. So we get the symmetric first order conditions;

$$\frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial c_j} + \gamma_j g_j'(c_j^*) + \delta = 0 \tag{48}$$

$$\frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{i\neq j} \lambda_i \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_j = 0 \tag{49}$$

$$\gamma_i f_i'(y_i^*) - \delta = 0 \tag{50}$$

and for each $i \neq j$,

$$-\lambda_i \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial c_i} + \gamma_i g_i'(c_i^*) + \delta = 0 \tag{51}$$

$$\frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{i\neq j} \lambda_i \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_i = 0 \tag{52}$$

$$\gamma_i f_i'(y_i^*) - \delta = 0 \tag{53}$$

By the equations (31) and (51), and the equations (34) and (48),
$\lambda_i = \lambda_j = -1$.

From (33) and (50), $\delta = \gamma f_i'(y_i^*) = \gamma f_j'(y_j^*)$.

Hence with (31), we get the following;

$$\frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial c_i} = -\gamma g_i'(c_i^*) - \delta = -\gamma(f_i'(y_i^*) + g_i'(c_i^*))$$

Define $D_{c_i}\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)$;

$$D_{c_i}\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*) \equiv \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial c_i}$$

Then,

$$\frac{(f_i'(y_i^*) + g_i'(c_i^*))}{D_{c_i}\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)} = -\frac{1}{\gamma}$$

The equation (32) becomes;

$$\begin{aligned}
&\frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \sum_{j\neq i} \lambda_j \frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma_i \\
&\quad = \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} + \sum_{j\neq i} \frac{\partial \bar{V}_j(c_j^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma \\
&\quad = \sum_{i\in N} \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X} - \gamma = 0
\end{aligned}$$

Define $D_X\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)$;

$$D_X\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*) \equiv \frac{\partial \bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{\partial X}$$

So, we have the following;

$$\sum_{i\in N} D_X\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*) = \gamma$$

Therefore,

$$\sum_{i\in N}\left\{\frac{D_X\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}{D_{c_i}\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)}(f_i'(y_i^*) + g_i'(c_i^*))\right\}$$

$$= \frac{(f_i'(y_i^*) + g_i'(c_i^*))}{D_{c_i}\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)} \sum_{i\in N} D_X\bar{V}_i(c_i^*, \sum_{h\in N} x_h^*)$$

$$= -\frac{1}{\gamma}\gamma = -1$$

This means;

$$\sum_{i \in N} \nu_i \left(c_i^*, \sum_{h \in N} x_h^* \right) (f_i'(y_i^*) + g_i'(c_i^*)) = 1$$

$$\nu_i(c_i^*, X) = \left| \frac{\frac{\partial H_i(c_i^*, X)}{\partial X}}{\frac{\partial H_i(c_i^*, X)}{\partial c_i}} \right|.$$

This is the Extended Condition of LBS for Uncertainty.

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Table 1

Country or Area	Carbon Emission in90 (Mil. Ton)	GDP in 90 (Bil.US$)	Carbon Intensity
EU 15	915	6,961	0.13
FSU 22	989	1,535	0.64
Japan	292	2,970	0.10
US	1,315	5,794	0.23
China	662	484	1.37
Korea	66	264	0.25
India	186	327	0.57

China includes Hong Kong and Macau.

Korea excludes DPRK.

Carbon Intensity is the carbon emission level per GDP ($e_i = E_i / GDP_i$).

Table 2

Country	Reduction Rate	Reduction (Mil.ton)	Initial Permits (Mil. ton)	P* (US$)	Equilibrium Cost (Mil. US$)
EU 15	0.08	73	841	9.65	575
FSU 22	0	0	989	9.65	-402
Japan	0.06	18	275	9.65	138
US	0.07	92	1223	9.65	563

P^* is the equilibrium price of carbon emission allowance

Table 3

Countries	Reduction Rate	Reduction (Mil.ton)	Initial Permits (Mil. ton)	P* (US$)	Equilibrium Cost (Mil. US$)
EU 15	0.08	73	841	6.65	424
FSU 22	0	0	989	6.65	-192
Japan	0.06	18	275	6.65	102
US	0.07	92	1223	6.65	457
China	0	0	662	6.65	-143
Korea	0	0	66	6.65	-8
India	0	0	186	6.65	-35

China includes Hong Kong and Macau.

Korea excludes DPRK.

Carbon Intensity is the carbon emission level per GDP ($e_i = E_i / GDP_i$).

