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A Negotiation Game Analysis of Airport and Airline Risk Sharing Contract

Katsuya Hihara*and Naoki Makimoto†

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Abstract

We analyze a linear risk sharing contract between airport and airline, called " Load Factor Guarantee Mechanism, " with the framework of generalized Nash bargaining solution for contracting phase and Cournot-type best response functions for effort-making game during the contract period. The slope of the linear contract, which contains the functions of risk sharing and incentive payment, is to be agreed upon so as to maximize the total welfare of both parties without any influence of negotiation power balance. The target load factor of the linear contract, which is a common target for efforts and a threshold for the payment, will be agreed upon in an equitable sharing manner based on the contract 's contribution to welfare gain and the bargaining power of each party in addition to the slope agreed upon as above[1].

key words

Airport-Airline Vertical Relationship, Risk Sharing, Incentive Design, Nash Bargaining Solution, Load Factor Guarantee Mechanism

JEL Classification Numbers:
C70,D81,D86,L93

1 Introduction

Airport and airline are both essential part of aviation and air traffic system to serve air travel passengers and cargo shippers. They could improve their service by jointly forming contractual relationship with good incentive design and risk sharing, and gain more revenues and profits.

In fact, under the recent volatile business environments, risk sharing mechanisms between airports and airlines were contracted to stabilize revenue fluctuation, to give incentives of enhancing efforts and improving service quality and to better serve the

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[1]Preliminary paper. Please do not quote with the written consent of the authors.

users of the routes at the airports[2]. Noto Airport Load Factor Guarantee Mechanism (LFGM) is one example of such contracts. Its payment structure is a simple linear function based on the realized state, not the effort. We can call such contract, a linear contract.

In this paper, we analyze such linear contracts from two objectives. How such contract is negotiated? What are the contents (slope and a reference point) of the negotiated linear contracts? We analyze a linear risk sharing contract between airports and airlines in two stages. For the first stage, we used the framework of generalized Nash bargaining game during a contract-negotiation process. For the second stage, we use the framework of Cournot-type best response function to effort-making game during the contract period.

Here is the map of this paper. First in section 2 we state the background and explain relevant literatures. In section 3 we explain our model in detail. In 4, we derive the equilibrium effort levels given the linear payoff contract. In section 5, based on the equilibrium effort levels, we show the contents of linear payoff contract both parties would agree upon by the Nash bargaining solution. In section 6, we explain the characteristics of the derived linear payoff contract and its implications to the real world business problem by using numerical examples. In the last section, 7, we state concluding remarks.

2 Literature Review

Because of the importance of airport and airline relationships in the air transport system, we have numerous studies from transport economics. For example, the recent study from the stand point of consumer welfare analyses is Barbot et al.(2011) and Oum and Fu (2009). They also studies from the view-points of market competition among many airport-airline vertical relationships and its effects on networking and pricing. Zhang et al. (2010) is a unique research about contractual revenue sharing between airport and airline, and its impacts on pricing and routes.

Also in the contract theory literatures, a lot of researchers studied the optimality of various contracts. Kim and Wang (1998) is a general analysis about the optimal contracts under double moral hazard settings. Linear contracts are very prevalent in the real world like the Noto LFGM contract, although it is piece-wise linear contract to be precise. Under moral hazard settings, analysis about optimality of linear contract include Holmstrom and Milgrom (1987) and Schattler and Sung (1993). Battachaiya (1995) is on optimality of linear contract for risk neutral agent and principal. The risk sharing research goes back to the pioneering work of Borch (1962).

Fukuyama et al. (2009) analyzed on another example of LFGM at Tottori Yonago

[2]Although the structures are not exactly the same with Noto case, we can find similar vertical contractual cases around the world. In US, for example, DOT granted money to local communities with airports to entice airlines ‘ operation into the region from 2002 by giving revenue guarantee or subsidy. In Europe, one example is that Charleroi airport, a secondary airport owned partially by Walloon regional government, made contract with Ryan Air to financially support the LCC ’s opening at the airport and to discount airport chargers among other things. As we describe later, with proper modification, we believe the structure and analyses on Noto case can be a foundation for other similar cases across the world, since the Noto case has wider scope and range of vertical relationship (coverage of upside as well as downside, direct risk sharing rather than simple subsidies or financial guarantees, for example)

airport at west side of Japan. They look into the parameter of linear contract with Nash bargaining solution. They do not analyze the function of risk sharing or incentive structure of the LFGM contract.

Hihara (2011) looked into the load factor guarantee mechanism as one of the examples of such risk sharing contracts with a double moral hazard model and showed linear payoff contracts can be the optimal contract. Also Hihara (2012) analyzed such mechanisms with incomplete contract framework and illustrated the effects of such contract on the parties ' effort/utility improvements.

Despite the vast amount of literature about airport and airline vertical relationship, such as Oum and Fu (2009), Zhang et al. (2010) and Barbot et al. (2011), however, negotiation process of such contracts between airports and airlines and the contents of such agreed contracts are, to our knowledge, neither modeled nor analyzed from the standpoints of risk sharing or incentive mechanism. Especially the linear contracts in which the payoff is a linear function of realized states are prevalent in real business environment. So the need to analyze such linear contracts is substantial.

Our analysis is not from the moral hazard standpoints. We focus more on the negotiation process and its results rather than the information asymmetry structure. Our model will deal with Nash bargaining settings with symmetric information structure. Also we will deal with the situation where both parties of a contract are risk averse. In double moral hazard settings like in Kim and Wang (1998), in most cases only one of the parties is risk averse and the other is risk neutral. Also our model looks into the detail of the risk sharing function and incentive structure of linear contract.

3 Settings

In this section, we set up a model to analyze the negotiation process and the efforts to be made by both parties during the contract period. Model consists of two stages. The first stage is a negotiation game on the contents of a linear contract of LFGM. The second stage is an effort making game of each party, which is a Cournot-type non-cooperative game, during the contract period after they signed the contract.

3.1 Model

Airport and airline can make effort to improve the load factor of the air transport route at the airport. Effort level e_p is that of airport and e_ℓ is that of airline. Herein after, the suffix p indicates airport and ℓ indicates airline. Definition range of e_p and e_ℓ is $\mathbb{R}_+ = [0, \infty)$. By making efforts e_p and e_ℓ, both incur necessary costs, which are indicated by functions $c_p(e_p)$ and $c_\ell(e_\ell)$. We assume that both $c_p(e_p)$ and $c_\ell(e_\ell)$ satisfy conditions of $c_p(0) = c_\ell(0) = 0$ and

$$c_k'(e_k) > 0, \quad k \in \{p, \ell\} \tag{1}$$

$$c_k''(e_k) > 0, \quad k \in \{p, \ell\} \tag{2}$$

$$\lim_{e_k \to \infty} c_k(e_k) = \infty, \quad k \in \{p, \ell\} \tag{3}$$

Random variable M, which indicate a load factor of the route, is assumed to be given by $M = \mu(e_p, e_\ell) + L$. Here $\mu(e_p, e_\ell)$ is a mean load factor that depends on the effort level of airport and airline. Random variable L indicates the spread of M. We assume $\mathrm{E}(L) = 0$ and $\mathrm{V}(L) = \sigma^2$. By these assumptions,

$$\mathrm{E}(M) = \mu(e_p, e_\ell), \qquad \mathrm{V}(M) = \sigma^2. \tag{4}$$

The risk in this study means the volatility of load factor M and is embodied in σ(standard deviation) above.

We assume mean load factor $\mu(e_p, e_\ell)$ satisfy the following.

$$\mu_k(e_p, e_\ell) > 0, \quad k \in \{p, \ell\}, \quad (e_p, e_\ell) \in \mathbb{R}_+^2 \tag{5}$$
$$\mu_{kk}(e_p, e_\ell) < 0, \quad k \in \{p, \ell\}, \quad (e_p, e_\ell) \in \mathbb{R}_+^2 \tag{6}$$
$$\mu_{p\ell}(e_p, e_\ell) = \mu_{\ell p}(e_p, e_\ell) < 0, \quad (e_p, e_\ell) \in \mathbb{R}_+^2 \tag{7}$$

Here we set the followings.

$$\mu_j(e_p, e_\ell) = \frac{\partial \mu(e_p, e_\ell)}{\partial e_j}, \qquad \mu_{jk}(e_p, e_\ell) = \frac{\partial^2 \mu(e_p, e_\ell)}{\partial e_k \partial e_j}, \qquad j, k \in \{p, \ell\}$$

(5) means that higher the effort levels, the higher the mean load factor. (6) means that the rate of increase of mean load factor decreases as effort levels increase. On the other hand, (7) means that the rate of increase of mean load factor decreases as the other party's effort t level increases. [3]. Furthermore, we assume $\underline{\mu} = \mu(0,0)$,$\overline{\mu} = \lim_{e_p, e_\ell \to \infty} \mu(e_p, e_\ell)$, $0 < \underline{\mu}$ and $\overline{\mu} < 1$. From (5), for any $e_p, e_\ell \geq 0$, we have $\underline{\mu} \leq \mu(e_p, e_\ell) \leq \overline{\mu}$.

Airport and airline have their independent profits based on the realized load factor. In this study, we assume the linear profit factions based on a realized load factor m as を $r_k(m) = \alpha_k m + \beta_k,\ k \in \{p, \ell\}$. β_k is a fixed profit level not dependent on load factor and α_ka parameter that shows the profit per unit load factor and that satisfies $\alpha_k > 0,\ k \in \{p, \ell\}$.

When parties agreed on a load factor guarantee mechanism contract, they have, in addition to the profit above, the payoff that is defined in the contract and a realized load factor. In this study, we assume the linear payoff function based on a realized load factor. This means that if the realized load factor is m, the payoff by the contract from airport to airline is given as follows[4];

$$q(m) = -\gamma(m - m_0). \tag{8}$$

If we set $m = m_0$, then we get $q(m_0) = 0$. This means $m = m_0$ is the target load factor at which the payoff by the contract is zero. The equation (8) shows the payoff function that is deprived of the upper and lower bound from the second year payoff function of the actual Noto airport load factor guarantee mechanism contract. In the real load factor guarantee mechanism contracts, including Noto case, it is likely that $\gamma > 0$. Since we think that the cost of efforts or profit functions have no difference between

[3](6) and (7) mean that as the amount of efforts increase, the rate of increase of average load factor decreases. The decreasing rate of increase of average load factor applies for both one's own efforts and the other's efforts. So we call such relationship as strategic substitution. In the Appendix A, we replace the condition of (7) with the following; $\mu_{p\ell}(e_p, e_\ell) = \mu_{\ell p}(e_p, e_\ell) = 0$, and try the same analysis.