Figure 1

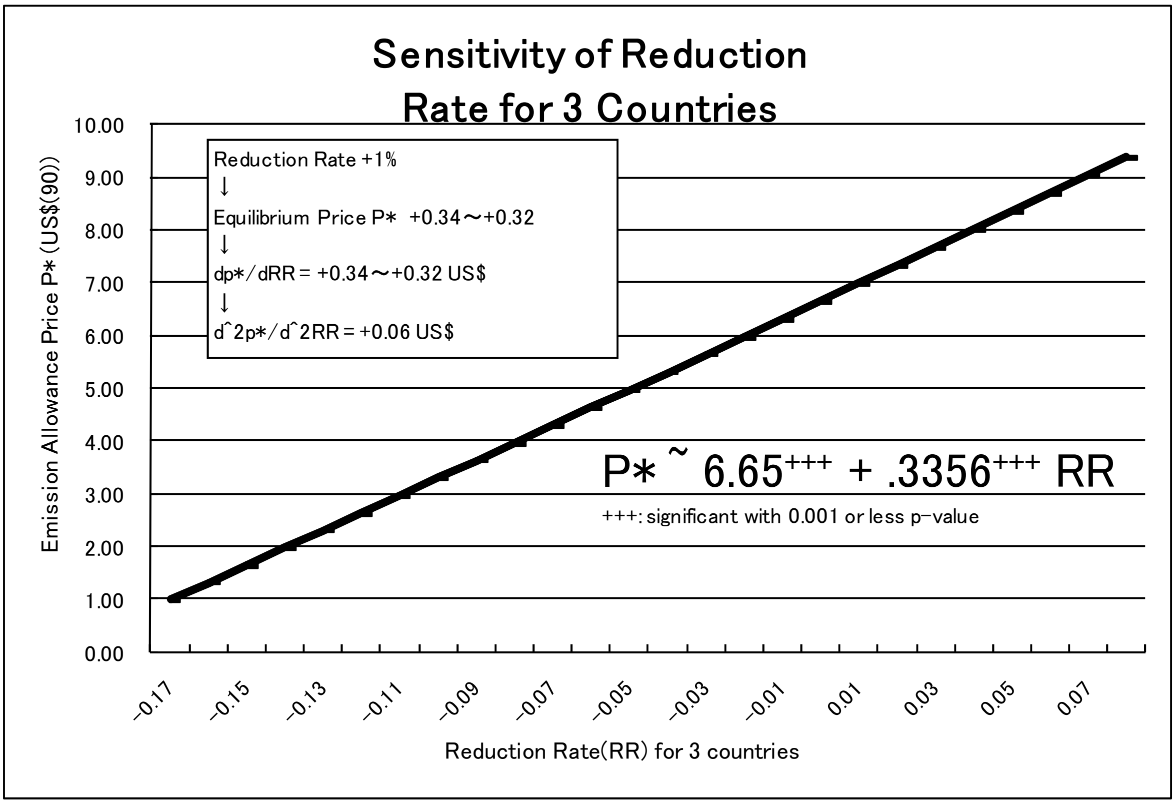

Figure 2

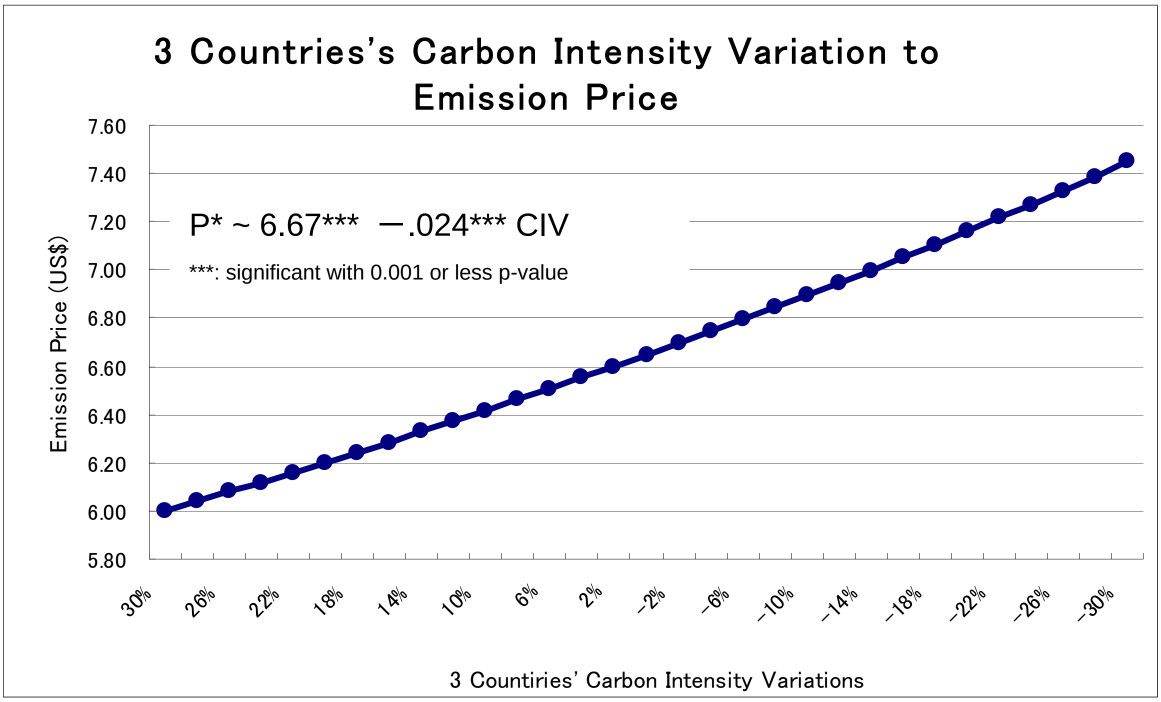