[4]In the case of $q(m) > 0$, airport pays to airline $q(m)$, and on the other hand in the case of $q(m) < 0$, airline pays to airport $-q(m)$.

airport and airline, here we do not think there is a justification for the assumption $\gamma > 0$. So we do our analysis without narrowing the range of γ to the positive range, $\gamma > 0$. Notice that the case of $\gamma = 0$ means there is always no payment whatever the value of a realized load factor, and thus it corresponds to the situation where the parties do not have any contract. Hereinafter, we call (γ, m_0) a linear payoff contract with its slope being γ and target load factor being m_0.

From the above, the profits of airport and airline when their effort levels are (e_p, e_ℓ) are as follows;

$$W_p(e_p, e_\ell, \gamma, m_0) = r_p(M) - q(M) - c_p(e_p) = (\alpha_p + \gamma)M - c_p(e_p) - \gamma m_0 + \beta_p \quad (9)$$

$$W_\ell(e_p, e_\ell, \gamma, m_0) = r_\ell(M) + q(M) - c_\ell(e_\ell) = (\alpha_\ell - \gamma)M - c_\ell(e_\ell) + \gamma m_0 + \beta_\ell. \quad (10)$$

Load factor M is random variable and hence the profits are also depending on the realization of the random variable M. For airport and airline, the fluctuation of profit or revenue is thought to be a risk factor. In this study, we include the profit fluctuation risk is included in their utilities. So we use, as their utility functions, mean-variance utility, in which the utility function is given by the mean value of profit subtracted by the variance of the profit fluctuation with risk averse parameter for each party. [5] Thus the utility functions for airport and airline are given by the following;

$$F_p(e_p, e_\ell, \gamma, m_0) = \mathrm{E}\left(W_p(\gamma, m_0; e_p, e_\ell)\right) - \lambda_p \mathrm{V}\left(W_p(\gamma, m_0; e_p, e_\ell)\right) \quad (11)$$

$$F_\ell(e_p, e_\ell, \gamma, m_0) = \mathrm{E}\left(W_\ell(\gamma, m_0; e_p, e_\ell)\right) - \lambda_\ell \mathrm{V}\left(W_\ell(\gamma, m_0; e_p, e_\ell)\right) \quad (12)$$

Here $\lambda_k,\ k \in \{p, \ell\}$ indicates the parameter for risk aversion for airport and airline. In the equations (9) and (10), the term involving uncertainty is M only. So from (4), (11) and (12) become

$$F_p(e_p, e_\ell, \gamma, m_0) = (\alpha_p + \gamma)\mu(e_p, e_\ell) - c_p(e_p) + \beta_p - \gamma m_0 - \lambda_p(\alpha_p + \gamma)^2\sigma^2 \quad (13)$$

$$F_\ell(e_p, e_\ell, \gamma, m_0) = (\alpha_\ell - \gamma)\mu(e_p, e_\ell) - c_\ell(e_\ell) + \beta_\ell + \gamma m_0 - \lambda_\ell(\alpha_\ell - \gamma)^2\sigma^2. \quad (14)$$

3.2 Bargaining on Contracts and Equilibrium Effort Levels

In the real process about the LFGM contract, airport and airline agree a contract at the beginning of each year and both make each effort on their business in order to increase the load factor at the year end. Then at the year end, the payoff is calculated

[5] utility function that satisfies monotonic increase and risk aversion is called risk averse utility function. If utility function $U(\cdot)$ is differentiable, these two conditions are described as $U'(\cdot) > 0$ and $U''(\cdot) < 0$. When an individual who has risk averse utility function $U(\cdot)$ has uncertain profit that is described as random variable X and either one of the following two conditions is satisfied, the model that depends only on the expectation and the variance of X can completely describe the problem of maximizing the expected utility of that individual.

(1) utility function $U(\cdot)$ is a quadratic function over the possible range of the profit

(2) uncertain profit (random variable) X follows a normal distribution

Hihara (2008) studied the time series analysis of load factor in Japanese domestic air passenger market, and found the structure of ARIMA (1,1,4). Based on the model, a load factor of the following year can be predicted with normal distribution. So the condition (2) is satisfied. So in this study, we use mean-variance utility function. For more details, please see Konno (1995), for example.

based on the realized load factor and is paid according to the contract. When we apply these process to the model in section 3.1, then we get the following procedure.

(1) airport and airline bargain and agree on contract (γ, m_0)

(2) following the agreed contract(γ, m_0), airport and airline, in order to maximize each utility $F_p(e_p, e_\ell, \gamma, m_0)$ and $F_\ell(e_p, e_\ell, \gamma, m_0)$, decide and execute the effort levels

(3) profits of airport $F_p(e_p, e_\ell, \gamma, m_0)$ and of airline $F_p(e_p, e_\ell, \gamma, m_0)$ are calculated

At the step(2), we assume airport and airline both decide their effort levels according to Cournot type best response, in which one take the other's effort level as given. From these responses, we have their equilibrium effort levels (e_p, e_ℓ) as Nash equilibrium.

Please notice that in the real process, after the step(3), we still have the following steps,

(4) according to mean $\mu(e_p, e_\ell)$ and variance σ^2, a realized load factor m is specified

(5) based on contact (γ, m_0) and a realized load factor m, payoff $q(m)$ is paid (received).

In our model, the uncertainty of load factor is included in the risk averse utility functions. As for the bargaining on contract and decision on effort levels, we can only consider the process up to the step (3).

4 Equilibrium Effort Levels

In this section, we form the problem of deciding the effort levels of airport and airline e_p and e_ℓ, when the contract (γ, m_0) is given, according to non-cooperative game framework and analyze how the equilibrium of such effort levels forms. First, in the section 4.1, we derive the best response curves (functions) of airport and airline. Based on the results, in section 4.2, we derive the equilibrium.

About the utility functions of airport and airline in (13) and (14), we set the following;

$$G_p(e_p, e_\ell, \gamma) = (\alpha_p + \gamma)\mu(e_p, e_\ell) - c_p(e_p) \tag{15}$$
$$G_\ell(e_p, e_\ell, \gamma) = (\alpha_\ell - \gamma)\mu(e_p, e_\ell) - c_\ell(e_\ell). \tag{16}$$

Then we get,

$$F_p(e_p, e_\ell, \gamma, m_0) = G_p(e_p, e_\ell, \gamma) + \beta_p - \gamma m_0 - \lambda_p(\alpha_p + \gamma)^2\sigma^2 \tag{17}$$
$$F_\ell(e_p, e_\ell, \gamma, m_0) = G_\ell(e_p, e_\ell, \gamma) + \beta_\ell + \gamma m_0 - \lambda_\ell(\alpha_\ell - \gamma)^2\sigma^2. \tag{18}$$

In equations (17) and (18), what depends on effort levels e_p and e_ℓ is limited to $G_p(e_p, e_\ell, \gamma)$ and $G_\ell(e_p, e_\ell, \gamma)$. So the equilibrium is decided by considering only $G_p(e_p, e_\ell, \gamma)$ and $G_\ell(e_p, e_\ell, \gamma)$.

Before going into further analysis, here we define several terms and explain preliminary results. Hereinafter, we describe the partial derivatives of $G_p(e_p, e_\ell, \gamma)$ and $G_\ell(e_p, e_\ell, \gamma)$ with respect to e_p and e_ℓ as follows for simplicity.

$$G_{i:j}(e_p, e_\ell, \gamma) = \frac{\partial G_i}{\partial e_j}(e_p, e_\ell, \gamma), \qquad G_{i:jk}(e_p, e_\ell, \gamma) = \frac{\partial^2 G_i}{\partial e_k \partial e_j}(e_p, e_\ell, \gamma),$$

$$i,j,k \in \{p,\ell\}$$

First we define

$$\begin{aligned}\Gamma_p &= \frac{c_p'(0)}{\mu_p(0,0)} - \alpha_p \\ \Gamma_\ell &= \alpha_\ell - \frac{c_\ell'(0)}{\mu_\ell(0,0)}.\end{aligned}$$

Then we get

$$\gamma \le \Gamma_p \Longleftrightarrow G_{p:p}(0,0,\gamma) = (\alpha_p+\gamma)\mu_p(e_p,e_\ell) - c_p'(e_p) \le 0 \quad (19)$$

$$\gamma \ge \Gamma_\ell \Longleftrightarrow G_{\ell:\ell}(0,0,\gamma) = (\alpha_\ell-\gamma)\mu_\ell(e_p,e_\ell) - c_\ell'(e_\ell) \le 0. \quad (20)$$

Notice that from (1) and (5), we have $\Gamma_p > -\alpha_p$ and $\Gamma_\ell < \alpha_\ell$. The relationships explained in the following (21)∼(28) can be easily shown from the definitions of $G_p(e_p,e_\ell,\gamma)$ and $G_\ell(e_p,e_\ell,\gamma)$ and the assumptions stated in section 3.1.

$$G_{p:p}(e_p,e_\ell,\gamma) = (\alpha_p+\gamma)\mu_p(e_p,e_\ell) - c_p'(e_p) < 0, \quad \gamma \le -\alpha_p, \quad (e_p,e_\ell) \in \mathbb{R}_+^2 \quad (21)$$

$$G_{p:pp}(e_p,e_\ell,\gamma) = (\alpha_p+\gamma)\mu_{pp}(e_p,e_\ell) - c_p''(e_p) < 0, \quad \gamma \ge -\alpha_p, \quad (e_p,e_\ell) \in \mathbb{R}_+^2 \quad (22)$$

$$G_{p:p\ell}(e_p,e_\ell,\gamma) = (\alpha_p+\gamma)\mu_{p\ell}(e_p,e_\ell) < 0, \quad \gamma > -\alpha_p, \quad (e_p,e_\ell) \in \mathbb{R}_+^2 \quad (23)$$

$$G_{\ell:\ell}(e_p,e_\ell,\gamma) = (\alpha_\ell-\gamma)\mu_\ell(e_p,e_\ell) - c_\ell'(e_\ell) < 0, \quad \gamma \ge \alpha_\ell, \quad (e_p,e_\ell) \in \mathbb{R}_+^2 \quad (24)$$

$$G_{\ell:\ell\ell}(e_p,e_\ell,\gamma) = (\alpha_\ell-\gamma)\mu_{\ell\ell}(e_p,e_\ell) - c_\ell''(e_\ell) < 0, \quad \gamma \le \alpha_\ell, \quad (e_p,e_\ell) \in \mathbb{R}_+^2 \quad (25)$$

$$G_{\ell:\ell p}(e_p,e_\ell,\gamma) = (\alpha_\ell-\gamma)\mu_{\ell p}(e_p,e_\ell) < 0, \quad \gamma < \alpha_\ell, \quad (e_p,e_\ell) \in \mathbb{R}_+^2 \quad (26)$$

$$\lim_{e_p\to\infty} G_{p:p}(e_p,e_\ell,\gamma) < 0, \quad \gamma \in \mathbb{R}, \quad e_\ell \ge 0 \quad (27)$$

$$\lim_{e_\ell\to\infty} G_{\ell:\ell}(e_p,e_\ell,\gamma) < 0, \quad \gamma \in \mathbb{R}, \quad e_p \ge 0 \quad (28)$$

4.1 The Best Response Curves

We define the best response effort levels of airport when the effort levels of airline e_ℓ and γ are given and the best response effort levels of airline when the effort levels of airport e_p and γ are given as follows;

$$\begin{aligned}e_p^*(e_\ell,\gamma) &= \mathrm{argmax}_{e_p\ge 0} G_p(e_p,e_\ell,\gamma) \\ e_\ell^*(e_p,\gamma) &= \mathrm{argmax}_{e_\ell\ge 0} G_\ell(e_p,e_\ell,\gamma).\end{aligned}$$

Also the best response curves are defined as follows[6];

$$\mathcal{C}_p(\gamma) = \{(e_p^*(e_\ell,\gamma),e_\ell) \mid e_\ell \ge 0\} \quad (29)$$

$$\mathcal{C}_\ell(\gamma) = \{(e_p,e_\ell^*(e_p,\gamma)) \mid e_p \ge 0\}. \quad (30)$$

First the best response curve of airport $\mathcal{C}_p(\gamma)$ is divided into three parts depending on the range of γ.

[6]The description of the right hand side of (29) and (30) shows the set of points (in fact continuous loci) over the two-dimensional plane $\mathbb{R}_+^2$. For example, the curve $\mathcal{C}_p(\gamma)$ in (29) shows the loci of the point$(e_p^*(e_\ell,\gamma),e_\ell)$ over the plane $\mathbb{R}_+^2$ when e_ℓ moves over the range $e_\ell \ge 0$. The descriptions in lemma 4.1 and lemma4.2 in later part are also the same. This means { coordinates of points | the range of movement of e_p or e_ℓ }.

(1) the case of $\gamma \leq -\alpha_p$: from (21), for any $e_\ell \geq 0$, $e_p^*(e_\ell, \gamma) = 0$
(2) the case of $-\alpha_p < \gamma \leq \Gamma_p$: from (19) and (23), we have

$$G_{p:p}(0, e_\ell, \gamma) \leq G_{p:p}(0, 0, \gamma) \leq 0, \qquad e_\ell \geq 0. \tag{31}$$

Also from (22), we have $G_{p:p}(e_p, e_\ell, \gamma) \leq G_{p:p}(0, e_\ell, \gamma)$. With the results and (31), for any $(e_p, e_\ell) \in \mathbb{R}_+^2$, we get $G_{p:p}(e_p, e_\ell, \gamma) \leq 0$. So for any $e_\ell \geq 0$, we have $e_p^*(e_\ell, \gamma) = 0$.
(3) the case of $\gamma > \Gamma_p$: fist we check the change of $G_{p:p}(0, e_\ell, \gamma)$ when e_ℓ changes. Noticing $\Gamma_p > -\alpha_p$ and from (23), $G_{p:p}(0, e_\ell, \gamma)$ is a decreasing function with respect to e_ℓ. Also from (19), we have

$$G_{p:p}(0, 0, \gamma) = (\alpha_p + \gamma)\mu_p(0, 0) - c_p'(0) > 0.$$

When we have

$$\lim_{e_\ell \to \infty} G_{p:p}(0, e_\ell, \gamma) < 0, \tag{32}$$

from the explanation above, there exists unique $\pi_p(\gamma) > 0$ that satisfies

$$G_{p:p}(0, \pi_p(\gamma), \gamma) = (\alpha_p + \gamma)\mu_p(0, \pi_p(\gamma)) - c_p'(0) = 0 \tag{33}$$

and we have

$$G_{p:p}(0, e_\ell, \gamma) \begin{cases} > 0, & 0 \leq e_\ell < \pi_p(\gamma) \\ \leq 0, & e_\ell \geq \pi_p(\gamma). \end{cases} \tag{34}$$

On the other hand, when (32) is not satisfied, for any $e_\ell \geq 0$ we have $G_{p:p}(0, e_\ell, \gamma) > 0$. But by defining $\pi_p(\gamma) = \infty$, we can still derive the results of (34). In either case, for e_ℓ satisfying $0 \leq e_\ell < \pi_p(\gamma)$, we have $G_{p:p}(0, e_\ell, \gamma) > 0$. Noticing (22) and (27), there exists unique $K_p(e_\ell, \gamma) > 0$ that satisfies

$$G_{p:p}(K_p(e_\ell, \gamma), e_\ell, \gamma) = (\alpha_p + \gamma)\mu_p(K_p(e_\ell, \gamma), e_\ell) - c_p'(K_p(e_\ell, \gamma)) = 0 \tag{35}$$

and $G_p(e_p, e_\ell, \gamma)$ reaches its maximum at $e_p = K_p(e_\ell, \gamma)$. Therefore, for e_ℓ satisfying $0 \leq e_\ell < \pi_p(\gamma)$, we get $e_p^*(e_\ell, \gamma) = K_p(e_\ell, \gamma)$. In the case of $\pi_p(\gamma) < \infty$, for e_ℓ satisfying $e_\ell \geq \pi_p(\gamma)$, we have, from (22) and (34), $e_p^*(e_\ell, \gamma) = 0$.

In summarizing the explanation above, we have the best response curve of airport in the following lemma 4.1.

Lemma 4.1 The best response curve of airport $\mathcal{C}_p(\gamma)$ is given by the following;

$$\mathcal{C}_p(\gamma) = \begin{cases} \{(0, e_\ell) \mid e_\ell \geq 0\}, & \gamma \leq \Gamma_p \\ \{(K_p(e_\ell, \gamma), e_\ell) \mid 0 \leq e_\ell < \pi_p(\gamma)\} \cup \{(0, e_\ell) \mid e_\ell \geq \pi_p(\gamma)\}, & \gamma > \Gamma_p. \end{cases} \tag{36}$$

Next, we derive the best response curve of airline $\mathcal{C}_\ell(\gamma)$. Just as the case of airport, we have divide our analysis into three parts depending on the value of γ.
(1) the case of $\gamma \geq \alpha_\ell$: from (24), for any $e_p \geq 0$, we have $e_\ell^*(e_p, \gamma) = 0$.
(2) the case of $\Gamma_\ell \leq \gamma < \alpha_\ell$: from (20) and (26), we have

$$G_{\ell:\ell}(e_p, 0, \gamma) \leq G_{\ell:\ell}(0, 0, \gamma) \leq 0, \qquad e_p \geq 0. \tag{37}$$

Also from(25), it can be shown that $G_{\ell:\ell}(e_p, e_\ell, \gamma) \leq G_{\ell:\ell}(e_p, 0, \gamma)$. With the result and (37), for any $(e_p, e_\ell) \in \mathbb{R}_+^2$, we have $G_{\ell:\ell}(e_p, e_\ell, \gamma) \leq 0$. So for any $e_p \geq 0$, we get $e_\ell^*(e_p, \gamma) = 0$.

(3) the case of $\gamma < \Gamma_\ell$: fist we check the change of $G_{\ell:\ell}(e_p, 0, \gamma)$ when e_p changes. Noticing $\Gamma_\ell < \alpha_\ell$ and (26), it is shown that $G_{\ell:\ell}(e_p, 0, \gamma)$ is a decreasing function with respect to e_p. Also from (20), we have

$$G_{\ell:\ell}(0, 0, \gamma) = (\alpha_\ell - \gamma)\mu_\ell(0, 0) - c'_\ell(0) > 0.$$

If the following is satisfied,

$$\lim_{e_p \to \infty} G_{\ell:\ell}(e_p, 0, \gamma) < 0 \tag{38}$$

from the explanation above, there exists unique $\pi_\ell(\gamma) > 0$ that satisfies

$$G_{\ell:\ell}(\pi_\ell(\gamma), 0, \gamma) = (\alpha_\ell - \gamma)\mu_\ell(\pi_\ell(\gamma), 0) - c'_\ell(0) = 0 \tag{39}$$

and we have

$$G_{\ell:\ell}(e_p, 0, \gamma) \begin{cases} > 0, & 0 \le e_p < \pi_\ell(\gamma) \\ \le 0, & e_p \ge \pi_\ell(\gamma). \end{cases} \tag{40}$$

On the other hand if (38) does not hold, for any $e_p \ge 0$, we have $G_{\ell:\ell}(e_p, 0, \gamma) > 0$. In this case, however, if we define$\pi_\ell(\gamma) = \infty$, we still can derive (40). In either case, for e_p satisfying $0 \le e_p < \pi_\ell(\gamma)$, $G_{\ell:\ell}(e_p, 0, \gamma) > 0$. So noticing (25) and (28), we have unique $K_\ell(e_p, \gamma) > 0$ that satisfies

$$G_{\ell:\ell}(e_p, K_\ell(e_p, \gamma), \gamma) = (\alpha_\ell - \gamma)\mu_\ell(e_p, K_\ell(e_p, \gamma)) - c'_\ell(K_\ell(e_p, \gamma)) = 0 \tag{41}$$

and $G_\ell(e_p, e_\ell, \gamma)$ reaches its maximum at $e_\ell = K_\ell(e_p, \gamma)$. So for e_p satisfying $0 \le e_p < \pi_\ell(\gamma)$, we have $e^*_\ell(e_p, \gamma) = K_\ell(e_p, \gamma)$. If $\pi_\ell(\gamma) < \infty$, for e_p satisfying $e_p \ge \pi_\ell(\gamma)$, from (25) and (40), we have $e^*_\ell(e_p, \gamma) = 0$.

Summarizing the explanation above, we derive the best response curve of airline in the following lemma 4.2.