Figure 3

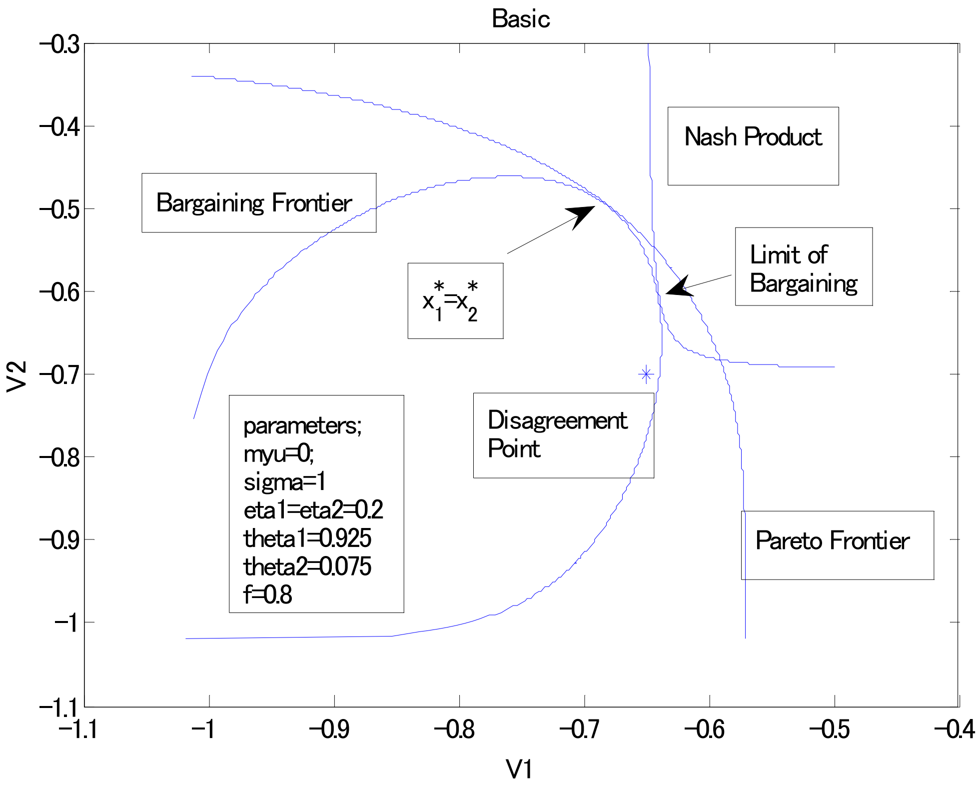

Figure4

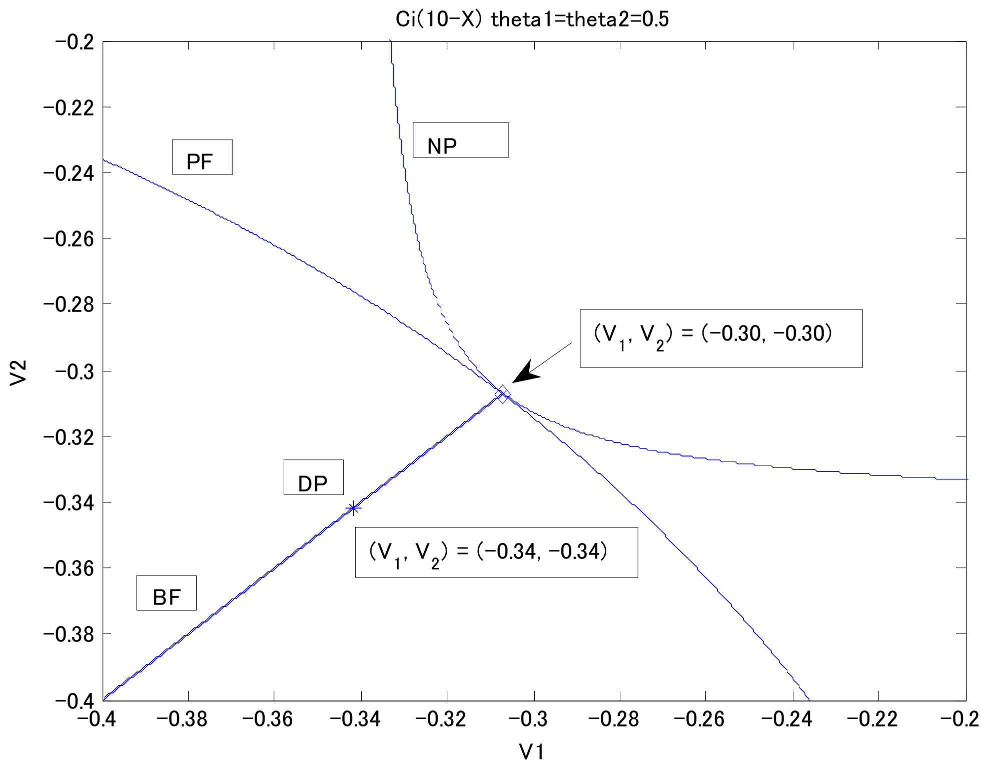