Lemma 4.2 The best response curve of airline $\mathcal{C}_\ell(\gamma)$ is given by the following;

$$\mathcal{C}_\ell(\gamma) = \begin{cases} \{(e_p, K_\ell(e_p, \gamma)) \mid 0 \le e_p < \pi_\ell(\gamma)\} \cup \{(e_p, 0) \mid e_p \ge \pi_\ell(\gamma)\}, & \gamma < \Gamma_\ell \\ \{(e_p, 0) \mid e_p \ge 0\}, & \gamma \ge \Gamma_\ell. \end{cases} \tag{42}$$

4.2 Derivation of Equilibrium Effort Levels

As stated above, airport and airline maximizing its utilities by taking the effort level of the other as given under the Cournot type non-cooperative game. So the equilibrium effort levels of airport and airline are given by the intersection point of the best response curves $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ derived in section 4.1 as Nash equilibrium.

Here we derive the equilibrium effort levels.

In preparation, we show the following lemma.

Lemma 4.3 In the case of $\Gamma_p < \Gamma_\ell$, for any $\gamma \in (\Gamma_p, \Gamma_\ell)$, there exists $\Gamma_1 \in (\Gamma_p, \Gamma_\ell)$ that satisfy

$$\pi_p(\gamma) \begin{cases} < K_\ell(0, \gamma), & \Gamma_p < \gamma < \Gamma_1 \\ \ge K_\ell(0, \gamma), & \Gamma_1 \le \gamma < \Gamma_\ell \end{cases} \tag{43}$$

and there exists $\Gamma_2 \in (\Gamma_p, \Gamma_\ell)$ that satisfy

$$\pi_\ell(\gamma) \begin{cases} \ge K_p(0, \gamma), & \Gamma_p < \gamma \le \Gamma_2 \\ < K_p(0, \gamma), & \Gamma_2 < \gamma < \Gamma_\ell. \end{cases} \tag{44}$$

(Proof) First in order show (43), we compare $\pi_p(\gamma)$ with $K_\ell(0,\gamma)$. From (23) and (31), we get $\lim_{e_\ell\to\infty} G_{p:p}(0,e_\ell,\Gamma_p) < G_{p:p}(0,0,\Gamma_p) = 0$. For γ that is close enough to Γ_p and satisfies $\gamma > \Gamma_p$, it is shown that (32) and $\pi_p(\gamma) < \infty$ hold. So if we do $\gamma \downarrow \Gamma_p$ in equation (33), we can confirm the following holds.

$$\lim_{\gamma\downarrow\Gamma_p} \pi_p(\gamma) = 0 \tag{45}$$

Also if we differentiate (33) with respect to γ and arrange the terms, we get

$$\pi_p'(\gamma) = -\frac{\mu_p(0,\pi_p(\gamma))}{(\alpha_p+\gamma)\mu_{p\ell}(0,\pi_p(\gamma))} > 0. \tag{46}$$

So we can check that for γ satisfying $\pi_p(\gamma) < \infty$ and $\gamma > \Gamma_p$, $\pi_p(\gamma)$ is a increasing function and that for larger γ we get $\pi_p(\gamma) = \infty$ [7]. On the other hand, if we set, at (41), $e_p = 0$ and $\gamma \uparrow \Gamma_\ell$, we can check that the following holds.

$$\lim_{\gamma\uparrow\Gamma_\ell} K_\ell(0,\gamma) = 0 \tag{47}$$

Also if we differentiate (41) with respect to γ and rearrange terms, we get

$$\begin{aligned}\frac{\partial K_\ell}{\partial\gamma}(e_p,\gamma) =& \frac{\mu_\ell(e_p,K_\ell(e_p,\gamma))}{(\alpha_\ell-\gamma)\mu_{\ell\ell}(e_p,K_\ell(e_p,\gamma)) - c_\ell''(K_\ell(e_p,\gamma))} < 0,\\ &\gamma < \Gamma_\ell, \quad 0 \le e_p < \pi_\ell(\gamma).\end{aligned} \tag{48}$$

So $K_\ell(0,\gamma)$ is a decreasing function of $\gamma < \Gamma_\ell$. From (45)$\sim$(48), it is shown that there exists $\Gamma_1 \in (\Gamma_p,\Gamma_\ell)$ that satisfies (43).

Next, in order to show (44), we compare $\pi_\ell(\gamma)$ with $K_p(0,\gamma)$. From (26) and (37), we get $\lim_{e_p\to\infty} G_{\ell:\ell}(e_p,0,\Gamma_\ell) < G_{\ell:\ell}(0,0,\Gamma_\ell) = 0$. So for γ close enough to Γ_ℓ and satisfying $\gamma < \Gamma_\ell$, (39) and $\pi_\ell(\gamma) < \infty$ hold. So if we do $\gamma \uparrow \Gamma_\ell$ in (39), we can check the following holds.

$$\lim_{\gamma\uparrow\Gamma_\ell} \pi_\ell(\gamma) = 0 \tag{49}$$

Also if we differentiate (39) with respect to γ and rearrange terms, we get

$$\pi_\ell'(\gamma) = \frac{\mu_\ell(\pi_\ell(\gamma),0)}{(\alpha_\ell-\gamma)\pi_{\ell p}(\pi_\ell(\gamma),0)} < 0. \tag{50}$$

So for γ satisfying $\pi_\ell(\gamma) < \infty$ and $\gamma < \Gamma_\ell$, $\pi_\ell(\gamma)$ is a decreasing function and for smaller γ, we have$\pi_\ell(\gamma) = \infty$ [8]. On the other hand, if we do $e_\ell = 0$、$\gamma \downarrow \Gamma_p$ in (35), we can confirm that the following holds.

$$\lim_{\gamma\downarrow\Gamma_p} K_p(0,\gamma) = 0 \tag{51}$$

If we differentiate (35) with respect to γ and rearrange terms, we get

$$\begin{aligned}\frac{\partial K_p}{\partial\gamma}(e_\ell,\gamma) =& \frac{\mu_p(K_p(e_\ell,\gamma),e_\ell)}{c_p''(K_p(e_\ell,\gamma)) - (\alpha_p+\gamma)\mu_{pp}(K_p(e_\ell,\gamma),e_\ell)} > 0,\\ &\gamma > \Gamma_p, \quad 0 \le e_\ell < \pi_p(\gamma).\end{aligned} \tag{52}$$

[7]We could have the case where for all $\gamma > \Gamma_p$ we always get $\pi_p(\gamma) < \infty$. Please see the derivation process of lemma 4.1.

[8]We could have the case where for all $\gamma < \Gamma_\ell$, we always get$\pi_\ell(\gamma) < \infty$. Please see the derivation process of lemma 4.2.

So $K_p(0,\gamma)$ is an increasing function of $\gamma > \Gamma_p$. From (49)∼(52), it is shown that there exists $\Gamma_2 \in (\Gamma_p, \Gamma_\ell)$ satisfying (44). □

Lemma 4.4 In the case of $\Gamma_p < \Gamma_\ell$, if Γ_1 and Γ_2 in lemma 4.3 satisfy $\Gamma_1 \leq \Gamma_2$, then, for any $\gamma \in [\Gamma_1, \Gamma_2]$, there exist $(e_p^\sharp, e_\ell^\sharp)$ that are in the ranges[9]

$$e_p^\sharp \leq \pi_\ell(\gamma), \qquad e_\ell^\sharp \leq \pi_p(\gamma) \tag{53}$$

and that satisfy the following.

$$e_p^\sharp = K_p(e_\ell^\sharp, \gamma) \tag{54}$$

$$e_\ell^\sharp = K_\ell(e_p^\sharp, \gamma) \tag{55}$$

(Proof) From $\gamma > \Gamma_p$, for e_ℓ satisfying $0 \leq e_\ell \leq \pi_p(\gamma)$, we can define $K_p(e_\ell, \gamma)$[10]. If we differentiate (35) with respect to e_ℓ and rearrange terms, we get

$$\frac{\partial K_p}{\partial e_\ell}(e_\ell, \gamma) = \frac{(\alpha_p + \gamma)\mu_{p\ell}(K_p(e_\ell, \gamma), e_\ell)}{c_p''(K_p(e_\ell, \gamma)) - (\alpha_p + \gamma)\mu_{pp}(K_p(e_\ell, \gamma), e_\ell)} < 0. \tag{56}$$

$\frac{\partial K_p}{\partial e_\ell}(e_\ell, \gamma)$ is a decreasing function of e_ℓ. Therefore there exists a unique inverse function of $K_p(e_\ell, \gamma)$ with respect to e_ℓ, namely $J(e_p, \gamma)$ that satisfy the following for given e_p.

$$K_p(J(e_p, \gamma), \gamma) = e_p \tag{57}$$

In the above, the range of $J_p(e_p, \gamma)$ is limited to the range that satisfies $K_p(\pi_p(\gamma), \gamma) \leq e_p \leq K_p(0, \gamma)$. From the definition, the following holds.

$$J(K_p(\pi_p(\gamma), \gamma), \gamma) = \pi_p(\gamma) \tag{58}$$

$$J(K_p(0, \gamma), \gamma) = 0 \tag{59}$$

If we differentiate (57) with respect to e_p and rearrange terms, along with (56), we get

$$\frac{\partial J}{\partial e_p}(e_p, \gamma) = \frac{1}{\frac{\partial K_p}{\partial e_p}(J(e_p, \gamma), \gamma)} < 0.$$

$J(e_p, \gamma)$ is a decreasing function of e_p. On the other hand, from $\gamma < \Gamma_\ell$, for e_ℓ satisfying $0 \leq e_p \leq \pi_\ell(\gamma)$, we can define $K_\ell(e_p, \gamma)$ [11]. If we differentiate (41) with respect to e_p and rearrange terms, we get

$$\frac{\partial K_\ell}{\partial e_p}(e_p, \gamma) = \frac{(\alpha_\ell - \gamma)\mu_{\ell p}(e_p, K_\ell(e_p, \gamma))}{c_\ell''(K_\ell(e_p, \gamma)) - (\alpha_\ell - \gamma)\mu_{\ell\ell}(e_p, K_\ell(e_p, \gamma))} < 0. \tag{60}$$

Noticing $K_p(\pi_p(\gamma), \gamma) \geq 0$, we have the following.

$$K_\ell(0, \gamma) \geq K_\ell(K_p(\pi_p(\gamma), \gamma), \gamma) \tag{61}$$

[9]The ranges include the case of $\pi_p(\gamma) = \infty$ or $\pi_\ell(\gamma) = \infty$.

[10]In the case of $\pi_p(\gamma) = \infty$, if we define $K_p(\pi_p(\gamma), \gamma) = \lim_{e_\ell \uparrow \pi_p(\gamma)} K_p(e_\ell, \gamma)$, then from (56), this limit always exists.

[11]In the case of $\pi_\ell(\gamma) = \infty$, we define $K_\ell(\pi_\ell(\gamma), \gamma) = \lim_{e_p \uparrow \pi_\ell(\gamma)} K_\ell(e_p, \gamma)$. From (60), this limit always exists.

From lemma 4.3, for $\gamma \in [\Gamma_1, \Gamma_2]$, we have the following.

$$K_\ell(0,\gamma) \le \pi_p(\gamma) \tag{62}$$
$$K_p(0,\gamma) \le \pi_\ell(\gamma) \tag{63}$$

From (58), (61) and (62), the following holds.

$$K_\ell(K_p(\pi_p(\gamma),\gamma),\gamma) \le K_\ell(0,\gamma) \le \pi_p(\gamma) = J(K_p(\pi_p(\gamma),\gamma),\gamma) \tag{64}$$

Also from (63) $K_\ell(e_p,\gamma)$ can be defined for $0 \le e_p \le K_p(0,\gamma)$. Along with (59), we have

$$K_\ell(K_p(0,\gamma),\gamma) \ge 0 = J(K_p(0,\gamma),\gamma). \tag{65}$$

(64) and (65) show that the relative value ranking of $K_\ell(e_p,\gamma)$ and $J(e_p,\gamma)$ change when e_p increases from $K_p(\pi_p(\gamma),\gamma)$ to $K_p(0,\gamma)$. So there exists $e_p^\sharp \in [K_p(\pi_p(\gamma),\gamma), K_p(0,\gamma)]$ that satisfies

$$K_\ell(e_p^\sharp,\gamma) = J(e_p^\sharp,\gamma). \tag{66}$$

From (64), we have $e_p^\sharp \le \pi_\ell(\gamma)$. If we define $e_\ell^\sharp$, from (55), $e_\ell^\sharp = K_\ell(e_p^\sharp,\gamma)$, then from (57) and (66) (54) holds. From (61) and (62), we have $e_\ell^\sharp \le \pi_p(\gamma)$. Therefore we have the contents of the lemma. □

With the explanation above, the equilibrium effort levels (e_p^*, e_ℓ^*) can be derived as follows.

Proposition 4.5 (1) In the case of $\Gamma_\ell \le \Gamma_p$: we get the unique equilibrium effort levels as follows.

$$\begin{aligned}
(e_p^*, e_\ell^*) &= (0, K_\ell(0,\gamma)), \quad && \gamma < \Gamma_\ell \\
(e_p^*, e_\ell^*) &= (0, 0), \quad && \Gamma_\ell \le \gamma \le \Gamma_p \\
(e_p^*, e_\ell^*) &= (K_p(0,\gamma), 0), \quad && \gamma > \Gamma_p
\end{aligned}$$

(2) In the case of $\Gamma_\ell > \Gamma_p$: if, for any $\gamma \in (\Gamma_p, \Gamma_\ell)$, $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ are crossing at one point [12], then there exists a unique equilibrium effort level. It is given as follows.

$$\begin{aligned}
(e_p^*, e_\ell^*) &= (0, K_\ell(0,\gamma)), \quad && \gamma < \Gamma_1 \\
(e_p^*, e_\ell^*) &= (e_p^\sharp, e_\ell^\sharp), \quad && \Gamma_1 \le \gamma \le \Gamma_2 \\
(e_p^*, e_\ell^*) &= (K_p(0,\gamma), 0), \quad && \gamma > \Gamma_2
\end{aligned}$$

Here Γ_1, Γ_2 are given in lemma 4.3 and $e_p^\sharp, e_\ell^\sharp$ are given in lemma 4.4.

(Proof) (1) the case of $\Gamma_\ell \le \Gamma_p$: From (36) and (42), for $\gamma < \Gamma_\ell$, we have $(0, K_\ell(0,\gamma)) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. So $(0, K_\ell(0,\gamma))$ is the equilibrium. Similarly for $\Gamma_\ell \le \gamma \le \Gamma_p$, we have $(0,0) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. For $\gamma > \Gamma_p$, we have $(K_p(0,\gamma), 0) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. So these

[12] In general, there are possibilities of cases where $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ have multiple crossing points and hence there exist multiple equilibria of effort levels. In that case, we need to deal with the problem of considering which equilibrium can be realized. In this paper, in order to avoid such complication and concentrate more on the wider setting of our analysis, which are to look into the negotiation of contract in addition to the equilibrium effort levels during the contract period, we proceed our analysis under the condition in which, for $\gamma \in (\Gamma_p, \Gamma_\ell)$, $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ make cross at one point.

are the equilibrium. From (36) and (42), we can check that each of these equilibria is unique.
(2) the case of $\Gamma_\ell > \Gamma_p$: From (36) and (42), for $\gamma \leq \Gamma_p$, we have $(0, K_\ell(0,\gamma)) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. For $\gamma \geq \Gamma_\ell$, we have $(K_p(0,\gamma), 0) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. So they are equilibrium.

Hereinafter, we concentrate only on the case of $\Gamma_p < \gamma < \Gamma_\ell$. From lemma 4.3, there exist Γ_1, Γ_2. For any $\gamma \in (\Gamma_p, \Gamma_\ell)$, $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ make cross at one point, we get $\Gamma_1 \leq \Gamma_2$. This can be checked by the following. If $\Gamma_2 < \Gamma_1$, for γ satisfying $\Gamma_2 < \gamma < \Gamma_1$, we get the following from (43) and (44).

$$0 < \pi_p(\gamma) < K_\ell(0,\gamma), \qquad 0 < \pi_\ell(\gamma) < K_p(0,\gamma)$$

Here, $(K_p(0,\gamma), 0)$ and $(0, K_\ell(0,\gamma))$ belong to $\mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. This is a contradiction of single crossing point assumption above. So it must be that $\Gamma_p < \Gamma_1 \leq \Gamma_2 < \Gamma_\ell$. From 4.4, there exists $(e_p^\sharp, e_\ell^\sharp)$.

(2-a) the case of $\Gamma_p \leq \gamma < \Gamma_1$, from (43) $(e_p^*, e_\ell^*) = (0, K_\ell(0,\gamma)) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. So $(0, K_\ell(0,\gamma))$ is the equilibrium.

(2-b) the case of $\Gamma_1 \leq \gamma \leq \Gamma_2$, there exists $(e_p^\sharp, e_\ell^\sharp)$ in lemma 4.4 and it belongs, from (53), to $\mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$ and is the equilibrium.

(2-c) the case of $\Gamma_2 < \gamma \leq \Gamma_\ell$, from (44) we get $(e_p^*, e_\ell^*) = (K_p(0,\gamma), 0) \in \mathcal{C}_p(\gamma) \cap \mathcal{C}_\ell(\gamma)$. So $(K_p(0,\gamma), 0)$ is the equilibrium.

If, for $\gamma \in (\Gamma_p, \Gamma_\ell)$, $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ make cross at one point, then in either case of (2-a)∼(2-c), the equilibrium is unique. Therefore, the proposition 4.5 is proven. □

5 Negotiation Game Analysis of Contract

In the section 4, we derived the equilibrium effort level when the contract (γ, m_0) is given. As stated in subsection 3.2, the real process is that parties first agree on a contract and based on the contract they choose the effort levels. In our setting of analysis, they make negotiation on the concrete contents of the contract while they are considering the equilibrium effort levels during the contract period.

In this section, based on the results in subsection 4, we analyze how the concrete contents of a contract are agreed upon by the negotiation game framework.

Here we consider a problem of deciding the contents of contract (γ, m_0) by the negotiation between airport and airline. When the slope γ of the payoff function (8) satisfies $\gamma = 0$, the payoff is always zero whatever the value of target load factor m_0. So the case of $\gamma = 0$ is equivalent to no contract case. Using this structure, we make the setting that the situation where they cannot agree on a contract by negotiation means the situation where they choose the contract of $(0, \cdot)$ [13]. We make $(0, \cdot)$ as a breaking point and the negotiation between airport and airline is assumed to be solved by the generalized Nash bargaining solution.

We define the utility functions of airport and airline when they choose the equilibrium effort levels (e_p^*, e_ℓ^*) as derived in subsection 4 when the contract (γ, m_0) is given.

$$F_p(\gamma, m_0) = G_p(\gamma) + H_p(\gamma) - \gamma m_0 \qquad (67)$$
$$F_\ell(\gamma, m_0) = G_\ell(\gamma) + H_\ell(\gamma) + \gamma m_0 \qquad (68)$$

[13] Hereinafter we state the no-contract case as $(0, \cdot)$ since $\gamma = 0$ means no payoff whatever the value of m_0.

Here, we set[14].

$$\begin{aligned} G_p(\gamma) &= (\alpha_p + \gamma)\mu(e_p^*, e_\ell^*) - c_p(e_p^*) &(69)\\ G_\ell(\gamma) &= (\alpha_\ell - \gamma)\mu(e_p^*, e_\ell^*) - c_\ell(e_\ell^*) &(70)\\ H_p(\gamma) &= \beta_p - \lambda_p \sigma^2 (\alpha_p + \gamma)^2\\ H_\ell(\gamma) &= \beta_\ell - \lambda_\ell \sigma^2 (\alpha_\ell - \gamma)^2. \end{aligned}$$

Also we set

$$F(\gamma) = F_p(\gamma, m_0) + F_\ell(\gamma, m_0). \quad (71)$$

Here notice that $G_k(\gamma), H_k(\gamma), F(\gamma)$ do not depend on m_0. Airport and airline can get at least the utility levels of $F_p(0,\cdot)$ and $F_\ell(0,\cdot)$ corresponding to the case of $(0,\cdot)$. So the contract that can be agreed upon satisfies the condition;

$$F_k(\gamma, m_0) > F_k(0,\cdot), \qquad k \in \{p, \ell\}. \quad (72)$$

The range of contract indicated by (72) is set by
$\Omega = \{(\gamma, m_0) \in \mathbb{R}^2 \mid F_k(\gamma, m_0) > F_k(0,\cdot),\ k \in \{p, \ell\}\}$.
The negotiation power balance between the negotiating parties is thought to be very important in reality. So we set the negotiation power of airport as $\tau \in (0,1)$according to the general negotiation game framework. Next we set the generalized Nash bargaining solution when the breaking points of airport and airline are $F_p(0,\cdot)$ and $F_\ell(0,\cdot)$. First by defining the generalized Nash product as follows.

$$L(\gamma, m_0) = \{F_p(\gamma, m_0) - F_p(0,\cdot)\}^\tau \{F_\ell(\gamma, m_0) - F_\ell(0,\cdot)\}^{1-\tau}$$

Then the generalized Nash bargaining solution is set by

$$(\gamma^*, m_0^*) = \mathrm{argmax}_{(\gamma, m_0)\in\Omega} L(\gamma, m_0). \quad (73)$$

Also we define the following.

$$\gamma^\sharp = \mathrm{argmax}_{\gamma\in\mathbb{R}} F(\gamma) \quad (74)$$

Then we get the following proposition.