Figure 5

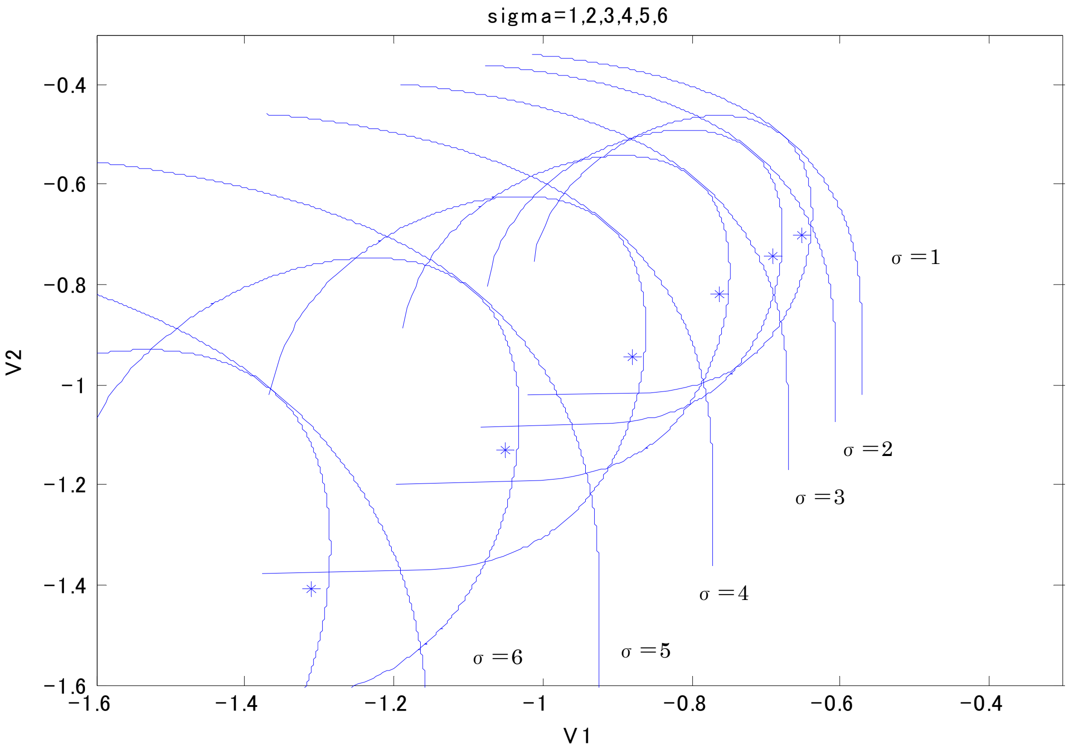

Figure 6