Proposition 5.1 In the case of $\gamma^\sharp \neq 0$, the generalized Nash bargaining solution (γ^*, m_0^*) in negotiation game between airport and airline to decide the contract is given by

$$\begin{aligned} \gamma^* &= \gamma^\sharp\\ m_0^* &= \frac{1}{\gamma^\sharp}[(1-\tau)\{G_p(\gamma^\sharp) + H_p(\gamma^\sharp) - G_p(0) - H_p(0)\}\\ &\qquad -\tau\{G_\ell(\gamma^\sharp) + H_\ell(\gamma^\sharp) - G_\ell(0) - H_\ell(0)\}]. \quad (75) \end{aligned}$$

On the other hand, in case of $\gamma^\sharp = 0$, $(0,\cdot)$ is the generalized Nash bargaining solution. There is no other generalized Nash bargaining solution other than the solutions above.

[14]The term $F_k, G_k,\ k \in \{p, \ell\}$ is used in (15)∼(18) in subsection 4 as $F_k(e_p, e_\ell, \gamma, m_0)$ and $G_k(e_p, e_\ell, \gamma)$. The functions of (67)∼(70) are derived when we put $(e_p, e_\ell) = (e_p^*, e_\ell^*)$ into (15)∼(18). Hereinafter we change the contents of these functions and keep using the same notation of such functions.

(Proof) First we set

$$\Delta F_k(\gamma) = G_k(\gamma) + H_k(\gamma) - G_k(0) - H_k(0), \qquad k \in \{p, \ell\}. \tag{76}$$

Then we can describe $L(\gamma, m_0)$ as follows.

$$L(\gamma, m_0) = \{\Delta F_p(\gamma) - \gamma m_0\}^\tau \{\Delta F_\ell(\gamma) + \gamma m_0\}^{1-\tau} \tag{77}$$

We can check the following holds.

$$\Omega = \{(\gamma, m_0) \mid -\Delta F_\ell(\gamma) < \gamma m_0 < \Delta F_p(\gamma)\} \tag{78}$$

From (77), $L(\gamma, m_0)$ is upper 凸 with respect to m_0 in Ω. The first order condition of optimality leads to the optimal target load factor m_0^* that satisfies

$$\begin{aligned} \frac{\partial L}{\partial m_0}(\gamma, m_0^*) &= \gamma\{\Delta F_p(\gamma) - \gamma m_0^*\}^{\tau-1}\{\Delta F_\ell(\gamma) + \gamma m_0^*\}^{-\tau}\{(1-\tau)\Delta F_p(\gamma) - \tau\Delta F_\ell(\gamma) - \gamma m_0^*\} \\ &= 0. \end{aligned}$$

Considering (76) and from (78), we get the following.

$$\gamma m_0^* = (1-\tau)\Delta F_p(\gamma) - \tau \Delta F_\ell(\gamma) \tag{79}$$

Notice that from $\tau \in (0,1)$, in the case of $\gamma \neq 0$, we get $(\gamma, m_0^*) \in \Omega$. Putting (79) into (77) and rearranging terms, we get the following.

$$L(\gamma, m_0^*) = \tau^\tau(1-\tau)^{1-\tau}\{\Delta F_p(\gamma) + \Delta F_\ell(\gamma)\} = \tau^\tau(1-\tau)^{1-\tau}\{F(\gamma) - F(0)\} \tag{80}$$

Therefore it is shown that $\gamma^* = \gamma^\sharp$. From (79), in the case of $\gamma^\sharp \neq 0$, m_0^* is given by (75). In the case of $\gamma^\sharp = 0$, for any m_0, we get $L(0, m_0) = 0$. So $(0, \cdot)$ is the generalized Nash bargaining solution. The γ that maximizes (80) is limited to $\gamma = \gamma^\sharp$. So it is shown that there is no other generalized Nash bargaining solution. □

From the characteristic of Nash bargaining solution, the contract (γ^*, m_0^*) as Nash bargaining solution gives pareto optimality to the parties. In our model, for any m_0, (γ^*, m_0) gives the pareto optimality.

Corollary 5.2 (1) For any $m_0 \in \mathbb{R}$, contract (γ^*, m_0) gives pareto optimality.
(2) In the case of $\gamma^* \neq 0$, for any $\gamma \neq \gamma^*$ and for any $m_0 \in \mathbb{R}$, contract (γ, m_0) does not give pareto optimality.

(Proof) (1) From the definition of γ^*, for any contract (γ', m_0') that is other than (γ^*, m_0), the following holds.

$$F_p(\gamma', m_0') + F_\ell(\gamma', m_0') = F(\gamma') \leq F(\gamma^*) = F_p(\gamma^*, m_0) + F_\ell(\gamma^*, m_0)$$

Therefore, $F_p(\gamma', m_0') > F_p(\gamma^*, m_0)$ and $F_\ell(\gamma', m_0') > F_\ell(\gamma^*, m_0)$ do not hold at the same time.
(2) In the case of $\gamma \neq \gamma^*$, we get $F(\gamma) < F(\gamma^*)$. From (67) and (68), we get the following.

$$G_p(\gamma) + H_p(\gamma) - \gamma m_0 + G_\ell(\gamma) + H_\ell(\gamma) + \gamma m_0 < G_p(\gamma^*) + H_p(\gamma^*) + G_\ell(\gamma^*) + H_\ell(\gamma^*)$$

So we have

$$G_\ell(\gamma)+H_\ell(\gamma)+\gamma m_0-\{G_\ell(\gamma^*)+H_\ell(\gamma^*)\} < G_p(\gamma^*)+H_p(\gamma^*)-\{G_p(\gamma)+H_p(\gamma)-\gamma m_0.\}$$

If $\gamma^* \neq 0$, we can choose m_0' that satisfies

$$\begin{aligned} G_\ell(\gamma)+H_\ell(\gamma)+\gamma m_0-\{G_\ell(\gamma^*)+H_\ell(\gamma^*)\} &< \gamma^* m_0' \\ &< G_p(\gamma^*)+H_p(\gamma^*)-\{G_p(\gamma)+H_p(\gamma)-\gamma m_0\} \end{aligned} \tag{81}$$

From (81), the followings hold.

$$F_p(\gamma, m_0) = G_p(\gamma)+H_p(\gamma)-\gamma m_0 < G_p(\gamma^*)+H_p(\gamma^*)-\gamma^* m_0' = F_p(\gamma^*, m_0') \tag{82}$$

$$F_\ell(\gamma, m_0) = G_\ell(\gamma)+H_\ell(\gamma)-\gamma m_0 < G_\ell(\gamma^*)+H_\ell(\gamma^*)-\gamma^* m_0' = F_\ell(\gamma^*, m_0') \tag{83}$$

Therefore, (γ_0, m_0) does not give the pareto optimality. □

Corollary 5.3 γ^* in (73) exists in the range of $-\alpha_p \leq \gamma^* \leq \alpha_\ell$.

(Proof) From (67), (68) and (71), if we define as follows;

$$\begin{aligned} G(\gamma) &= (\alpha_p+\alpha_\ell)\mu(e_p^*, e_\ell^*) - c_p(e_p^*) - c_\ell(e_\ell^*) \\ H(\gamma) &= -(\lambda_p+\lambda_\ell)\sigma^2\left(\gamma - \frac{\lambda_\ell\alpha_\ell-\lambda_p\alpha_p}{\lambda_p+\lambda_\ell}\right)^2 + \frac{(\lambda_\ell\alpha_\ell-\lambda_p\alpha_p)^2}{\lambda_p+\lambda_\ell}\sigma^2 \\ &\quad - \lambda_p\alpha_p^2\sigma^2 - \lambda_\ell\alpha_\ell^2\sigma^2 + \beta_p + \beta_\ell \end{aligned}$$

then we have $F(\gamma) = G(\gamma)+H(\gamma)$. With the assumptions $\lambda_p, \lambda_\ell > 0$, we get

$$-\alpha_p < \frac{\lambda_\ell\alpha_\ell-\lambda_p\alpha_p}{\lambda_p+\lambda_\ell} < \alpha_\ell.$$

This means that $H(\gamma)$ monotonically increases over the range of $\gamma < -\alpha_p$, and monotonically decreases over the range of $\gamma > \alpha_\ell$.

Next we check the increase/decrease of $G(\gamma)$. First in the case of $\Gamma_\ell \leq \Gamma_p$ and $\gamma < \Gamma_\ell$, or in the case of $\Gamma_\ell > \Gamma_p$ and $\gamma \leq \Gamma_1$, from the proposition 4.5, the equilibrium is given by $(e_p^*, e_\ell^*) = (0, K_\ell(0,\gamma))$. From (41), we get

$$(\alpha_\ell - \gamma)\mu_\ell(0, K_\ell(0,\gamma)) - c_\ell'(K_\ell(0,\gamma)) = 0.$$

Noticing (48), we have the following.

$$G'(\gamma) = \frac{\partial K_\ell}{\partial\gamma}(0,\gamma)\left\{(\alpha_p+\alpha_\ell)\mu_\ell(0, K_\ell(0,\gamma)) - c_\ell'(K_\ell(0,\gamma))\right\} > 0, \qquad \gamma < -\alpha_p$$

Therefore for $\gamma < -\alpha_p$, $G(\gamma)$ increases monotonically.

Finally we analyze in the case of $\Gamma_\ell \leq \Gamma_p$ and $\gamma > \alpha_\ell$, or in the case of $\Gamma_\ell > \Gamma_p$ and $\gamma \geq \Gamma_2$. In these cases, the equilibrium is given by $(e_p^*, e_\ell^*) = (K_p(0,\gamma), 0)$. From (35), we have

$$(\alpha_p + \gamma)\mu_p(K_p(0,\gamma), 0) - c_p'(K_p(0,\gamma)) = 0.$$

Noticing (52), we have the following.

$$G'(\gamma) = \frac{\partial K_p}{\partial\gamma}(0,\gamma)\left\{(\alpha_p+\alpha_\ell)\mu_p(K_p(0,\gamma), 0) - c_p'(K_p(0,\gamma))\right\} < 0, \qquad \gamma > \alpha_\ell$$

So for $\gamma > \alpha_\ell$, $G(\gamma)$ decreases monotonically.

Therefore $F(\gamma)$ increase monotonically for $\gamma < -\alpha_p$ and decreases monotonically for $\gamma > \alpha_\ell$. Hence the proposition is proven. □

6 Analysis on Contract

In our study we focus on the linear payoff function as indicated in (8) as contract structure. Parameters to be decided about the contract are two, namely slope γ^* and target load factorm_0^* of the linear payoff function.

Proposition 4.5 states that when contract is decided by negotiation game, first from (74), slope γ^* is agreed and then target load factor is from (75), agreed. The slope γ^* 9is agreed upon so as to maximize the combined utility levels of airport and airline, and it does not depend on the negotiation power balance τ. Also from the explanation in subsection 4, the equilibrium effort levels of airport and airline depend only on the agreed slope γ^* and not on the agreed load factor m_0^*. Therefore γ^* has the function to incentivize parties' effort levels so as to maximize the combined utility levels of both parties.

On the other hand, the target load factor m_0^* in (75) is agreed upon in a manner to distribute the increase of the utility level by agreeing a contract in accordance with the negotiation power balance τ. The agreed load factor depends on negotiation power balance. In this sense, the agreed target load factor m_0^* has the function of distributing the maximized combined utility levels by agreeing the slope γ^* in a balanced manner between airport and airline. This situation is easily confirmed by seeing that in corollary 5.2, the contract with $\gamma = \gamma^*$ gives the pareto optimality regardless of the value of m_0.

The negotiation game in general involves the situation where the interests of two parties are in conflict with each other. The result of negotiation game does not always guarantees the maximization of the combined utility levels of the parties. In our model, γ^* maximizes the combined utility levels. In this sense, the linear payoff function structure is playing an important role. In our linear payoff function model, the fact that the terms of $-\gamma m_0$ and γm_0 that are included in the utility functions of airport and airline are cancelled out in the combined utility levels enables the separation of γ and m_0 and the maximization of the combined utility levels by γ only. Also this is possible by the fact that our utility functions are the mean-variance utility functions. Without either one of the two characteristics, linear payoff function or the mean-variance utility function, the clear solution of γ^* in (74) is not possible.

At corollary 5.3, we show $-\alpha_p \leq \gamma^* \leq \alpha_\ell$. If we have a contract of $\gamma > \alpha_\ell$, $\alpha_\ell - \gamma < 0$ in (70). So airline does not have any incentive to make effort to increase the mean load factor. On the other hand, if we have a contract of $\gamma < -\alpha_p$, airport does not have an incentive to make effort. In such two cases, only one of the two ends up in making efforts. We think this is why γ^* is in the range of corollary 5.3.

Although the range for $\gamma^\sharp$ in (74) is shown in corollary 5.3, we cannot explicitly derive it in more general model. In order to look into the concrete characteristics, we use numerical examples by picking up some concrete models in the following.

First, with $\overline{\mu} > \underline{\mu} > 0$ and $\delta, \xi > 0$, $\mu(e_p, e_\ell)$ is given as concrete functions as follows.

$$\mu(e_p, e_\ell) = \overline{\mu} - (\overline{\mu} - \underline{\mu})e^{-\delta e_p - \xi e_\ell} = \overline{\mu} - \Delta\mu e^{-\delta e_p - \xi e_\ell}; \qquad \Delta\mu = \overline{\mu} - \underline{\mu} > 0$$

Also with $\kappa, \eta, \zeta, \nu > 0$, $c_p(e_p)$ and $c_\ell(e_\ell)$ is set as concrete functions as follows.

$$\begin{aligned} c_p(e_p) &= \kappa(e^{\eta e_p} - 1) \\ c_\ell(e_\ell) &= \zeta(e^{\nu e_\ell} - 1) \end{aligned}$$

In this case, we can easily check that (1)∼(7) hold. If we calculate the concrete example according to the explanation in subsection 4, we get the followings.

$$
\begin{aligned}
\Gamma_p &= \frac{\kappa\eta}{\Delta\mu\delta} - \alpha_p \\
\Gamma_\ell &= \alpha_\ell - \frac{\zeta\nu}{\Delta\mu\xi} \\
\pi_p(\gamma) &= \frac{1}{\xi}\ln\frac{(\alpha_p+\gamma)\Delta\mu\delta}{\kappa\eta}, \qquad \gamma \geq \Gamma_p \\
K_p(e_\ell,\gamma) &= \frac{1}{\delta+\eta}\left\{-\xi e_\ell + \ln\frac{(\alpha_p+\gamma)\Delta\mu\delta}{\kappa\eta}\right\}, \qquad \gamma \geq \Gamma_p, \quad 0 \leq e_\ell \leq \pi_p(\gamma) \\
\pi_\ell(\gamma) &= \frac{1}{\delta}\ln\frac{(\alpha_\ell-\gamma)\Delta\mu\xi}{\zeta\nu}, \qquad \gamma \leq \Gamma_\ell \\
K_\ell(e_p,\gamma) &= \frac{1}{\xi+\nu}\left\{-\delta e_p + \ln\frac{(\alpha_\ell-\gamma)\Delta\mu\xi}{\zeta\nu}\right\}, \qquad \gamma \leq \Gamma_\ell, \quad 0 \leq e_p \leq \pi_\ell(\gamma)
\end{aligned}
$$

Also in the case of $\Gamma_\ell > \Gamma_p$, Γ_1 and Γ_2 in lemma 4.3 can be calculated as unique solution that satisfies

$$
\begin{aligned}
\left\{\frac{\Delta\mu\delta}{\kappa\eta}(\alpha_p+\Gamma_1)\right\}^{1/\xi} &= \left\{\frac{\Delta\mu\xi}{\zeta\nu}(\alpha_\ell-\Gamma_1)\right\}^{1/(\xi+\nu)}, \qquad \Gamma_1 \in (\Gamma_p,\Gamma_\ell) \\
\left\{\frac{\Delta\mu\xi}{\zeta\nu}(\alpha_\ell-\Gamma_2)\right\}^{1/\delta} &= \left\{\frac{\Delta\mu\delta}{\kappa\eta}(\alpha_p+\Gamma_2)\right\}^{1/(\delta+\eta)}, \qquad \Gamma_2 \in (\Gamma_p,\Gamma_\ell).
\end{aligned}
$$

Further, since $K_p(e_\ell,\gamma)$ and $K_\ell(e_p,\gamma)$ are both linear functions of e_ℓ or e_p, we can compare the slope of the two linear functions. For any $\gamma \in (\Gamma_p,\Gamma_\ell)$, it is shown that $\mathcal{C}_p(\gamma)$ and $\mathcal{C}_\ell(\gamma)$ make cross at one point only.

The solutions for (54) and (55) are calculated as

$$
\begin{aligned}
e_p^\sharp &= \frac{1}{\eta\nu+\eta\xi+\delta\nu}\left\{(\nu+\xi)\ln\frac{(\alpha_p+\gamma)\Delta\mu\delta}{\kappa\eta} - \xi\ln\frac{(\alpha_\ell-\gamma)\Delta\mu\xi}{\zeta\nu}\right\} \\
e_\ell^\sharp &= \frac{1}{\eta\nu+\eta\xi+\delta\nu}\left\{-\delta\ln\frac{(\alpha_p+\gamma)\Delta\mu\delta}{\kappa\eta} + (\delta+\eta)\ln\frac{(\alpha_\ell-\gamma)\Delta\mu\xi}{\zeta\nu}.\right\}
\end{aligned}
$$

Then we have the equilibrium effort levels by putting these results into proposition 5.1.

Now we further put concrete numbers into the examples of functions above. Based on the real revenue scale and fare range, we set the parameters for base case as follows.

$$
\begin{array}{llll}
\underline{\mu} = 0.4, & \overline{\mu} = 0.85, & \delta = 1, & \xi = 1 \\
\kappa = 0.189, & \eta = 0.1, & \zeta = 0.189, & \nu = 0.1 \\
\alpha_p = 1.0, & \alpha_\ell = 1.4, & \beta_p = 0, & \beta_\ell = 0 \\
\lambda_p = 0.25, & \lambda_\ell = 0.25, & \sigma = 0.04, & \tau = 0.35
\end{array} \tag{84}
$$

The concrete contract calculated from parameters in (84) gives us slope of linear payoff function $\gamma^* = 0.1997$ and target load factor $m_0^* = 0.6480$. If we calculate the concrete contract by changing only one parameter in (84) at a time without changing the other parameters, we get the results of contracts and mean load factors as in the table 1.

In general, we can observe from the table 1 that one parameter change brings not large change into contracts. This tendencies is more significant in the cases of slope

Table 1: Impact of Parameter Value Change on Contract

Parameter Value Change	γ^*	m_0^*	$\mu(e_p^*, e_\ell^*)$
Base Case	0.1997	0.6480	0.8315
$\sigma = 0.08$	0.1997	0.6452	0.8315
$\tau = 0.7$	0.1997	0.6364	0.8315
$\alpha_p = 1.05$	0.1747	0.6519	0.8319
$\lambda_\ell = 0.275$	0.2007	0.6480	0.8315
$\zeta = 0.1984$	0.1997	0.6433	0.8311

γ^* of linear payoff functions. At equilibrium, mean load factor is around 83%. This is much higher than the mean load factor $\mu(0,0) = \underline{\mu} = 40\%$ without any effort. On average in these cases, payoff is made from airline to airport.

Next we see the situation when the values of parameters are changed. When the negotiation power of airport doubles ($\tau = 0.7$), slope is unchanged [15], and target load factor decreases. The contract becomes more favorable to airport in the case, since the payment target to airport is lowered. When the index parameter α_p of airport's independent profit increases by 5% ($\alpha_p = 1.05$), slope decreases but target load factor increases. The decrease of slope makes the unit of payment from airport to airline decrease. The productivity of airport increases because of the parameter change. So this could mean that the decreased slope slows down the payment from more productive airport, thus makes incentives to encourage more efforts of airport relatively, and changes the equilibrium so as to maximize the combined utility levels of the two parties.

These kinds of observations are based on some specific models and some concrete parameter values. In order to look into the more general characteristics of contracts with linear payoff functions, further varieties of models and more detailed numerical analyses will be needed.