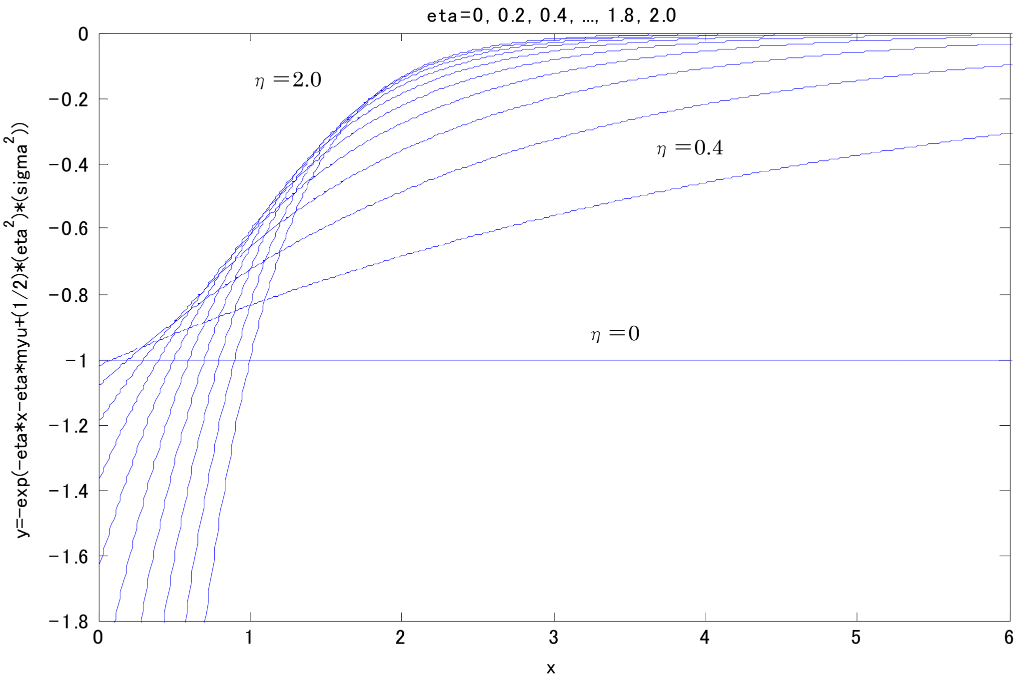

Figure 7

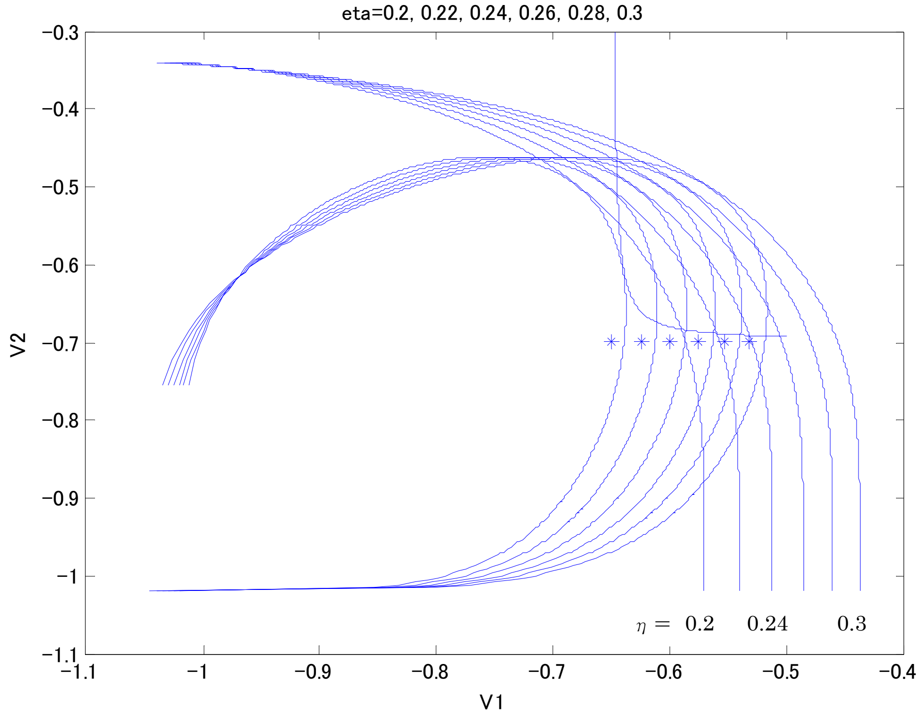

Figure 8