7 Concluding Remarks

In this paper, we analyze the contract with linear payoff function (a simplified version of Noto airport LFGM contract) with the models of two-stage games. At the first stage, airport and airline negotiate the contents of a contract with linear payoff functions. At the second stage, we set they play efforts-making games under the Cournot-type non-cooperative game settings.

The results show that the agreed (optimal) slope of the linear payoff function does not depend on negotiation power balance and it is agreed so as to maximize the combined utility levels of the two players. This contract brings them the pareto optimality. Also it is shown that the agreed (optimal) load factor is decided based on the utility level increase by the contract and negotiation power balance between the contract parties in addition to the agreed slope.

In the contract of linear payoff functions, the slope has the function of parameter that converts the state = realized load factor into payment. In this sense, slope is

[15] From proposition 5.1, γ^* does not depend on τ.

indirectly rewarding the effort through the state = realized load factor and it has the incentives to encourage efforts of the contract parties.

Also the payoff of the contract has the function to stabilize the profit fluctuation. So the contract has the function of risk allocation/sharing with such mitigating payment to profit fluctuating risk. Slope of such linear payoff contract sets the scale of payment. As already stated, slope is agreed upon so as t maximize the combined utility levels.

The target load factor of such linear payoff risk sharing contract sets the common target for the two parties, and it has function to decide the direction of reward payment (incentive to encourage effort) and risk mitigating payment. The generalized Nash bargaining solution brings the agreed load factor that allocates the utility increase by the contract in a "balanced" way along with the negotiation power balance and the agreed slope. By "balanced," we mean that the allocation by the agreed load factor does not tolerate one-side victory for either party.

Our study utilizes the good maneuverability of linear payoff structure and mean-variance utility and derives the analytically clear solution of contract from two-stage game between two risk averse players. We think these results contribute to the more clear understanding of risk allocation function and incentive structure design of risk sharing linear contract between airport and airline. Also our analysis of risk sharing mechanism and incentive structure of such contracts gave useful guidance to airports and airlines who might consider making linear risk sharing contracts and policy-makers who would like to evaluate such risk sharing contracts from the standpoints of their effects on entire aviation networks.

On the other hand, we have the remaining points for further research. The analytical solution of upper/lower bounds of piece-wise linear payoff function in the actual Not airport LFGM contract, and analysis of the zero payment special range around target load factor are the example of such points. Also analysis of airfare and route structure with these risk sharing contracts between airport and airline is another important area for future research from the public policy standpoints.

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Appendix：
The Case of $\mu_{p\ell}(e_p, e_\ell) = \mu_{\ell p}(e_p, e_\ell) = 0$

In the main part of the study, we derived the equilibrium under the assumption of

$$\mu_{p\ell}(e_p, e_\ell) = \mu_{\ell p}(e_p, e_\ell) < 0,$$

which is indicated by (7). The assumption by (7) mean that as the level of efforts increases, the rate of increase of average load factor decreases. This assumption seems realistic. On the other hand, there may be a case such that the rate of increase does not depend on the other's effort levels. This means;

$$\mu_{p\ell}(e_p, e_\ell) = \mu_{\ell p}(e_p, e_\ell) = 0 \tag{85}$$

And such situation might be closer to the real situation. Therefore in this appendix, we replace (7) with (85) as our assumption and derive the equilibrium. In the following, we assume all the assumptions in the section 3 hold except for (7). We use the same notations in the section of 4 unless we specifically indicate. The analysis follows the same process as in the section of 4 and the section of 5. First we derive the best response functions (curves) by airport and airline, and then we calculate the equilibrium effort levels as the crossing of the two best response functions (curves).

By $\mu_{p\ell}(e_p, e_\ell) = \mu_{\ell p}(e_p, e_\ell) = 0$, (23) and (26) are modified as

$$G_{p:p\ell}(e_p, e_\ell, \gamma) = 0, \qquad \gamma \in \mathbb{R}, \quad (e_p, e_\ell) \in \mathbb{R}^2_+ \tag{86}$$

$$G_{\ell:\ell p}(e_p, e_\ell, \gamma) = 0, \qquad \gamma \in \mathbb{R}, \quad (e_p, e_\ell) \in \mathbb{R}^2_+. \tag{87}$$

From (86), $G_{p:p}(e_p, e_\ell, \gamma)$ does not depend on e_ℓ. So in the following, we use the notation of $G_{p:p}(e_p, \cdot, \gamma)$. Similarly, from (87), $G_{\ell:\ell}(e_p, e_\ell, \gamma)$ does not depend on e_p. So we use the notation of $G_{\ell:\ell}(\cdot, e_\ell, \gamma)$.

First, we derive three cases for the best response curves of airport $\mathcal{C}_p(\gamma)$ depending on the ranges of γ.

(1) the case of $\gamma \le -\alpha_p$: from (21), for any $e_\ell \ge 0$, we have $e_p^*(e_\ell, \gamma) = 0$.

(2) the case of $-\alpha_p < \gamma \le \Gamma_p$: from (19) and (22), for any $e_p \ge 0$, $G_{p:p}(e_p, \cdot, \gamma) \le G_{p:p}(0, \cdot, \gamma) \le 0$. So we have $e_p^*(e_\ell, \gamma) = 0$.

(3) the case of $\gamma > \Gamma_p$: noticing $\Gamma_p > -\alpha_p$ and from (19), we have

$$G_{p:p}(0, \cdot, \gamma) = (\alpha_p + \gamma)\mu_p(0, \cdot) - c_p'(0) > 0. \tag{88}$$

Therefore, noticing (22) and (27), we have unique $e_p^\flat > 0$ that satisfies

$$G_{p:p}(e_p^\flat, \cdot, \gamma) = (\alpha_p + \gamma)\mu_p(e_p^\flat, \cdot) - c_p'(e_p^\flat) = 0. \tag{89}$$

$G_p(e_p, e_\ell, \gamma)$ is at maximum at $e_p = e_p^\flat$. Therefore, for any $e_\ell \ge 0$, we have $e_p^*(e_\ell, \gamma) = e_p^\flat$.

Lemma 7.1 The best response curve of airport $\mathcal{C}_p(\gamma)$ is given by the following;

$$\mathcal{C}_p(\gamma) = \begin{cases} \{(0, e_\ell) \mid e_\ell \ge 0\}, & \gamma \le \Gamma_p \\ \{(e_p^\flat, e_\ell) \mid e_\ell \ge 0\}, & \gamma > \Gamma_p. \end{cases} \tag{90}$$

Next, we derive the best response curve of airline $\mathcal{C}_\ell(\gamma)$.

(1) the case of $\gamma \ge \alpha_\ell$: from (24), for any $e_p \ge 0$, we have $e_\ell^*(e_p, \gamma) = 0$.

(2) the case of $\Gamma_\ell \le \gamma < \alpha_\ell$: from (20) and (25), for any $e_\ell \ge 0$, we have $G_{\ell:\ell}(\cdot, e_\ell, \gamma) \le G_{\ell:\ell}(\cdot, 0, \gamma) \le 0$. Therefore $e_\ell^*(e_p, \gamma) = 0$.

(3) the case of $\gamma < \Gamma_\ell$: noticing $\Gamma_\ell < \alpha_\ell$ and from (20),

$$G_{\ell:\ell}(\cdot, 0, \gamma) = (\alpha_\ell - \gamma)\mu_\ell(\cdot, 0) - c_\ell'(0) > 0. \tag{91}$$

So noticing (25) and (28), we have unique $e_\ell^\flat > 0$ that satisfies the following;

$$G_{\ell:\ell}(\cdot, e_\ell^\flat, \gamma) = (\alpha_\ell - \gamma)\mu_\ell(\cdot, e_\ell^\flat) - c_\ell'(e_\ell^\flat) = 0. \tag{92}$$

$G_\ell(e_p, e_\ell, \gamma)$ is at its maximum at は $e_\ell = e_\ell^\flat$. Therefore for any $e_p \ge 0$, we have $e_\ell^*(e_p, \gamma) = e_\ell^\flat$.

Lemma 7.2 The best response curve of airline $\mathcal{C}_\ell(\gamma)$ is give by the following;

$$\mathcal{C}_\ell(\gamma) = \begin{cases} \{(e_p, e_\ell^\flat) \mid e_p \geq 0\}, & \gamma < \Gamma_\ell \\ \{(e_p, 0) \mid e_p \geq 0\}, & \gamma \geq \Gamma_\ell. \end{cases} \tag{93}$$

With the argument above, we have the equilibrium as follows;

Proposition 7.3 (1) the case of $\Gamma_\ell \leq \Gamma_p$: the equilibrium is unique and is given as;

$$(e_p^*, e_\ell^*) = (0, K_\ell(0,\gamma)), \qquad \gamma < \Gamma_\ell \tag{94}$$

$$(e_p^*, e_\ell^*) = (0, 0), \qquad \Gamma_\ell \leq \gamma \leq \Gamma_p \tag{95}$$

$$(e_p^*, e_\ell^*) = (K_p(0,\gamma), 0), \qquad \gamma > \Gamma_p. \tag{96}$$

(2) the case of $\Gamma_\ell > \Gamma_p$: the equilibrium is unique and is given as;

$$(e_p^*, e_\ell^*) = (0, K_\ell(0,\gamma)), \qquad \gamma < \Gamma_p \tag{97}$$

$$(e_p^*, e_\ell^*) = (e_p^\flat, e_\ell^\flat), \qquad \Gamma_p \leq \gamma \leq \Gamma_\ell \tag{98}$$

$$(e_p^*, e_\ell^*) = (K_p(0,\gamma), 0), \qquad \gamma > \Gamma_\ell. \tag{99}$$

(proof) (1) Depending on the range of γ, proof is given by lemma 7.1 and lemma 7.2. (2) In the cases of $\gamma < \Gamma_p$ and $\gamma > \Gamma_\ell$, proof is given by lemma 7.1.

In the case of $\Gamma_p \leq \gamma \leq \Gamma_\ell$, on the two dimension plane of (e_p, e_ℓ), $\mathcal{C}_p(\gamma)$ is a perpendicular line $e_p = e_p^\flat > 0$ and $\mathcal{C}_\ell(\gamma)$ is a horizontal line $e_\ell = e_\ell^\flat > 0$. So $\mathcal{C}_p(\gamma)$ and $C_\ell(\gamma)$ cross each other only at $(e_p, e_\ell) = (e_p^\flat, e_\ell^\flat)$. Therefore, the equilibrium is unique and it is $(e_p^\flat, e_\ell^\flat)$. □

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